CALCULATION OF PRODUCTIVITY OF LABORATORY-TYPE COURSES

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A mathematical model is presented for calculation of productivity (student credit hours per-full-time equivalent faculty) for combined lecture-laboratory-recitation courses. Analysis of computer-generated plots of productivity versus enrollment for typical combined lecture-lab courses leads to the following conclusions: (1) Productivity levels off rather than continuously increasing as enrollment increases, suggesting that large lecture sections cannot be justified. (2) The productivity increase with enrollment is not monotonic, but has a sawtooth appearance, suggesting that forbidden intervals of enrollment would maximize productivity. (3) In lab-optional courses, productivity is maximized by discouraging lab enrollment after the first lab section is filled.

The overall productivity of an instructional unit such as a department is shown to be calculable as the sum of course productivities, each multiplied by a weighting factor equal to the fraction of total teaching time devoted to that course.

Potential applications of the model to course and curriculum design are discussed. The effects on productivity of varying any of the parameters in the model—for example, lecture and lab contact and credit hours, lab capacity, average lecture size, and fraction of students taking the lab—can be quantitatively predicted.

Key words: productivity; laboratory

It is well-known that courses involving laboratory and recitation components are labor-intensive compared to lecture-only courses, but the manner in which the various parameters describing a lecture-lab-recitation course combine to determine the overall "productivity" of that course is complicated. In this paper a mathematical model is presented for the calculation of productivity of combined lecture-laboratory-recitation courses. In addition to laboratory sciences, the model is appropriate for courses such as music, art, and industrial

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education—that is, any course in which instructor weekly *contact* hours exceeds the course *credit* hours. Illustrative applications of the model are presented as computer-generated plots of productivity versus enrollment. Although the basic model applies to a single course or course sequence, the proper weighting of individual course productivities in the calculation of overall departmental productivity is also discussed. Some implications for the design of courses, the structuring of departmental curricula, and the allocation of faculty time are drawn from the analysis.

The commonly used definition of productivity (which the author begrudgingly accepts) is the ratio of student credit hours (SCH) produced to the full time equivalent faculty (FTEF) used. For example, a 3-credit-hour lecture course with 30 students produces 90 SCH. If a normal teaching load is 12 credit hours per semester, or 24 credit hours per academic year, this course requires 3/24 = 0.125 FTEF. The productivity of this course, then, is 90/0.125 = 720 SCH/FTEF. The calculation is more complicated if the course has a laboratory component, since there may be more than one lab section per lecture, not all students may take the lab, and the definition of FTEF must recognize *contact* as well as credit hours. For the model presented here, it is assumed that the FTEF is defined in terms of a standard weekly contact hour load. One FTEF then is the number of weekly contact hours taught by a full-time faculty member multiplied by the number of sessions in the academic year taught by a full-time faculty member. The examples presented here are for a two-semester year but the model is equally applicable to the quarter and trimester systems.

The model is given by equation 1.

$$P \equiv \frac{\text{SCH}}{\text{FTEF}} = \frac{N_a L\,E + N_b F_b L\,E}{(1/W)\,[n_a\,L + n_b\,(F_b\,L\,E/C_b)^* + n_c(F_c\,L\,E/C_c)^*]} \qquad (1)$$

Where:

- N_i = number of course credit hours for the i^{th} type of section.
- n_i = number of weekly faculty contact hours for the i^{th} type of section.
- C_i = capacity of i^{th} type of section.
- L = number of concurrently offered lecture sections.
- E = enrollment in the lecture section, or average enrollment in multiple lecture sections.
- F_i = fraction of students taking an optional section of type i. $F = 1$ for required labs, recitations, etc.
- $(F_iLE/C_i)^*$ = number of sections of type i, where $i \neq a$. The superscript asterisk indicates the fraction is rounded up to the next whole integer.
- i = a, for lecture sections
  - b, for laboratory sections
  - c, for recitation, discussion, quiz, etc. sections.

W = weekly contact hour standard load multiplied by the number of academic sessions taught per year. This product defines 1.00 FTEF.

The model has been constructed so that the average lecture enrollment and number of lecture sections are fundamental quantities. The total lecture enrollment for a course is then $L \cdot E$, and the number of lab or recitation sections required to handle that total enrollment is given as $(F_i \cdot L \cdot E/C_i)$ rounded up to the next whole integer. Thus, the number of lab and recitation sections are not independent variables except indirectly through manipulation of section capacities (C_i), total course enrollment ($L \cdot E$), and fraction of students taking optional-type sections (F_i). Similarly, enrollments for labs and recitations, and SCH for labs are derived quantities. This formulation, although somewhat arbitrary, corresponds most closely with the practical applications to schedule planning and course design for which the author's department has used the model.

The model assumes that lab and recitation sections will fill to capacity before new sections are opened, i.e., only the minimum number of sections necessary are used. It is assumed that recitation sections generate no SCH, however, an additional term could be added to the numerator to account for recitation SCH if necessary. It is not required that the same individual be teaching lecture, lab, and recitation sections or, for example, all of the lab sections of a course; only the total FTEF used enters the calculation. Although the model as presented covers the most complex course in the author's experience, additional terms can easily be added if there are more than three types of sections associated with a course.

It is interesting to note the simple form that the model reduces to for a single section of a lecture-only course,

$$P = W \cdot E \tag{2}$$

and for a single section of a lab-only course,

$$P = (N_b/n_b)\, W \cdot E \tag{3}$$

Equation 2 points out that lecture course productivity is independent of the course credit hours. This is because both the numerator and denominator in the ratio defining productivity increase proportionally to credit hours. Equation 3 shows that the productivity of a lab course is a simple function of the credit to contact hour ratio for the lab.

The principal value of the model is in analyzing the effects on productivity when parameters describing a combined lecture-lab course are independently varied. Examples are presented in Figures 1–4. These are computer-generated plots of productivity versus enrollment per

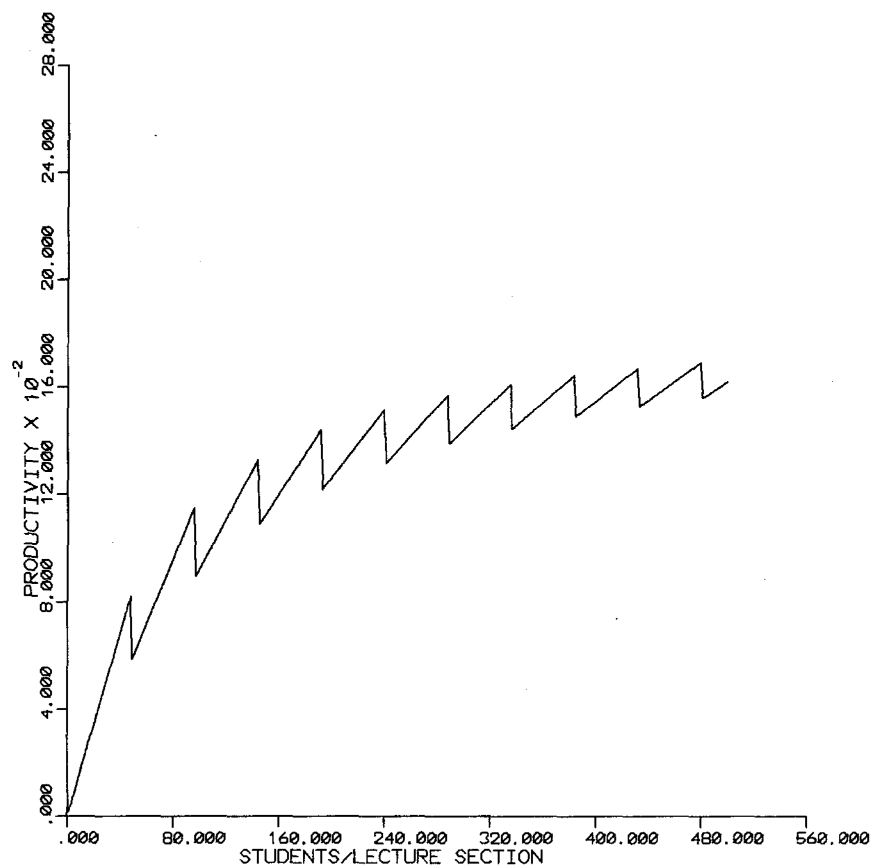

FIGURE 1. Productivity for 5-credit course with 4 hours of lecture (one section) and 3 hours of lab (required) per week, lab capacity of 48, and the workload standard defined as 24 weekly contact hours per two-semester academic year. In terms of the symbols used in equation 1: $N_a = 4$, $N_b = 1$, $n_a = 4$, $n_b = 3$, $C_b = 48$, $L = 1$, $F_b = 1$, $W = 24$. Since this is a lab-required course clearly an equivalent formulation would be to set $N_a = 5$ and $N_b = 0$. The enrollment, E, is plotted as the independent variable. A drop in productivity occurs with every 48 students corresponding to the opening of a new lab section.

lecture section. The parameters describing the courses are specified in the figure legends. Several interesting facts emerge from these plots.

In all cases there is a leveling off in productivity as lecture size becomes very large. This is clearly illustrated in Figure 1 and by the bottom plot of Figure 2. As shown by equations 2 and 3, such a leveling off is not found when either a lecture-only or a lab-only course section is allowed to increase without limit. However, leveling off is charac-

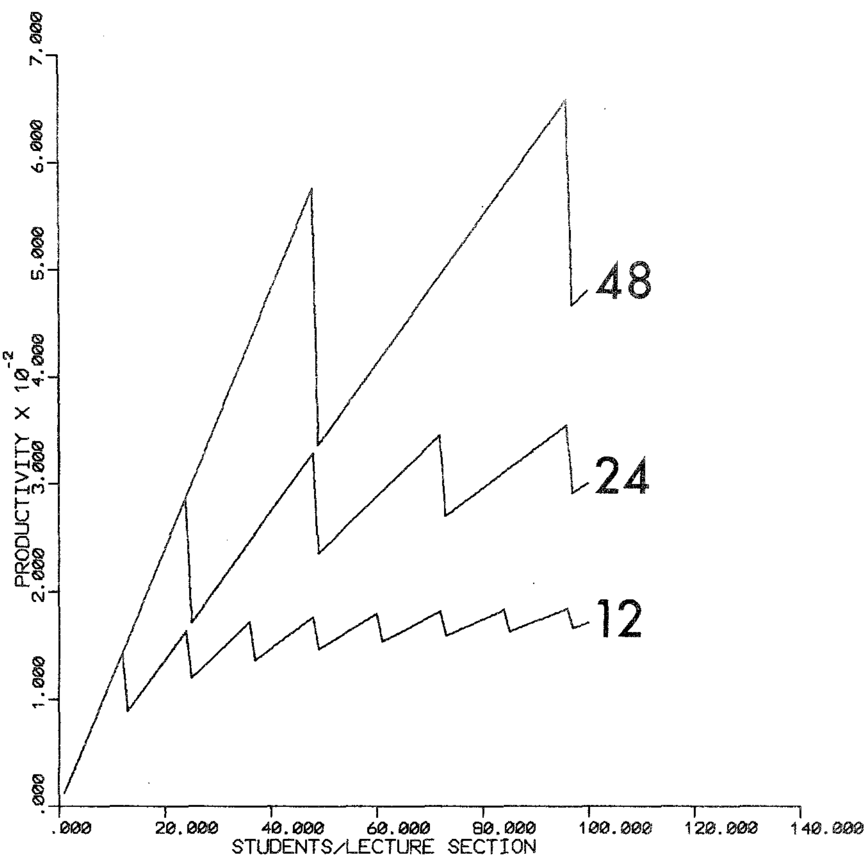

FIGURE 2. Effect on productivity of varying the lab capacity from 12, 24, and 48 in the 4-credit course with 2 hours of lecture and 6 hours of required lab per week. $L = 1$, $W = 24$. Lab capacities, C_b, are indicated beside the plots.

teristic of all combined lecture-lab courses; it is an inevitable consequence of the increasing number of low-productivity lab sections required to handle the students. At high enrollments, the total productivity of the lecture-lab course is dominated by the poor productivity of the lab component. The important application of this phenomenon is in realizing that for high-enrollment lecture-lab courses which are on or near the plateau part of the productivity curve, very large lecture sections cannot be justified. The pedagogical adventages of smaller lecture sections can be had for a very small price in decreased productivity.

Another characteristic of combined lecture-lab courses is that the productivity curves have a sawtooth appearance. A sudden drop in productivity occurs at the point at which a new lab section must be

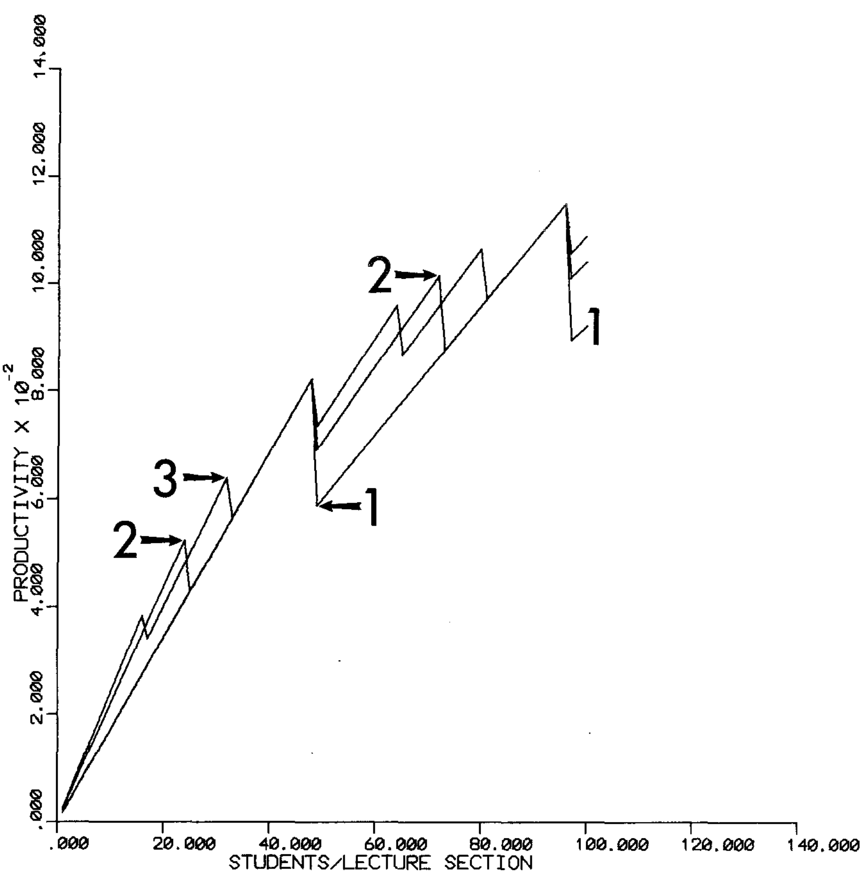

FIGURE 3. Productivity of the same course as in Figure 1 for varying number of lecture sections, *L*, as indicated.

opened to accommodate one additional student. The magnitude of this drop in productivity depends on the class, but is often as much as 40 percent. Furthermore, a large enrollment increase is necessary to return productivity to the previous high point. For example, in the upper curve of Figure 2, the first productivity peak occurs at 576 SCH/FTEF with 48 students; the productivity drops to 336 SCH/FTEF at 49 students and does not return to 576 SCH/FTEF until the lecture size has increased to 84 students. This suggests that it would be wise to define "forbidden intervals" of enrollment for some courses in order to avoid the low productivity valleys. The magnitude of the productivity drop gradually decreases as successive lab sections are opened. Although, as can be seen in Figure 2, the magnitude of the drop is greater for courses with high capacity labs, the percentage decrease in productivity is nearly independent of lab capacity. The ratio of the productivities just after to just before the first drop, P_{min}/P_{max}, can be derived from equation 1. For the case $L = 1$, F = 1, and $n_c = 0$ we obtain,

$$\frac{P_{min}}{P_{max}} = \frac{(C_b + 1)}{C_b} \frac{(n_a + n_b)}{(n_a + 2n_b)} \tag{4}$$

Equation 4 reveals that the productivity drop is nearly independent of lab capacity because $(C_b + 1)/C_b \simeq 1$ for ordinary values of C_b. The productivity ratio is seen to depend only on the relative hours of lecture versus lab contact time. The percentage decrease in productivity upon adding the first additional lab is $(1 - P_{min}/P_{max})100$. Representative values of the percentage decrease are: 20 percent, $n_b = 1/3(n_a)$; 33 percent, $n_b = n_a$; 43 percent, $n_b = 3(n_a)$. The values approach a limit of 50 percent for $n_b >> n_a$.

The rather dramatic effect of varying the lab capacity is shown in Figure 2, in which three courses with identical lecture credits and contact times but different lab capacities are compared. The leveling off begins markedly earlier as the size of the lab decreases. The factors which determine where the limiting-productivity plateau occurs can be understood by considering the envelope curve traced out by the productivity maxima, P_{max}. For the case where $F = 1$, $L = 1$ and $n_c = 0$, equation 1 can be rewritten as,

$$P_{max} = \frac{(N_a + N_b)\, W\, E}{n_a + n_b\,(E/C_b)} \tag{5}$$

where the function is discontinuous, i.e., only defined for values of E equal to integral multiples of C_b. In the high enrollment limit, the equation reduces to

$$P_{max} \simeq \frac{(N_a + N_b)\, W\, C_b}{n_b} \tag{6}$$

which defines the productivity plateau. Thus, the limiting productivity is seen to be directly proportional to three things: (1) the ratio of course credit to lab contact hours, (2) the teaching load corresponding to 1 FTEF, and (3) as shown in Figure 2, the lab capacity.

The effect of varying the number of lecture sections is shown in Figure 3. Productivity is as high or slightly higher when multiple lecture sections enroll S students each as compared to a single section with S students enrolled. Although one would always be somewhat better off if all sections were combined into one large section, at some point the leveling-off effect mentioned above would make further productivity increases negligible (and pedagogical problems intolerable).

In many combined lecture-lab courses, the lab is optional. Figure 4

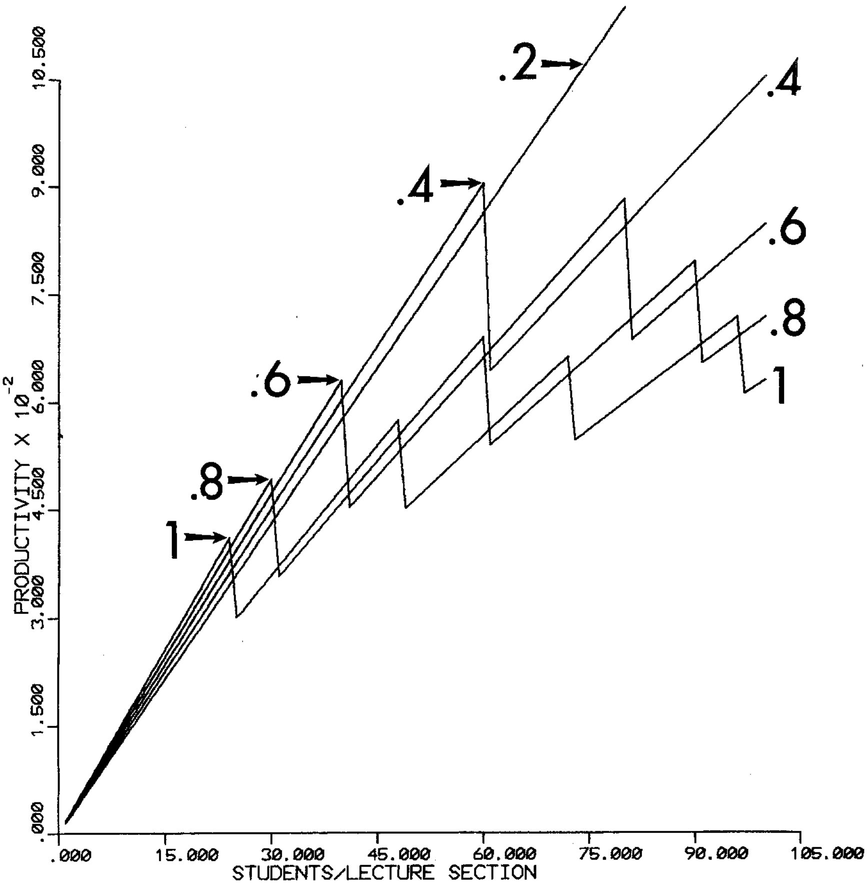

FIGURE 4. Effect of varying the fraction of students, F_b, who take the lab portion of a combined lecture-lab course with the lab optional. F_b = 0.2, 0.4, 0.6, 0.8, 1.0 as indicated. The other course parameters are: $N_a = 4$, $N_b = 1$, $n_a = 4$, $n_b = 3$, $C_b = 24$, $L = 1$, $W = 24$.

shows how productivity in such courses depends on the fraction of students taking the lab. As long as there is only one lab section, the more students taking the lab, the higher the productivity is. However, as soon as multiple lab sections must be opened, the productivity becomes highest for the case with the smallest fraction of students taking the lab. The complexity of this inversion of the productivity dependence on F as enrollment is increased highlights the need for an explicit mathematical function to describe the situation.

If it is desired to calculate the total productivity of a department, a properly weighted average of all departmental courses must be computed. This is done as follows. The total productivity, P_t, is defined in terms of the department's total credit hours, $(SCH)_t$, and total faculty useage, $(FTEF)_t$,

$$P_t = \frac{(SCH)_t}{(FTEF)_t} \tag{7}$$

Equation 7 can be rewritten in terms of individual course productivities, P_i; course credit hours, $(SCH)_i$; and course faculty useage, $(FTEF)_i$; as follows:

$$P_t = \frac{\sum_i (SCH)_i}{(FTEF)_t} = \sum_i \frac{(FTEF)_i}{(FTEF)_t} \frac{(SCH)_i}{(FTEF)_i} = \sum_i \frac{(FTEF)_i}{(FTEF)_t} P_i \tag{8}$$

The last term in equation 8 shows that when the total departmental productivity is calculated, the proper weighting factor for the P_i is the fraction of total FTEF useage dedicated to the i^{th} course. While it is intuitively reasonable that P_t should be proportional to the P_i, it is not immediately clear that P_i should, as indicated by equation 8, be proportional to the fraction of faculty time required, especially for courses with low P_i. That is, it appears that low P_i could be compensated for by large commitments of faculty time. The paradox is explained by realizing that, while P_i can increase without limit, the fractions $(FTEF)_i/(FTEF)_t$ must sum to unity. Therefore, if the weighting factor is large for a class of low P_i, the factor must be correspondingly smaller for the classes of higher P_i, and the total productivity will be pulled back down. Once the P_i are calculated by equations 1–3, equation 8 allows a department to quantitatively determine what mix of course offerings and faculty commitments it can field consistent with some specified level of overall productivity. In particular, the impact or "cost" of the labor-intensive courses—such as those commonly associated with honors, graduate, and laboratory programs—can be quantitated. This analysis, considered along with the productivity-plateau effect discussed above, forces a department to recognize that, although it may be very proud of the instructional value of its lab-intensive course, it can afford only a limited number of such courses, since they must all be "carried" by some other high-productivity courses when the average departmental productivity is calculated.

The courses analyzed in Figures 1–4 correspond to actual courses in the author's department and are illustrative of the usefulness of the model. Although these particular examples did not include courses with recitation sections, such sections are included in the model. It is clear that any activity such as recitations, which consume many FTEF while producing few or no SCH, has to lower productivity. The extension of the model to handle even more-complicated course types is straightforward; additional terms of similar types must be added to the top and/or bottom of equation 1.