

CASCADE PROCESS IN STRONG MAGNETIC AND ELECTRIC FIELDS
UNDER ASTROPHYSICAL CONDITIONS

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The self-sustained electron-positron cascade process is analyzed in connection with pulsars. The electric and magnetic fields of the electron-positron plasma surrounding a neutron star and the radio emission of such a system are analyzed.

Éidman [1] has analyzed the self-sustained electron-positron cascade process in which the energy which the particles lose in pair production is replenished by acceleration of the charges in an external electric field. Under astrophysical conditions, however, the particles are much more likely to be moving in both magnetic and electric fields.

In a strong magnetic field, we know that a γ ray can be directly transformed into an electron-positron pair (see, e.g., [2-4]). To take this effect into account we adopt a very simple model: We assume that the particles are moving in a slightly curved magnetic field H_0 in the region $-L < x < L$ and that there is also an electric field \vec{E}_1 ($E_1 \ll H_0$) parallel to this magnetic field. In the absence of charges, the electric field is $E_1 = E_0/[1 - (x/L)]$, $-L < x < L$ (x is the coordinate along the magnetic line of force). We denote the positron density by n_1 and the electron density by n_2 ; then with a sufficiently large radius of curvature of the magnetic line of force, we can assume that the electric field is described by

$$\vec{E} = e(x-L) \left(-\frac{E_0}{eL} + n_1 - n_2 \right). \quad (1)$$

Here the densities n_1 and n_2 satisfy

$$\begin{aligned} \frac{dn_1}{dt} &= G - \frac{n_1 c}{L} \mathcal{1} \left(-\frac{E_0}{eL} + n_1 - n_2 \right) - \frac{v_g}{L} n_1, \\ \frac{dn_2}{dt} &= G - \frac{n_2 c}{L} \mathcal{1} \left(\frac{E_0}{eL} - n_1 + n_2 \right) - \frac{v_g}{L} n_2. \end{aligned} \quad (2)$$

Here $\mathcal{1}(y) = \begin{cases} 1 & \text{for } y > 0 \\ 0 & \text{for } y < 0 \end{cases}$, v_g is the particle drift velocity, and G describes the pro-

duction of pairs per unit time due to γ rays emitted by the particles in their motion along the magnetic line of force (see below). Equations (2) take into account the circumstance that positively charged particles leave this interaction region, moving along the magnetic lines of force at a velocity near the speed of light c , if $n_1 - n_2 - E_0/eL > 0$, while the electrons, on the other hand, leave, correspondingly, if $n_1 - n_2 - (E_0/eL) < 0$. Furthermore, the charges escape from this region as a result of drift. If we assume that the density of the energetic particles is not too high, we can ignore pair production as well as annihilation in particle collisions in (2).

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To write an equation for G , we note that since the fields \vec{H}_0 and \vec{E} under these conditions are slightly inhomogeneous and quite intense, with $E \ll H_0$, we can use the results of [5]. It follows from that paper that a charge under these conditions moves primarily along the magnetic line of force at a velocity near the speed of light c , and it drifts at a low velocity across \vec{H} . During this motion it radiates in a synchrotron manner, governed by the radius of curvature ρ of the magnetic line of force (this is the so-called magnetodrift radiation). The charge energy ε and the radiation frequency ω_γ are given by

$$\frac{\varepsilon}{mc^2} = \frac{1}{\sqrt{1-\beta^2}} = \left(\frac{3|E|c^2}{2e} \right)^{1/4}, \quad (3)$$

$$\omega_\gamma = \Omega_c (1-\beta^2)^{-3/2}, \quad (4)$$

where $\beta = v/c$, v is the velocity of the particle ($\beta \rightarrow 1$), $\Omega_c = c/\rho$ and $\vec{E} \parallel \vec{H}_0$. All the energy which the particle acquires in the field E is expended on this radiation:

$$e|E|c = \frac{2}{3} \frac{e^2}{c} \frac{\Omega_c^2}{(1-\beta^2)^2}. \quad (5)$$

If the energy of the photons which are emitted, $\hbar\omega_\gamma$, is quite high, then these photons, moving in the field \vec{H}_0 , can themselves produce pairs of particles [2-4]. Accordingly, the particle production should be described by ($\varepsilon_\gamma = \hbar\omega_\gamma$)

$$G \simeq \frac{ec|E|}{\hbar\omega_\gamma} (n_1 + n_2) l(\varepsilon_\gamma - 2mc^2) \Big|_{t=t-t_3}. \quad (6)$$

Relation (6) takes into account the circumstance that at a given time t the pairs are produced by those γ rays which were emitted at time $t-t_3$, and the quantity ct_3 determines the mean free path of the γ rays in the field $\vec{H}_0(r)$. It should be kept in mind that at the time of emission the γ ray is moving approximately parallel to \vec{H}_0 , and for this ray to convert into a pair the magnetic line of force must rotate through a large angle from the direction \vec{k}_γ [2-4] (here $k_\gamma = \omega_\gamma/c$ is the momentum of the γ ray). Relation (6) takes into account the circumstance that γ rays with energies below $2mc^2$ cannot produce pairs. Here we have

$$l(\varepsilon_\gamma - 2mc^2) = \begin{cases} 1 & \text{for } \varepsilon_\gamma > 2mc^2 \\ 0 & \text{for } \varepsilon_\gamma < 2mc^2. \end{cases}$$

To find a qualitative explanation of the behavior described by (3)-(6) we assume $|E| \simeq eL|n_1^* - n_2|$, $n_1^* = n_1 - n_0$ and $n_0 = E_0/eL$. Then introducing the new dimensionless variables $\eta = (n_1^* + n_2)/n_0$, $\xi = (n_2 - n_1^*)/n_0$, $t_1 = ct/L$ and $\tau = ct_3/L$; and using (2)-(6), we find, for $\xi(t) \neq 0$,

$$\frac{d\xi}{dt_1} = \alpha |\xi(t_1 - \tau)|^{1/4} [1 + \eta(t_1 - \tau)] \cdot l(|\xi(t_1 - \tau)| - \beta) \cdot l(\tau) \cdot l(\xi - \tau) - \beta_g - \beta_g \eta - \frac{\eta}{2} - \frac{\xi}{2} \operatorname{sgn} \xi - 1(-\xi) \equiv G - \beta_g - \beta_g \eta - \frac{\eta}{2} - \frac{\xi}{2} \operatorname{sgn} \xi - 1(-\xi), \quad (7)$$

$$\frac{d\eta}{dt_1} = -\frac{\eta}{2} \operatorname{sgn} \xi - \frac{\xi}{2} - \beta_g \xi + \beta_g + 1(-\xi). \quad (8)$$

The range of definition of ξ , η is $\eta \geq \xi - 2$ for $\xi > 1$ and $\eta \leq -\xi$ for $\xi < 1$.

Here

$$z = \frac{2e^2 L^2 n_0^{1/4}}{hc \rho^{1/2}} \left(\frac{2}{3L} \right)^{3/4}, \quad g = \frac{\rho}{L}, \quad \delta = \frac{2}{3Ln_0} \left(\frac{2mc}{\hbar \sqrt{\rho}} \right)^{4/3}, \quad \beta_g = \frac{v_g}{c},$$

$$\operatorname{sgn} \xi = \begin{cases} 1 & \text{for } \xi > 0 \\ -1 & \text{for } \xi < 0, \end{cases} \quad \xi \equiv \xi(t).$$

Equation (7) takes into account the circumstance that the retardation time satisfies the conditions $0 < t_3 < L/c$ (the dimension of this system is on the order of L). We note that the quantity η determines the total number of particles, while ξ gives the electric field E . Equations (7) and (8) should be supplemented with a relation giving the delay between the emission of a γ ray and its transformation into a pair. The mean free path of the γ ray, d_0 , in a homogeneous field $\vec{H}_\perp \perp \vec{k}_\gamma$ is given in [3, 4]:

$$d_0 = \frac{2\hbar}{\alpha_1 mc} \frac{H_{kp}}{H} [T(\chi)]^{-1}, \quad (9)$$

where $\alpha_1 = e^2/\hbar c$, $H_{kp} = m^2 c^3/e\hbar$ and $\chi = (1/2) (\hbar \omega_\gamma/mc^2) (H_\perp/H_{kp})$. The function $T(\chi)$ can be written approximately as $T(\chi) \approx 0.16 \chi^{-1} K_{1/3}^2(2/3\chi)$, where $K_{1/3}^2(2/3\chi)$ is the Macdonald function [4]. It follows, in particular, that the reaction $\hbar \omega_\gamma + H_\perp = e^+ + e^- + H_\perp$ can occur only at sufficiently large values of the γ energy and at sufficiently strong magnetic fields H .

In the case under consideration here the γ ray is propagating in an inhomogeneous magnetic field, and at the time of emission we have $H_\perp \approx 0$. Accordingly, if the γ ray is to be absorbed in pair production, it must traverse a certain finite path, given in order of magnitude by

$$\int_{c(t-t_3)}^{ct} \frac{dr}{d(r)} = 1, \quad (10)$$

here $d(r)$ is found from (9) if H_\perp is replaced by some function $H_\perp(r)$ which determines the value of H_\perp over the entire path traversed by the γ ray [for $r = c(t - t_3)$, we have $H_\perp \approx 0$].

For the qualitative analysis below we assume $H_\perp(r) \approx Hr/\rho$, $r \leq L$, and, in general, $\rho \approx L$. Then Eq. (10) can be rewritten in terms of dimensionless variables as

$$1 = \frac{b}{|\xi(t_1 - \tau)|^{3/4}} \int_{t_1 - \tau}^{t_1} K_{1/3}^2 \left(\frac{d_1}{|\xi| |t_1 - \tau|^{3/4} r_1} \right) dr_1, \quad (11)$$

where

$$r_1 = r/L, \quad d_1 = \frac{4\sqrt{\rho} mc H_{kp}}{3\hbar (3n_0 L/2)^{3/4} L H}, \quad b = \frac{0.16 e^2 m^2 c L^{1/4}}{\hbar^3 \sqrt{\rho} (3n_0/2)^{3/4}}.$$

It is easy to see that Eqs. (7) and (8) become incorrect in the case $\xi = 0$. The integral curves of Eqs. (7) and (8) near the $\xi = 0$ axis behave in the following manner: For $\eta < 2\beta$, the integral curves intersect the $\xi = 0$ axis from left to right; for $\eta > 2\beta$, they approach the $\xi = 0$ axis from both directions. Taking this circumstance into account, it is natural to complete the definition of the system by assuming the following equation holds for $\xi(t) = 0$, $\eta > 2\beta$:

$$\frac{d\tau_1}{dt_1} = G - \beta_g - \beta_g \gamma - \tau_1 - 1. \quad (12)$$

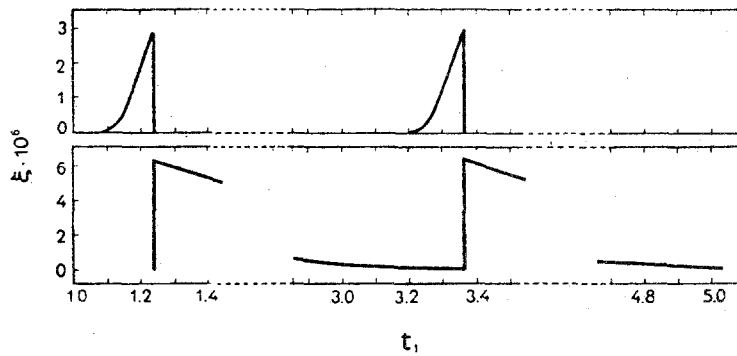


Fig. 1

System (7), (8), (11), (12) describes an oscillatory regime. We will first determine the qualitative nature of this phenomenon, and then we will give the results of a numerical calculation carried out by Ya. I. Al'ber and Z. N. Krotova. We assume that the system is initially in the state described by $\xi = 0$, $\eta = \eta_0 > 2\beta$, $G = 0$. In this state, charges of both signs leave the interaction region. In this stage of the process, the system is described by Eq. (12), which corresponds to motion downward along the $\xi = 0$ axis from the point $\eta = \eta_0$ to the point $\eta = 2\beta$. At the point $\xi = 0$, $\eta = 2\beta$, the integral curve departs from the $\xi = 0$ axis and enters the region $\xi > 0$, $-\xi < \eta < -\xi(1 + 2\beta) + 2\beta$, as long as we have $G = 0$. Now the electrons leave the interaction region even more rapidly, the field E intensifies, and energetic γ rays appear (this is the second stage of the process). After the first energetic γ rays have traversed a distance on the order of their mean free path, pair production begins [the corresponding roots τ of Eq. (11) and G become nonvanishing]. Here η begins to increase rapidly, and ξ begins to decrease (the field E decreases). The integral curve rapidly returns to the axis $\xi = 0$, $\eta > 2\beta$, and begins to move upward along the $\xi = 0$ axis [see (12)], as long as the condition $G \neq 0$ is satisfied. As soon as G vanishes, the pair production stops, and the integral curve begins to descend along the $\xi = 0$ axis down to the point $\eta = 2\beta$ [see (12)]; thereafter, the process occurs in the manner described above. The system thus executes a periodic motion. Figure 1 shows the results of a numerical solution of Eqs. (7), (8), (11), and (12) for the following parameter values: $L \approx \rho = 10^6$ cm, $H_0 = 10^{12}$ G, $\Omega = 10$ sec $^{-1}$, and $E_1 = (1/3)10^7$ esu, i.e., $n_0 = 10^5$ cm $^{-3}$, $\alpha = 10^5$, $b = 5 \cdot 10^8$, and $d_1 = 10^{-3}$. We note that in this case the time which the system spends in the state $\xi = 0$ (i.e., $E = 0$) is 10 times as long as the time it spends in the state with $\xi \neq 0$.

Let us now attempt to apply this mechanism to pulsars.* We first note that during the rotation of a magnetic rotator (a neutron star) in vacuum, the electric field component E_1 directed along the magnetic field \vec{B} can have the form corresponding to that discussed [see (1)]; i.e., the sign can change along the magnetic line of force. For example, if the angle δ between \vec{B}_0 and $\vec{\Omega}$ is small ($\vec{\Omega}$ is the angular rotation velocity and \vec{B}_0 is the magnetic induction of the neutron star), then we have $E_1 \approx -(B_0 \Omega a / c) \cos^3 \theta$ ($r > a$, where a is the radius of the neutron star, θ is the angle between \vec{B} and $\vec{\Omega}$, and r is the distance from the center of the neutron star).

Now taking into account the circumstance that a neutron star is surrounded by an electron-positron plasma, we must determine the structure of the electromagnetic fields surrounding the neutron star. We work from [7], where the field of a magnetic rotator in a conducting medium was analyzed. To use the results of [7], we must know the conductivity of the plasma surrounding the neutron star. In this case of an electron-positron plasma, in which the energy loss of the particles is due to magnetodrift radiation [see (3)-(5)], we can evaluate the electrical conductivity σ_2 of the plasma from

$$\sigma_2 = \frac{e^2 n \sqrt{1 - \beta^2}}{m v_{\text{eff}}} = \frac{3c^2 n (1 - \beta^2)^2}{2\Omega_r^2} \quad (14)$$

*The pair-production mechanism for pulsars was first used by Sturrock [6].

where v_{eff} is the ratio of the energy radiated per unit time, from (5), to the energy of the particle, i.e., $v_{\text{eff}} = (2/3)(e^2 \Omega_c^2 / mc^3 (1 - \beta^2)^{3/2})$, where $\Omega_c = c/\rho$ and n is the particle density. In accordance with (5), we can write Eq. (14) as ($E_{\text{co}} \neq 0$)

$$z_0 = \frac{enc}{E_{\text{co}}}, \quad \vec{E}_{\text{co}} \parallel \vec{B}. \quad (15)$$

We first consider time-independent fields due to the constant component of the magnetic induction of the neutron star. In contrast with the system examined in [7], in the present case, because of the possible spontaneous production of plasma in strong \vec{B} , \vec{E}_{co} ($\vec{E}_{\text{co}} \parallel \vec{B}$) fields, the system is primarily in a state with $\vec{E}_{\text{co}} = 0$. The field \vec{E}_0 which arises periodically is much weaker than the corresponding field in an ordinary plasma or in vacuum [7] (according to the numerical calculation, we have $E_{\text{co}} < 5 \cdot 10^{-6} E_1$, where E_1 is the field in a vacuum). Furthermore, as will become clear from the discussion below, we have $E_{\text{co}} \neq 0$ only in a small region of the plasma surrounding the neutron star. Accordingly, to find the approximate structure of the magnetic field around the neutron star under these conditions we can assume $E_{\text{co}} \approx 0$.

We work from the Maxwell equations in a conducting medium with a conductivity σ_2 ($r > a$):

$$\text{rot } \vec{B} = \frac{4\pi\sigma_2}{c} \vec{E}, \quad \text{rot } \vec{E} = 0, \quad E = -\nabla\varphi. \quad (16)$$

It follows from the symmetry of the problem that in the spherical coordinate system r, ϑ, φ with axis coinciding with the rotation axis of the neutron star the fields \vec{B} and \vec{E} are independent of the angle φ . Then from the requirement $E_{\text{co}} = 0$ or $\vec{B} \text{ rot } \vec{B} = 0$ we find ($\vec{B} = \{B_r, B_\vartheta, B_\varphi\}$, $\vec{E} = \{E_r, E_\vartheta, E_\varphi\}$, $E_\varphi = 0$)

$$\frac{B_r}{\sin \vartheta} \frac{\partial}{\partial r} (B_\varphi \sin \vartheta) - B_\vartheta \frac{\partial}{\partial r} (r B_\varphi) = 0. \quad (17)$$

Since B_r and B_φ cannot vanish simultaneously (at the $r = a$ boundary the field component B_r must be continuous), we have $B_\vartheta = 0$. Also using the conditions $\partial \vec{B} / \partial \varphi = 0$ and $E_\varphi = 0$, we find that we have $\text{rot } \vec{B} = 0$ or that we have $\vec{E} = 0$ if $\sigma_2 \neq 0$.

Then, by analogy with [7], we find the following results for $\sigma_2 \ll \sigma_1$ (σ_1 is the conductivity of the neutron star):

$$\vec{B}_\parallel \approx B_\parallel \{\cos \vartheta_1 - \sin \vartheta, 0\}, \quad (18)$$

$$\vec{E} \approx -\frac{1}{c} [\vec{V} \vec{B}_\parallel] = -\frac{B_\parallel \Omega r}{c} \left\{ \sin^2 \vartheta, \frac{1}{2} \sin 2\vartheta, 0 \right\} \quad \text{for } r < a,$$

$$\vec{B}_\parallel \approx \frac{B_\parallel a^3}{r^3} \left\{ \cos \vartheta, \frac{1}{2} \sin \vartheta, 0 \right\}, \quad \vec{E} \approx 0 \quad \text{for } r > a, \quad (19)$$

where $\vec{V} = [\vec{\Omega} \vec{r}]$, a is the radius of the neutron star, $B_\parallel = B_0 \cos \delta_0$, and δ_0 is the angle between $\vec{\Omega}$ and the homogeneous magnetic induction \vec{B} of the neutron star.

We note the following in connection with Eqs. (16) and (17).

Since there is a discontinuity in E_ϑ at the boundary, it follows from the condition $\text{rot } \vec{E} = 0$ that a strong radial electric field of the type $E_r^* = (B_\parallel \Omega a^2 / 6c) \delta(r - a) (3 \cos^2 \vartheta - 1)$ is concentrated near the boundary (there is an electric double layer concentrated at the boundary). In this case the first equation

should be written in the form $\text{rot } \vec{B} = (4\pi/c)(\sigma_2 \vec{E} + \vec{j}_{st})$, $\vec{j}_{st} = (\vec{j}_r, 0, 0)$, $j_r = -\sigma_2 \vec{E}_r^*$. Since, on the other hand, we must have $E_{CO} = 0$ within the plasma (see the discussion above), we must move the boundary between the rotating and fixed media right up to the surface of the neutron star. Accordingly, within the framework of the present analysis, this circumstance can be taken as an argument that the plasma is not entrained in the rotation of the neutron star. Here, of course, we are not thinking of a thin plasma sheath, whose interaction with the surface of the neutron star is governed by factors ignored here (e.g., nonelectromagnetic forces; if d_1 is the height of the irregularities on the surface of the neutron star, then this transitional sheath cannot be thinner than d_1).

Since the conductivity σ_2 is assumed high, the equations for the variable parts of the fields due to $B_\perp = B_0 \sin \delta_0$ are [7]

$$\begin{aligned} \vec{B}_\perp &\simeq B \{ \sin \vartheta \cos \mu, \cos \vartheta \cos \mu, -\sin \mu \}, \\ \vec{E}_\perp &= -\frac{1}{c} [\vec{V} \vec{B}_\perp] = B_\perp \frac{\Omega r}{c} \left\{ \frac{1}{2} \sin 2\vartheta, -\sin^2 \vartheta, 0 \right\} \cos \mu \end{aligned} \quad (20)$$

for $r < a$, or

$$\begin{aligned} \vec{B}_\perp &\simeq B_\perp e^{-(r-a)/d} \left\{ \sin \vartheta \cos \mu_1, 0, -\frac{a}{d} \sin^2 \vartheta (\cos \mu_1 - \sin \mu_1) \right\}, \\ \vec{E}_\perp &= -\frac{1}{c} [\vec{V} \vec{B}_\perp] \simeq -\frac{B_\perp \Omega a}{c} \{ 0, \sin^2 \vartheta \cos \mu_1, 0 \}, \quad (\vec{E}_\perp \cdot \vec{B}_\perp) = 0 \end{aligned} \quad (21)$$

for $r > a$, where $\sigma_1 \gg \sigma_2$, $d = c / \sqrt{2\pi\Omega\sigma_2}$ is the skin thickness, $d \ll a$, $\mu = \varphi - \Omega t$, and $\mu_1 = \mu + (r-a)/d$.

Equations (20) and (21) contain only the maximum field components. It would seem to follow from Eq. (21) that an electromagnetic field \vec{E}_\perp , \vec{B}_\perp would cause the plasma to rotate as if it were a solid object. The kinetic energy per unit volume of the plasma associated with this motion is on the order of $W_\Omega = m(\Omega a)^2 n / \sqrt{1-\beta^2}$, where n is the particle density. However, this rotation must occur in a constant magnetic field \vec{B}_\perp ($\vec{E}_\perp = 0$) with an energy density $B_\perp^2 / 8\pi$ much larger than W_Ω . Accordingly, the magnetic field B_\perp does not lead to this process, and the assumption that the plasma at $r > a$ does not move as a whole is not internally contradictory in this sense. We again emphasize that this circumstance is a consequence of the spontaneous production of the plasma, i.e., the condition $\vec{E}_\perp = 0$. Otherwise, the field due to B_0 would itself entrain the plasma. We also note that Eqs. (19) and (21) imply the existence of a certain field $E_{CO} = (\vec{B}_\perp \cdot \vec{B}_\perp)$, but this field is weak under the condition $a \gg d$ and is localized in a small region near the boundary.

The equations for the fields in the region $r > a$ have a remarkable feature, which may yield an explanation for the pulsed nature of the emission of pulsars (see the discussion below). It follows from (21) that in the case $\tan \mu_1 = 1$ the magnetic field in the skin is weak (it does not exceed \vec{B}_\perp with $\delta_0 = 1$). If, on the other hand, we have $\tan \mu_1 \neq 1$ ($d/a \ll 1$), then the component B_{1z} is strong, and it will hinder the escape of particles from the region surrounding the neutron star. In the region $\tan \mu_1 = 1$, on the other hand, the particles leave the system more rapidly, in accordance with the onset here of the oscillatory regime discussed above, which is in turn established throughout the space around the neutron star by the plasma.

To apply the results obtained earlier [see (7), (8), (11)] to pulsars, we must set $E_0 = \Omega a B_0 / c$ (E_0 is the electric field component parallel to the magnetic field in the case in which the neutron star is rotating in a vacuum), we must assume the dimension of the system to be $L = a$ (a is the radius of the neutron star), and we must set $n_0 = E_0 / eL$. The particles in the oscillatory system are moving primarily in meridional planes along the lines of force of $\vec{B}_\perp + \vec{B}_\perp$ in the region $\tan \mu_1 = 1$.

It follows from a numerical solution of Eqs. (7), (8), and (11) that the electric field $E \approx E_{CO}$ which arises in the second stage of the oscillatory process vanishes very rapidly. For the parameter values listed above, the field E_{CO} vanishes over a time Δt on the order of $5 \cdot 10^{-5} L/c$, i.e., $\Delta t \approx 10^{-9}$ sec (with $L \approx 10^6$ cm). This vanishing of the field E_{CO} occurs because of an abrupt change in the current due to motion along \vec{B} of the charges along the magnetic lines of force [see (1) and (2)]. The scale time for this process is on the order of Δt . This current is sharply bounded spatially by the surface of the neutron star. If this surface is smooth, this process of the vanishing of the field E_{CO} can become an effective source of radio emission. It is not difficult to show that the magnitude of the volume radiating coherently, V , is $V \approx \pi a \lambda^2$ where $\lambda = 2\pi c/\omega$ is the emitted wavelength, ω is the emission frequency, $2\pi/\omega \approx \Delta t$, $2\pi/\omega \approx c/d_1$, d_1 is the thickness of the transitional region, which is governed in particular by the height of the irregularities on the surface of the neutron star (see the discussion above). In other words, in a given direction \vec{R} there is efficient emission from a disk (of radius \sqrt{a} and thickness λ) at the boundary of the neutron star; the axis of this disk lies parallel to \vec{R} . The emission mechanism is analogous to the emission during β decay. The total emission intensity is

$$I \approx \frac{e^2 n^2 \dot{V}^2 \omega \nu}{c} \approx 2\pi^3 e^2 n^2 a \lambda^3 c, \quad (22)$$

where $V = \pi a \lambda^2$, ν is the number of events in which the field E_{CO} vanishes per second ($\nu \approx c/a$), n is the particle density. Setting $n \approx n_0 \approx E_0/e a \approx 10^{10}$ cm $^{-3}$, $\lambda \approx 10^3$ cm, and $a \approx 10^6$ cm (see the discussion above), we find $I \approx 10^{10}$ ergs/sec, in accordance with experimental data. Since we have $(\omega_H/\omega) = (eB\sqrt{1-\beta^2})/m c \omega \gg 1$, $\omega_0^2/\omega^2 = (4\pi e^2 n \sqrt{1-\beta^2})/m \omega^2 \ll 1$ in this case, the radio emission should freely escape the generation region. This emission is pulsed. Emission occurs in a given direction \vec{R} when this direction falls in the meridional region governed by $\tan \mu_1 \approx 1$. The pulse width is characterized by the ratio d/a (the pulse repetition period is π/Ω).

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