FLARE STARS IN THE PLEIADES

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We have collected data on 45 new flare stars in the Pleiades, discovered mainly during the observational season 1968-1969 at the Tonantzintla, Asiago, Byurakan, Budapest, and Alma-Ata Observatories (Table 1). Together with the 100 flare stars in the previous lists of the Tonantzintla Observatory the total number of flare stars discovered in the region of the Pleiades has now reached 146. One of them (H II 2411) belongs to the Hyades. Of the remaining 145 stars, 123 have shown one flare, 16 have shown two flares, and 6 more than two flares.

A special analysis of flare stars has been carried out and it was found that the total number of flare stars in the Pleiades should be greater than 600. The distribution of flare stars can be satisfactorily represented by the sum of two Poisson distributions with different mean frequencies.

Almost all the members of the Pleiades with $V \geq 13.3$ are flare stars. A sharp boundary between photographically observed flare stars and nonflaring stars occurs at $V = 13.29$. The mean frequency of large flares with amplitudes in excess of $0^m.6$ is of the order of $4 \cdot 10^{-4} h^{-1}$ for the majority of the stars.

The total mass of the Pleiades is greater than the dynamic mass determined from the virial theoren (400 $M_\odot$). The difference is due to an outer envelope consisting of stars of lower luminosity (mostly flare stars).

1. **Introduction.** Phenomena connected with the origin and development of stars and stellar systems have attracted considerable attention during the last decade [1].

The evolution of stars is usually discussed theoretically [2]. Analyses of this kind are often based on models of the internal structure, and attempts are made to calculate the model parameters as functions of time. The models themselves are based on the hypothesis of the thermonuclear origin of the energy emitted by the stars. Without denying the great value of the various researches carried out in this area, we must recall that astrophysics is, above all, an observational science. One is therefore justified in demanding that the evolutionary features should be determined mainly through generalization and detailed analysis of observational data. Theoretical ideas can, of course, play an auxiliary role, but it is desirable to minimize the number of hypotheses used in this process of generalization and analysis.

In fact, the first steps in this direction have already been taken in astrophysics. Consider the problem of the group origin of stars [3,4]. This problem has not even been formulated by theoreticians. Moreover, it has been regarded as almost self-evident that, as a rule, stars are formed independently of each other. It was only after observational data led to the discovery of stellar associations that it was possible to establish that stars originated in groups. In precisely the same way, the existence of T associations has led to the conclusion [3,4] that newly formed stars enter the main sequence at different points. Further studies of these associations have led to a number of further conclusions with regard to the early phases of stellar evolution. An example of observations of major importance for the problem of stellar origin is the discovery of the Haro-Herbig objects [5] and surprising changes that occur in them. However, these changes have so far remained outside the scope of modern theories.

In this paper, we analyze certain data on flare stars in the Pleiades. Professor Haro [6] was the first to appreciate and estimate the major importance of flare stars in the problem of stellar evolution. He showed that data on flare stars in clusters and associations suggested that the earliest evolutionary stage, i.e., the stage of RW Aurigae (or T Tauri), was followed by another stage during which one of the most important characteristics of the star was its ability to undergo large-amplitude flares from time to time (up to the fifth or even seventh magnitude in the ultraviolet).

Having understood the importance of these stars, Haro et al. [7] continued to investigate them in clusters and associations, thus providing an important basis for more accurate analysis of the various facts relating to them. Most of the conclusions reported by Haro et al. were subsequently confirmed by Rosino et al. [8].

Systematic observations of flare stars in stellar aggregates have also been developing in recent years at the Byruakin Observatory [9–12]. In the following sections we try to deduce some concrete results from existing data, largely those referring to the Pleiades. Our discussion will include data on flares observed in the Pleiades during the 1968–1969 season. More complete data obtained as a result of the Byurakan observations, and the corresponding photographs, will be published elsewhere.

§2. **Statistical analysis of observational data on flare stars in associations and clusters.** Observations have shown that at least some stars in stellar aggregates exhibit flares from time to time, but no flare periodicity has been observed for each flare star. On the contrary, there are indications that flares are highly irregular in time. This suggests that the time distribution of flares is similar to the distribution of random events and must therefore satisfy a law similar to the Poisson distribution. The fact is, however, that the validity of the Poisson distribution is very difficult to verify for any particular star because this would require observations on at least a few hundred successive flares. This is, of course, a very difficult problem and it is only for the star HII 2411 which is projected onto the Pleiades that 48 flares have been observed [13].

Table 1

New Flare Stars

No	Star (HII)	α (1900)	δ (1900)	m_{pg}	Δm_{pg}	Date of Flare (1968–1969)	Telescope	Reference
1	2	3	4	5	6	7	8	9
101		$3^h33^m.2$	24°25′	17.8	6.5 U	23/12	26″	[19]
101		"	"	"	4.8 U	24/12	"	"
101		"	"	"	5.6	26/10	21	+
102		34.8	24 50	19.0	4.5 U	25/11	26	[19]
103		36.9	23 08	16.2	0.8 U	16/12	"	"
104		38.7	24 13	19.0	5.1 U	17/12	"	"
105		41.7	23 23	16.4	2.5	26/10	"	"
106		42.1	23 11	18.4	4.9 U	16/11	"	"
107	2208	43.3	24 16	15.3	0.7 U	16/11	"	"
107	2208	"	"	"	1.8 U	18/12	"	"
108		43.9	25 06	14.8	1.3 U	12/1	"	"
108		"	"	"	0.7 U	20/1	"	"
109	2927	45.1	24 25	14.8	1.7 U	18/12	"	"
110	3019	45.4	23 47	14.6	1.0 U	22/12	"	"
111	3104	45.7	22 53	14.7	4.0 U	25/11	"	"
112		47.8	24 07	>18.2	>4.0 U	13/1	"	"
113	624	39.4	24 32	16.0	1.5	1967 28/11	21	+
114		46.0	24 21	17.5	2.0	1967 29/11	"	+
115		46.2	24 15	17.7	3.0	1967 29/11	"	+
116		33.5	25 10	18.7	3.4	17/11	"	+
117		33.7	25 53	18.4	4.2	16/10	40	+
118		37.3	24 20	17.5	3.5	16/10	"	+
119		37.8	23 25	>21.0	>4.6	16/10	"	+
120		40.7	23 28	17.8	1.6	18/10	"	+
121		41.0	24 09	18.1	6.0	21/11	21	+
122		41.0	23 10	17.8	1.8	18/10	40	+
123		42.5	24 45	16.3	1.0	20/12	21	+
124		45.8	21 45	15.0	0.5	19/12	"	+
125		46.0	23 40	19.3	5.9	18/10	40,21	+
126		46.4	21 50	14.7	1.2	19/12	21	+
127		48.5	25 22	16.5	2.0	17/11	"	+
128		36.1	25 34	17.0	>1.0	24/12	"	++
129		36.0	23 16	~20.0	>6.4	11/1	26	+++
130		43.5	25 22	~15.4	~0.5	11/1	"	+++
131	924	$3^h40^m.3$	23°19′	16.7	0.9	17/10	24	×
132		41.7	22 30	18.5	2.7	18/10	"	×
133		30.5	24 23	18.2	2.8	18/10	"	×
134		37.4	22 49	17.1	1.0	20/10	"	×
135		45.2	24 05	17.0	1.7	24/10	"	×
136		38.5	24 57	19.0:	3.1:	24/10	"	×
137		53.0	23 27	16.8	1.2	24/10	"	×
138		33.2	23 41	17.8	4.5	28/10	"	×
139		38.7	23 12	17.8	3.3	28/10	"	×
140	1547	41.8	24 30	16.2	1.1	23/12	"	×
141		31.1	24 27	16.5	2.4	19/1	"	×
142		48.4	23 04	17.2	1.7	19/1	"	×
143		39.3	24 57	17.5	2.3	30/10	"	×
144		44.0	24 48	(17.5	(3.0	30/10	"	×
145		41.9	22 06	19.2:	3.7:	25/11	"	×
146		41.3	21 59	(19	(3.2	26/11	"	×

Note: The crosses (+, ++ and +++) indicate flare stars discovered, respectively, at the Byurakan, Alma-Ata, and Budapest (Konkoly) Observatories, the data on which are published here for the first time. The symbol × represents flare stars discovered at the Asinago Observatory, as reported by Rosino [20].

Table 2

New Flares of Known Flare Stars

1	2	3	4	5	6	7	8	9
8	357	$3^h38^m.5$	23 51′	14.5	0.6 U	16/11	26″	[19]
8	"	"	"	"	1.0 U	"	"	"
8	"	"	"	"	1.2	27/10	24	×
14	906	40.2	24 22	15.9	2.2 U	16/11	26	[19]
15		40.9	23 59	18.0	2.5	26/11	24	×
21	1653	42.0	24 25	14.6	1.4 U	24/11	26	[19]
21	"	"	"	"	1.0 U	28/12	"	"
40		37.2	24 22	18.0	3.5	25/9	21	
88	2193	43.2	23 15	15.2	2.1 U	26/12	26	[19]
93	2602	44.2	23 41	16.4	3.1 U	22/12	"	"

The corresponding number for all the remaining flare stars is much smaller. At the same time, even this star has not been continuously observed and, therefore, it is not possible to establish any statistical results on the intervals between successive flares. However the absence of continuous observations and the more or less irregular distribution of these observations, which is determined by factors unconnected with the given star (time of year, time of day, weather, release of telescope for other work, etc.), enable us to simplify the statistical analysis of the observed flare data. In fact, observers have investigated the behavior of each star for less than 2 or 3% of the total time. This small fraction is, in turn, subdivided into small intervals which are more or less randomly distributed along the time axis. The activity of the star during the remaining time is not observed, and the corresponding possible flares are lost.

It can therefore be assumed, to a high degree of accuracy, that under these conditons the probability of detection of k flares during an effective observational time t is satisfactorily described for each star by the Poisson distribution

$$p_k = \frac{e^{-\nu t}(\nu t)^k}{k\,!}, \tag{1}$$

where ν is the mean flare frequency and t is the total time covered by the observations.

There are no reasons to suppose that ν should be the same for all stars in a given aggregate. In fact, there should be a distribution law $f(\nu)$ for the values of ν corresponding to different flare stars in a given system. The determination of this function should be the aim of the observations.

Let the number of stars with frequencies between ν and $\nu + d\nu$ be denoted by $Nf(\nu)d\nu$, where N is the total number of flare stars in the system under investigation. The distribution of the number of flares for randomly selected stars in this system will be given by

$$p_k = \frac{t^k}{k!}\int_0^\infty f(\nu)\, e^{-\nu t}\, \nu^k d\nu, \tag{2}$$

and the empirical expectation of the number of stars which have survived k flares in a time t in a system will be Np_k.

Distribution laws of the type given by Eq. (2), which are, in fact, superpositions of Poisson distributions with different frequencies, have been investigated in great detail in the theory of probability and in statistics. Auxiliary formulas and tabulations are available [14, 15] and this facilitates the determination of $f(\nu)$.

If it is desired to determine $f(\nu)$ from observations, the first approximate method that can be used is to equate the observed number n_k of stars undergoing k flares in a time t with the corresponding mathematical expectation $\overline{n_k} = Np_k$. Observations can, in principle, be used to determine N and hence to predict the results of further studies of flares in a given system [16].

§3. New observations on flare stars in the Pleiades. The original paper by Haro [17] published in 1968 contained the entire material on flare stars in aggregates up to 1965. These data showed that the mean number of flare stars per hour of observations in the Pleiades was not very different from the corresponding number for one of the richest associations, i.e., the Orion association. Moreover, the first attempt at a statistical analysis of the flares observed in the Pleiades, based on the data reported in [17], led to an unexpected result [16]. It was found that all, or almost all, stars in this cluster which were weaker than V = 13.25 should be flare stars. Their total number was estimated as approximately 320.

It therefore became necessary to verify this conclusion about the unusually large number of flare stars in the Pleiades on the basis of observational data.

The new observations of Haro and Chavira [18], which merely confirmed this conclusion, became known to us soon after. These observations, carried out at the Tonantzintla Observatory in 1965–1967, have added 39 new flare stars to the known 61 flares in the Pleiades. The observations of 1968–1969 have produced even more data. During this period, the Tonantzintla, Byurakan, and Asiago Observatories discovered 43 new flare stars [19, 20]. Another three flares in the Pleiades were discovered at Konkoly and Alma Ata Observatories. Lists of new flare stars and new flares which have occurred are given in Tables 1 and 2, repectively. The columns give, respectively, the running number (continuing the numeration of Haro), the number (according to the Hertzsprung et al. catalog [21]), the coordinates, the stellar magnitude at minimum (in photographic rays), the photographic or ultraviolet (U) amplitude of the flare, the date of the flare, the telescope (diameter), and the reference. The number of known flare stars has thus increased to 145. We have used these extensive data for the statistical analysis of flare stars in the Pleiades, using the method described in §2.

Table 3

Telescope (observatory)	Number of flares detected	Effective time of observation (hours)	Number of flares per hour
21" Byurakan	9	62	0.15
26" Tonantzintla	22	138	0.16
26" Konkoly	2	11.7	0.17
40" Byurakan	6	6	1.00

§4. **The effect of the telescope and the method of observations on the number of observed flares.** Not all flares are accessible to a telescope of given diameter. During a given time the telescope will record flares with small amplitudes in bright stars, and only flares with large amplitudes in weak stars. The terms, "bright" and "weak", are relative designations, depending on the quality of the telescope. The exposure time also depends on the telescope diameter. For telescopes with diameters of 20–40" and high-sensitivity plates, the optimum exposure in photographic rays is between 5 and 10 min. Further increase in the exposure results in a gain as far as the limiting magnitude is concerned and weaker stars become accessible, but then fast flares are lost. In the final analysis the total number of flares which can be recorded is practically the same. Moreover, the longer exposure results in a time averaging and an artifical lowering of the true flare amplitude.

The choice of the spectral region for the observations is quite important. Since the flare amplitude increases toward the ultraviolet, the use of an ultraviolet filter should ensure the detection of flares with small amplitudes in bright stars, but flares in weak stars are lost. Although, in the case of observations without the filter, a reduction in the amplitude leads to a loss of a certain fraction of flares in bright stars, it nevertheless seems to us that photographic observations without a filter should be more effective in searches for new flare stars, and especially weak flare stars.

In the statistical analysis of the observed flares one should not, in principle, use data obtained with telescopes of different diameters because the probability of detection of low luminosity flare stars will change with the limiting stellar magnitude. For example, in the case of the 1-m Schmidt telescope at the Byurakan Observatory an observational period of 6 hr resulted in the discovery of six flare stars, i.e., one flare per hour, whereas during 1 hr of observations with the 21" Schmidt telescope there were 0.15 flare. Comparison of parallel observations performed with the two telescopes has shown that practically all the flares detected with the aid of the 40" telescope produced a trace on the photographs obtained with the 21" telescope (owing to the large amplitudes), but they were missed since only the weak image corresponding to the light maximum was visible, so that it was not possible to establish reliably that the flare had taken place. It became evident that the 40" Schmidt telescope recorded a much broader range of flares than the 21".

For the purposes of statistics, it is always desirable to have data obtained either with the same telescope or with telescopes having comparable diameters. This is confirmed by Table 3, which compares data for the 1968–1969 season. It shows that the number of flares per hour is nearly the same for telescopes of comparable diameter.

§5. **Distribution of the number of observed flares for a randomly selected star in a real case.** Since not all the flares are usually recorded but only a certain percentage, the distribution law given by Eq. (2) must be suitably transformed to take this into account.

Let q(m) be the fraction of all flare stars with magnitude m at minimum, which can be detected with a given telescope. It is clear that q(m) depends on the distribution of the flare amplitudes. Let the frequency distribution of the flares for these stars be $f_m(\nu)$. The mean number of flares of the given star in time dt will then be νdt, and the number of large flares among them which can be recorded will be $q(m)\nu dt$.

The probability of recording k flares for a randomly selected flare star with magnitude m at minimum is, in this case, given by

$$p_k(m) = \frac{q^k t^k}{k!}\int_0^\infty e^{-q\nu t} f_m(\nu)\nu^k d\nu. \qquad (3)$$

Let P_k be the probability that k flares are observed in a time t with a given telescope. We then have

$$P_k = \int_{m_0}^\infty p_k(m)\, a(m)\, dm, \qquad (4)$$

where $a(m)$ is the magnitude distribution function for the flare stars and m_0 is the magnitude of the brightest flare star in the aggregate.

Substituting for $p_k(m)$ into this expression, we obtain

$$P_k = \frac{t^k}{k!}\int_{m_0}^\infty a(m)dm \int_0^\infty e^{-q\nu t} f_m(\nu)\, q^k \nu^k d\nu. \qquad (5)$$

Let us replace $q\nu$ in this expression with ν', and change the order of integration. We then have

$$P_k = \frac{t^k}{k!}\int_{m0}^\infty \frac{a(m)}{q(m)} dm \int_0^\infty e^{-\nu' t}\nu'^k f_m\left(\frac{\nu'}{q}\right) d\nu' =$$

$$= \frac{t^k}{k!}\int_0^\infty e^{-\nu' t}\nu'^k d\nu' \int_m^\infty \frac{a(m)}{q(m)} f_m\left(\frac{\nu'}{q}\right) dm. \qquad (6)$$

It will be convenient to write

$$\int_{m_0}^\infty \frac{a(m)}{q(m)} f_m\left(\frac{\nu'}{q}\right) dm = f_1(\nu'). \qquad (7)$$

Finally, from Eq. (6) we have the following distribution law:

$$P_k = \frac{t^k}{k!}\int_0^\infty f_1(\nu')\, e^{-\nu' t}\nu'^k\, d\nu'. \tag{8}$$

This distribution is analogous to Eq. (2) with the difference that, in the present case, the expression for p_k includes the new frequency distribution function $f_1(\nu')$.

§6. The effect of flare-frequency dispersion on the estimated total number of flare stars. The distribution law given by Eq. (2) leads to a very important inequality for the mathematical expectation of the number of still undiscovered flare stars. Before we proceed to its derivation, let us consider an imaginary case where all the stars have the same frequency ν, and all the flares are equally accessible to observation with a given telescope. The mathematical expectation of the number of observed flares in a time t is then

$$\bar{n}_k = Ne^{-\nu t}\frac{(\nu t)^k}{k!}. \tag{9}$$

By writing this equation separately for k = 0, 1, 2, we immediately have

$$2\,\bar{n}_0\bar{n}_2 = \bar{n}_1^2, \tag{10}$$

and hence

$$\bar{n}_0 = \frac{\bar{n}_1^2}{2\bar{n}_2}. \tag{11}$$

If we replace, approximately, the mathematical expectations with the numbers of stars which have flared once and twice, we can deduce from this formula the value of $\bar{n}_0$, i.e., the number of flare stars whose flares have not been detected. Thus, of the 145 flare stars known at present in the Pleiades, 123 have shown one flare and 16 have shown two flares. Substituting these numbers for $\bar{n}_1$ and $\bar{n}_2$, we obtain

$$\bar{n}_0 = 473,$$

and the total number of flare stars (recorded and unrecorded) should be close to N = 600, which is valid if Eq. (9) is valid.

This large number of flare stars reinforces the earlier conclusion in [16] that all, or practically all, stars in the Pleiades which lie below a certain absolute magnitude are flare stars.

Next, substituting in turn k = 0 and k = 1 in Eq. (9) and dividing the resulting expression by $\bar{n}_0$, we obtain

$$\nu t = \frac{\bar{n}_1}{\bar{n}_0}. \tag{12}$$

For the aggregate in the Pleiades the above data taken in conjunction with Eq. (12) yield $\nu t \cong 0.26$.

If we assume that the total effective time of observations was approximately 750 hr (we do not know precisely the effective observational time in [20] and have assumed that it was ~100 hr), we find that the mean frequency of flares in the Pleiades is 0.00035 hr^{-1} $\nu^{-1} \cong 2900$ hr).

Let us now consider the general case of Eq. (2), when there are flares with different mean flare frequencies. We have already shown that this formula can be used even when the telescope does not record all the flares.

Table 4

k	n_k	n_k (obs.)
0	474	?
1	123	123
2	16	16
3	2	2
4	1	1
5	1	1
6	1	1
7	1	0
8	0.5	0
9	0.3	1

According to the Schwartz inequality, we have

$$\left(\int fg\,d\nu\right)^2 \leqslant \int f^2 d\nu \cdot \int g^2 d\nu. \tag{13}$$

It will be convenient to substitute

$$f = \nu\sqrt{e^{-\nu t}f(\nu)},$$
$$g = \sqrt{e^{-\nu t}f(\nu)}.$$

Instead of inequality (13) we then have

$$\left(\int_0^\infty \nu e^{-\nu t} f(\nu)\,d\nu\right)^2 \leqslant \int_0^\infty \nu^2 e^{-\nu t} f(\nu)\cdot\int_0^\infty e^{-\nu t} f(\nu)\,d\nu. \tag{14}$$

Multiplying both sides of this inequality by t^2, we obtain

$$p_1^2 \leqslant 2p_0p_2. \tag{15}$$

If we now multiply the last inequality by N^2, we obtain

$$\bar{n}_0 \geqslant \frac{\bar{n}_1^2}{2\bar{n}_2}. \tag{16}$$

Thus, by using the formula given by Eq. (11), which is valid for equal mean flare frequencies, we obtain in the general case the lower limit for the mathematical expectation of the number of stars for which the flares have not as yet been recorded. To obtain an idea about the change in $\bar{n}_0$ resulting from the presence of dispersion among the mean frequencies from the above lower limit, let us consider another imaginary case for which

$$f(\nu) = \frac{1}{b}e^{-b\nu}. \tag{17}$$

From Eq. (2) we can readily show that, in this case,

$$\bar{n}_0 = \frac{\bar{n}_1^2}{\bar{n}_2}, \tag{18}$$

i.e., this value is larger by a factor of two than predicted by Eq. (11). However by reducing the value of b, we obtain in the limit uniform distribution of all frequencies if Eq. (17) is valid. It may thus appear that however great the frequency dispersion, $\bar{n}_0$ cannot exceed the value given by Eq. (18).

Table 5

Δm_U	Number of flares	
	14^m2-16^m0	16^m1-17^m5
1^m0-1^m9	19	6
2.0–2.9	13	21
3.0–3.9	7	5
4.0–4.9	2	6
5.0–5.9	0	3

However, this is not the case. It is possible to imagine a distribution which has a sharp maximum or tends to infinity around the frequency $\nu = 0$. This case is very different from the case represented by Eq. (17). Moreover, although this imaginary case seems artificial at first sight, it has a deep physical significance. Stars for which the flare frequency is nearly zero are practically undetectable. All the bright stars in the Pleiades for which flares have not been observed can be conveniently regarded as "flare stars" with frequency close to zero.

Inequality (16) can therefore be strengthened if the number of flare stars includes stars which are practically nonflare. However, at this stage of our investigation this assignation of bright stars to the class of flare stars is of no interest because our first problem is to determine the stars which undergo flares over practicable observational periods (not more than a few thousand hours).

If on this basis we exclude from our calculations the possible maximum of the function $f(\nu)$ near $\nu = 0$, the value predicted by Eq. (18) will be practically the upper limit for the number of all nonflare stars.

We thus have, finally,

$$\frac{\bar{n}_1^2}{2\bar{n}_2} \leqslant \bar{n}_0 \leqslant \frac{\bar{n}_1^2}{\bar{n}_2}. \tag{19}$$

We may therefore suppose that the total number of flare stars in the Pleiades which can be detected by telescopes of, say, 20–30" in diameter, lies somewhere between 600 and 1000.

We have already noted that our calculated values of $\bar{n}_0$ are only lower limits for the true values. The position may become more complicated for the following reasons. Flare activity may vary with time for some of the stars by analogy with the situation in the case of the sun. We assume, for simplicity, that a fraction of the stars undergo no flare activity over a period of a few years. Since the observations of flare stars in the Pleiades cover only a decade, it is possible that a large group of flare stars may not be included in our calculations. In other words, we must remember the possibility of slow changes in the frequency ν with time for an individually selected star. As a result, the inequality given by Eq. (16) may become stronger still.

§7. Representation of observations by a combination of two Poisson distributions with two mean frequencies. The number of stars in the Pleiades which have shown repeated flares is very small. The possibility of determining the function $f(\nu)$ is therefore very limited. The observed number ($n_2 = 16$) is such that its difference from the true value may be 25% or more. Hence, the value $\bar{n}_0 = \bar{n}_1/2\bar{n}_2$ may be subject to a comparable relative error. With this lack of information, the best that can be done is to try to represent all the observed values of n_1 by a combination of two Poisson distributions with different frequencies.

It turns out that good agreement can be obtained by assuming that there are two groups, one large with total number $N_1 = 615$, and the other small with $N_2 = 7$. The corresponding frequencies are such that $\nu_1 t = 0.26$ and $\nu_2 t = 6$, where t is the total effective time of observation.

In fact, under these conditions, the mathematical expectation of the number of stars for which n_k flares have been observed, where

$$n_k = N_1 \frac{e^{-\nu_1 t}(\nu_1 t)^k}{k!} + N_2 \frac{e^{-\nu_2 t}(\nu_2 t)^k}{k!}, \tag{20}$$

is tabulated in Table 4.

Column 3 of Table 4 gives the observed values of n_k.

This excellent agreement between observations and the calculated mathematical expectations is, of course, the result of the fact that we were able to choose the values of four parameters (N_1, N_2, $\nu_1 t$, $\nu_2 t$) for the relatively short table of observational data.

The value of $N = N_1 + N_2 = 622$ adopted for the total number of flare stars in the Pleiades is somewhat greater than the value of N = 618 obtained in §6 from Eq. (11). This is perfectly acceptable because it was shown above that Eq. (11) is valid for the case of equal mean flare frequencies, and gives the lower limit for the mathematical expectation of the number of flare stars for which flares have not as yet been detected in the general case, when the system contains stars with different mean flare frequencies.

We must augment the foregoing with the following notes.

1. Although further observations leading to new values of n_k may force us to somewhat change the adopted values of N_1, N_2, $\nu_1 t$, $\nu_2 t$, there is no doubt that, in addition to the main mass of stars in the Pleiades, which indicates relatively rare flares ($\nu_1^{-1} \cong$ $\cong 2900$ hr), there is a small group of stars which undergoes frequent flares ($\nu_2^{-1} \cong 120$ hr).

2. The group of stars which undergo rare flares may actually have a relatively broad frequency spectrum. There are, however, no data at present which would enable us to judge the frequency bandwidth because for any frequency dispersion it is still possible to obtain good agreement with observations by assuming only one value for ν_1. As regards the group of seven stars with high frequencies, most of them, if not all, have already been observed. In all probability, four stars showing more than three flares are present

Table 6

m_{pg}	n	n_1	n_2	$n_{n\geq3}$	n_0	N	νt	ν^{-1} (hours)
$14^m\!.2-16^m\!.7$	50	37	9	4	76	126	0.487	1540
16.8—18.2	50	45	3	2	338	388	0.133	5640
18.3—>21.5	45	41	4	—	210	255	0.195	3850

in this group. Moreover, it is possible that at least one or two representatives of this group are among the 16 stars which have undergone two flares.

§8. **Amplitude distribution of the flares.** It is important to know the amplitude distribution for the statistics of flare stars. The observed amplitude distribution of flares will correspond to the true distribution only if one is sure that all the flares above a certain threshold amplitude have been recorded. For the rough photographic observations used in the present case, this threshold amplitude may be assumed to be $A = 0^m\!.6$, but this is valid only for the brighter flare stars.

To derive the amplitude distribution of the flares we have therefore used only those flare stars whose light at minimum was not less than $17^m\!.5$ in photographic rays. The total number of such stars in the list turns out to be 80, and the total number of their flares 112. However, 30 of these flares have an amplitude less than 1^m in the ultraviolet, and not all such flares may have been detected by the telescopes, especially in the case of the weakest of the selected stars. We therefore decided to consider amplitudes corresponding to $A \geq 1^m\!.0$ in the ultraviolet rays. The use of amplitudes in the ultraviolet was due to the fact that most of the flares were discovered in these rays. Approximate ultraviolet corrections to the amplitudes were introduced for flares discovered in photographic rays and included in our statistics. To simplify the calculations we assumed that for all the flare stars $U - B = +1^m$ at minimum, and that the color of the flare was $U - B = -1^m$.

All the 82 flares which we used were divided into two equal groups in accordance with their brightness in photographic rays, i.e., $14^m\!.2-16^m\!.0$, $16^m\!.1-17^m\!.5$.*

Inspection of Table 5 will show that the distributions are considerably different for flare stars of different brightness. In particular, for the brighter stars (group 1) the maximum number of flares has amplitudes in the range $1^m\!.0-1^m\!.9$, whereas for the second group this maximum lies in the next interval. We have the impression that the weaker stars more frequently show flares with larger amplitudes.

This is also indicated by the following fact. Out of all the flares in the Pleiades which were recorded, 157 have amplitudes $\geq 1^m\!.0$ in the ultraviolet, and of these 42 have amplitudes $\geq 5^m\!.0$. Moreover, the brightest star for which such a flare was recorded had a brightness of $16^m\!.7$. Of the 157 flares noted above, 51 were observed for stars brighter than $16^m\!.7$, and the remainder for the weaker stars. Thus, out of the 52 flares of bright stars with amplitude $\geq 1^m$, including the above star of $16^m\!.7$, only one has an amplitude $\geq 5^m\!.0$, whereas among the 105 flares of weaker stars there are 41 with such a large amplitude. The observational conditions were such that flares of weak stars with small amplitude e.g. an amplitude of 2^m in stars of 20^m) could not be observed. However, it may be supposed that the percentage of lost flares cannot be much greater than 50. Therefore, even if we assume that for the weaker stars we should have observed 210 flares during the same time, it was nevertheless found that one out of five flares had an amplitude $\geq 5^m\!.0$. This cannot be a result of random fluctuations, and indicates that the mean amplitude of flares increases with decreasing brightness.

The observed increase of amplitude with decreasing brightness appears to support the conclusion that the mean energy liberated during each flare is not very dependent on the luminosity of the flare stars. (We are indebted to V. S. Oskanyan for pointing this out).

§9. **Dependence of flare frequency on the brightness of flare stars.** It is shown in §7 that the observed distribution of flares among flare stars for which n_k flares have been observed (k = 1, 2, etc.) can be represented by the sum of two Poisson distributions with different mean frequencies. This in itself shows that the flare stars in the Pleiades have different flare frequencies. At the same time, it is important that stars which flare up relatively more frequently are the brightest objects. This is an indication of the fact that the flare frequency depends on brightness. In this connection, we tried to obtain directly the flare frequencies for stars of different brightness, using the data at our disposal. All the known flare stars in the Pleiades were divided into three roughly equal groups: $14^m\!.2-16^m\!.7$, $16^m\!.8-18^m\!.2$, $18^m\!.3-{>}21^m\!.5$. Using Eqs. (11) and (12), we calculated the total number N of flare stars and νt independently for each group. Next, since practically all the observations on the region of the Pleiades which have been used for the discovery of flare stars were carried out with the wide-angle Schmidt cameras, we assumed that the effective time of observations was the same (about 750 hr) for all three groups, and calculated the mean flare frequency. The results of the calculation are given in Table 6.

Although the calculated values of N and ν^{-1} given in lines 2 and 3 of Table 6 are subject to considerable errors, because the corresponding values of n_2 are small, nevertheless, the higher value of the mean flare frequency for the brighter stars given in the first line of the table may be regarded as a real effect. This fact de-

*The photographic amplitudes of flare stars were taken from the catalog [21]. In the remaining cases we used magnitudes as reported by the observers. In some cases, when the stellar magnitude is followed by the symbol > or (, the stellar magnitudes were estimated from the Palomar charts.

Table 7

r'	$\overline{m}_{pg}$	n	n_1	n_2	$n_{k\geqslant 3}$	n_0	N	νt	ν^{-1} (hours)
0— 46.6	17.2	49	34	10	5	58	107	0.596	1280
46.7— 85.1	17.5	48	45	3	—	337	385	0.134	5600
85.2—155.7	17.8	48	44	3	1	323	371	0.135	5520

serves considerable attention and we shall return to it in the following section. On the other hand, there is no doubt that, in the case of weak stars (this refers particularly to line 3 of Table 6), a considerable fraction of the flares, i.e., flares with small amplitude, is lost in the observations. Therefore, the values of ν^{-1} given in the various lines of Table 6 cannot be compared.

The data of Table 6 are confirmed by the analogous data shown in Table 7. These refer to three groups of stars at different distances from the center of the aggregate in projection onto the celestial sphere. The groups are chosen so that there is an almost equal number of flare stars in each of them. The first column of the table shows the limiting distances from the center (in minutes of arc), and the second column gives the mean photographic magnitude of the flare stars for the given group.

Comparison of the first two columns of Table 7 shows that there is a definite relation between the mean distances from the center and the mean stellar magnitudes. The data of this table can also be considered, therefore, as confirming the observed dependence of flare frequency on the brightness of the flare stars.

The total number of flare stars in Tables 6 and 7 is considerably greater than the value N = 618 obtained in §6. It seems to us that this is a further argument in favor of the fact that the values of N and ν^{-1} obtained directly from the resultant data using Eqs. (11) and (12) are actually only the lower limits for the required quantities.

We will not go into a more detailed discussion of the observed relation between the flare frequency and the stellar magnitude at minimum for the following reason. The only significant conclusions are those based on the comparison of comparable quantities. If we are concerned with the comparison of flare frequencies in a given amplitude interval, it is important to remember that our data suffer from an underestimate of low-amplitude flares in weak stars. On the other hand, if we are interested in flares for which the energy release lies within given limits, we must remember that a flare whose amplitude amounts to one magnitude for 14^m stars is roughly equivalent to a flare of five magnitudes in the case of $18^m.5$ stars. If we are not recording flares of less than 0.7 magnitude for 14^m stars, we should not consider flares with amplitude less than four magnitudes in the case of $18^m.5$ stars.

Before we compare the frequencies for stars of equal brightness, we must agree about the particular sets of flares in stars of different magnitude in which we are interested.

§10. Stars nearing the end of flare activity. The observational data show that the brightest flare stars have considerable flare activity. We note the magnitudes of the four stars which have experienced at least four flares.

All four stars belong to the group of the brightest flare stars, and three of them lie near the boundary (V = 13.30) separating the region of flare stars from nonflare stars along the axis of visual stellar magnitudes. Although one could suspect that the detection of flares for the brighter stars is easier than for the weaker stars, observational selection cannot explain such a striking result.

We also note the mean amplitudes of the three brightest stars shown in Table 8. These mean amplitudes are so small in comparison with the mean amplitudes of stars which have undergone one or two flares ($3^m.3$ and $2^m.3$, respectively) that it seems very probable that for flare stars near the above boundary between flare and nonflare objects, i.e., for flare stars for which the flare activity has nearly ceased, there is a reduction in the mean amplitude.

This conclusion means that as the observed flare activity ends, the amplitude gradually decreases. Hence, stars which have not shown flares of appreciable amplitude may be regarded as objects in which the flare amplitude lies below the limit detectable by the rough photographic method which we have employed (e.g. less than $0^m.4$ in U). It would therefore be very interesting to use more accurate methods of investigating the behavior of stars with V = 13.25 (or somewhat brighter) with the aim of detecting small flares.

To establish what happens to a star just before the termination of flare activity, we shall proceed as follows. Let us set up a list of stars in the Pleiades in order of decreasing visual brightness in accordance with the photometric measurements of Johnson and Mitchell [22]. Table 9 shows ten successive stars in this list (we have excluded HII 1794 and HII 1850, which form a binary system with integral magnitude V = 13.36 [22]) on either side of the boundary separating flare and nonflare stars. Column 3 of the table gives the number of observed flares for the corresponding stars and column 4 the mean flare amplitudes in U.

From this table we may draw the following two conclusions.

1. There are sharp differences in the stellar magnitudes between flare and nonflare stars (we are, of course, concerned with flares of amplitude not less than 0.5 magnitude).

2. If the flare stars at the end of their activity were to have flare frequency close to the value for the entire aggregate, we should detect a flare for only one out of every four stars. In fact, flares have been observed for all five stars which have come to the end of their flare

Table 8

No.	HII	m_V	$\overline{\Delta m_U}$	Sp	Number of flares	References
8	357	13.46	1.3	K6Ve	9	[8, 18, 19]
14	906	15.24	2.1	K7-M0Ve	6	[18, 19]
17	1306	13.39	1.2	dK5(e)	5	[18, 22]
21	1653	13.31	0.9	K4.5e	4	[18, 19]

activity. This means that the flare frequency is higher for these objects.

§11. **Possible correction to the dynamic mass of the Pleiades.** The Pleiades should contain at least 600, mostly weak, flare stars. However, there are bright nonflare members of the Pleiades whose total mass is quite considerable.

Observations show that none of the stars with V < 13.30 in the Pleiades exhibits a flare. The total mass of all the bright stars in the Pleiades up to V = 13.30 is approximately 260 $M_\odot$ [16]. Moreover, the dynamic mass of the Pleiades determined from the virial theorem can be estimated as 400 $M_\odot$ [23, 24]. Even if we allow for the possibility of error in the measured mean square velocity we must still conclude that the result cannot exceed 450 $M_\odot$.

Therefore, the flare stars account for a total mass between 140 and 190 solar masses. Moreover, there is no difficulty in estimating their mean mass from the absolute magnitudes of the flare stars through the mass-luminosity relation. This turns out to be one-third of the solar mass. We may, therefore, take 200 $M_\odot$ as the lower limit for the total mass of all the flare stars in the Pleiades.

In fact, our calculations do not include flare stars which are inaccessible to telescopes of intermediate size even for large flares. Consequently, it is probable that the total mass of all the flare stars is much greater than 200 $M_\odot$. Therefore, whereas in [16] we considered that the flare stars are included in the total mass of the Pleiades determined dynamically, we now come to doubt this possibility.

However, the total mass determined with the aid of the virial theorem applied to the members of the cluster taken from the catalog [21] does not include the mass of the spherical layer outside a 1° radius around Alcyone. At the same time, this spherical layer contains not less than half the flare stars discovered so far. The attraction of this spherically symmetric layer can have no effect on the motion of the Hertzsprung members occupying a spherical volume of radius 1°.

The dynamic mass of the Pleiades can include only half or even less of the total mass of the flare stars i.e., no contradiction with the dynamic mass is obtained. At the same time, the dynamic mass determined so far is not the total mass of the Pleiades. To obtain the total mass we must augment the dynamic mass by the total mass of roughly half of all the flare stars, i.e., not less than 100 $M_\odot$.

§12. **Conclusions.** Statistical analysis of the 145 flare stars in the Pleiades, including the 85 stars recently discovered, leads to the following conclusions.

1. The Pleiades contain more than 600 flare dwarfs with visual magnitudes greater than $13^{m}.25$.

Table 9

HII	V	n	HII	V	n	$\overline{\Delta m_U}$
2588	13.10	0	1531	13.30	2	0.6
3187	13.12	0	1653	13.31	4	0.9
380	13.19	0	1306	13.39	5	1.2
451	13.25	0	3104	13.41	1	4.0
945	13.29	0	3019	13.45	1	1.0

2. There is a sharp boundary on the visual magnitude axis between flare stars and stars for which flares are not observed. All, or practically all, members of the Pleiades for which V ≥ 13.29 show no flares. It must be remembered, however, that the photographic method which we are employing is very rough and, in practice, allows us to record only flares with A > $0^{m}.5$. It is possible that the application of more accurate methods to the "boundary stars", i.e., the detection of flares with small amplitudes, will lead to a shift of this boundary toward weaker stars.

3. The mean frequency of detection of flares for the great majority of the stars is of the order of 0.0004 hr^{-1}. The result is of the same order both for the Pleiades and the Orion associations. In the Pleiades the flare stars whose brightness lies near the boundary V = 13.29 have a considerably higher frequency. At the same time, the mean amplitude is lower than for the other flare stars.

It must be remembered that the estimated frequencies are, to some extent, conventional and depend on the sensitivity of the observational method. We may suppose, however, that if we were able to record flares with A = $0^{m}.1$ the mean frequencies would be much higher.

4. The true total mass of all the stars in the Pleiades should be somewhat greater than the dynamic mass determined from the virial theorem because a fraction of weak flare stars forms an outer spherical envelope which has no influence on the motion of the brighter stars which lie in the interior and are used to determine the dynamic mass. The correction to the dynamic mass may reach 100 $M_\odot$.

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of recorded flares achieved during the last year is actually the result of close international cooperation in this area.

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