*Transport in Porous Media* **22:** 53–72, 1996.

# Dimensional Analysis of Pore Scale and Field Scale Immiscible Displacement

R. HILFER[1]* and P. E. ØREN[2]
[1]*Institut für Physik, Universität Mainz, 55099 Mainz, Germany*
[2]*Statoil Research Centre, Postuttak, N-7005 Trondheim, Norway*

(Received: 27 July 1994)

**Abstract.** A basic re-examination of the traditional dimensional analysis of microscopic and macroscopic multiphase flow equations in porous media is presented. We introduce a 'macroscopic capillary number' $\overline{\mathrm{Ca}}$ which differs from the usual microscopic capillary number Ca in that it depends on length scale, type of porous medium and saturation history. The macroscopic capillary number $\overline{\mathrm{Ca}}$ is defined as the ratio between the macroscopic viscous pressure drop and the macroscopic capillary pressure. $\overline{\mathrm{Ca}}$ can be related to the microscopic capillary number Ca and the Leverett $J$-function. Previous dimensional analyses contain a tacit assumption which amounts to setting $\overline{\mathrm{Ca}} = 1$. This fact has impeded quantitative upscaling in the past. Our definition for $\overline{\mathrm{Ca}}$, however, allows for the first time a consistent comparison between macroscopic flow experiments on different length scales. Illustrative sample calculations are presented which show that the breakpoint in capillary desaturation curves for different porous media appears to occur at $\overline{\mathrm{Ca}} \approx 1$. The length scale related difference between the macroscopic capillary number $\overline{\mathrm{Ca}}$ for core floods and reservoir floods provides a possible explanation for the systematic difference between residual oil saturations measured in field floods as compared to laboratory experiment.

**Key words:** dimensional analysis, similarity theory, scaling groups, two-phase flow, residual oil saturation, mobilization of residual oil, capillary desaturation curves, capillary forces, viscous forces, gravity forces, macroscopic force balance, capillary numbers, immiscible displacement.

## 1. Introduction

After waterflooding an oil reservoir a significant fraction $S_{\mathrm{o}r}$ of oil remains microscopically trapped inside the reservoir. The fraction $S_{\mathrm{o}r}$ is called the residual oil saturation, and it ranges typically between 25% and 50% for water wet reservoirs [1–5]. The trapped oil can be mobilized if the viscous forces overcome the capillary retention forces [6].

Many more or less equivalent dimensionless ratios, or 'scaling groups' have been proposed to quantify the relative importance of viscous and capillary forces (see, e.g., Table 1 in [6] or Table 2 on page 71 in [3] for overviews). The dimen-

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* Present address: Institute of Physics, University of Oslo, PO Box 1048, 0316 Oslo, Norway.

sionless ratio of viscous to capillary forces is called the *capillary number*, and it is commonly expressed as

$$\mathrm{Ca} = \frac{\mu u}{\sigma} = \frac{k\Delta p}{\sigma\overline{\phi}L}, \tag{1}$$

where $u$ denotes an average microscopic velocity, $\mu$ is the viscosity, $\sigma$ the surface tension, $k$ the permeabilty, $\overline{\phi}$ the average porosity, and $\Delta p$ the pressure drop over the distance $L$. While the first expression in Equation (1) involves only fluid parameters, the second expression invokes Darcy's law $\overline{\phi}u = -(k\Delta p)/(\mu L)$ to express the flow velocity $u$ through the macroscopic viscous pressure drop.

Displacement experiments in a variety of porous media including micromodels show a strong correlation between the residual oil saturation $S_{\mathrm{o}r}$ and capillary numbers Ca (see [1,3,4,7] and references therein). The resulting curves $S_{\mathrm{o}r}$ (Ca) are called capillary number correlations, recovery curves or capillary desaturation curves. All such capillary desaturation curves exhibit a critical capillary number $\mathrm{Ca}_c$ below which the residual oil saturation remains constant. This critical capillary number $\mathrm{Ca}_c$ marks the point where the viscous forces begin to dominate the capillary forces. Surprisingly, all experimentally observed values for $\mathrm{Ca}_c$ are much smaller than 1. This surprise has been formulated by Dullien [1] who discusses $\mathrm{Ca}_c$ for the measurements of [8] and [9] on unconsolidated glass beads:

> It is certainly peculiar that when the viscous and capillary forces acting on a blob are equal the capillary number that is supposed to be equal to the ratio of viscous-to-capillary forces is equal to $2.2 \times 10^{-3}$ (F.A.L. Dullien in [1], p. 450).

For unconsolidated sand $\mathrm{Ca}_c$ is often reported to be $\mathrm{Ca}_c \approx 10^{-4}$ while for sandstone $\mathrm{Ca}_c \approx 3\times10^{-6}$ and for limestone $\mathrm{Ca}_c \approx 2\times10^{-7}$ [3]. The exceedingly small values of $\mathrm{Ca}_c$ as well as their dependence on the type of porous medium strongly suggest that the capillary number defined in Equation (1) cannot be an adequate measure of the macroscopic balance between viscous and capillary forces.

Given this obvious failure of Equation (1), the question arises how to define a dimensionless ratio of viscous to capillary forces whose value will generally be close to unity when mobilization of residual oil sets in. The present paper attempts to answer this question by means of a dimensional analysis of microscopic and macroscopic equations of motion governing multiphase flow in porous media.

Let us conclude this introduction with a brief overview over the organization of the rest of this paper. We begin our discussion with the microscopic equations of motion for fluid flow at the pore scale. Subsequently the well known microscopic dimensional analysis leading to the familiar dimensionless numbers of fluid dynamics is repeated. We then turn to the accepted macroscopic equations of motion which are routinely used in reservoir simulation studies. Our macroscopic dimensional analysis differs fundamentally from the traditional analysis [10–13]. Instead of using Darcy's law to normalize the pressure field we use a representative pressure from the capillary pressure curve. This gives rise to macroscopic dimensionless numbers. We then relate these macroscopic dimensionless scaling groups to the traditional microscopic groups. In conclusion, we present the application of our analysis to the problem of quantitatively predicting residual oil saturation, gravitational relaxation times and the width of a gravitational segregation front.

## 2. Pore Scale Equations of Motion

Microscopic equations of motion for two-phase flow in porous media are commonly given as Stokes (or Navier–Stokes) equations for two incompressible Newtonian fluids with no-slip and stress-balance boundary conditions at the interfaces. In the following the wetting fluid (water) will be denoted by a subscript w while the nonwetting fluid (oil) is indexed with o. The solid phase (rock), indexed as r, is assumed to be porous and rigid. It fills a closed subset $G_\mathrm{r} \subset \mathbf{R}^3$ of three-dimensional space. The pore space is filled with the two fluid phases described by the two closed subsets $G_\mathrm{w}(t), G_\mathrm{o}(t) \subset \mathbf{R}^3$ which are in general time dependent. The pore space boundary and the fluid-fluid interface are obtained as

$$\begin{aligned} \partial_\mathrm{r} &= (G_\mathrm{r} \cap G_\mathrm{w}(t)) \cup (G_\mathrm{r} \cap G_\mathrm{o}(t)) \quad (\text{rock} - \text{surface}), \\ \partial_\mathrm{ow}(t) &= G_\mathrm{o}(t) \cap G_\mathrm{w}(t) \quad (\text{oil} - \text{water interface}), \end{aligned} \tag{2}$$

where all time dependences have been indicated explicitly. The standard formulation of pore scale equations of motion for two incompressible and immiscible fluids flowing through a porous medium are the Navier–Stokes equations

$$\begin{aligned} \rho_\mathrm{w}\frac{\partial \mathbf{v}_\mathrm{w}}{\partial t} + \rho_\mathrm{w}(\mathbf{v}_\mathrm{w}^T \cdot \boldsymbol{\nabla})\mathbf{v}_\mathrm{w} &= \mu_\mathrm{w}\Delta\mathbf{v}_\mathrm{w} + \rho_\mathrm{w} g \boldsymbol{\nabla} z - \boldsymbol{\nabla} p_\mathrm{w}, \\ \rho_\mathrm{o}\frac{\partial \mathbf{v}_\mathrm{o}}{\partial t} + \rho_\mathrm{o}(\mathbf{v}_\mathrm{o}^T \cdot \boldsymbol{\nabla})\mathbf{v}_\mathrm{o} &= \mu_\mathrm{o}\Delta\mathbf{v}_\mathrm{o} + \rho_\mathrm{o} g \boldsymbol{\nabla} z - \boldsymbol{\nabla} p_\mathrm{o}, \end{aligned} \tag{3}$$

and the incompressibility conditions

$$\boldsymbol{\nabla}^T \cdot \mathbf{v}_\mathrm{w} = 0, \qquad \boldsymbol{\nabla}^T \cdot \mathbf{v}_\mathrm{o} = 0, \tag{4}$$

where $\mathbf{v}_\mathrm{w}(\mathbf{x}, t), \mathbf{v}_\mathrm{o}(\mathbf{x}, t)$ are the velocity fields for water and oil, $p_\mathrm{w}(\mathbf{x}, t), p_\mathrm{o}(\mathbf{x}, t)$ are the pressure fields in the two phases, $\rho_\mathrm{w}, \rho_\mathrm{o}$ the densities, $\mu_\mathrm{w}, \mu_\mathrm{o}$ the dynamic viscosities, and $g$ the gravitational constant. The vector $\mathbf{x}^T = (x, y, z)$ denotes the coordinate vector, $t$ is the time, $\boldsymbol{\nabla}^T = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$ the gradient operator, $\Delta$ the Laplacian and the superscript $T$ denotes the transposition. The gravitational force is directed along the $z$-axis and it represents an external body force.

The standard formulation is completed by specifying an initial fluid distribution $G_\mathrm{w}(t = 0), G_\mathrm{o}(t = 0)$ and boundary conditions. The latter are usually no-slip boundary conditions at solid-fluid interfaces

$$\mathbf{v}_\mathrm{w} = 0 \quad \text{at } \partial_\mathrm{r}, \qquad \mathbf{v}_\mathrm{o} = 0 \quad \text{at } \partial_\mathrm{r}, \tag{5}$$

as well as for the fluid-fluid interface

$$\mathbf{v}_{\mathrm{w}} = \mathbf{v}_{\mathrm{o}}, \qquad \text{at } \partial_{\mathrm{ow}}(t), \tag{6}$$

combined with stress-balance across the fluid-fluid interface

$$\tau_{\mathrm{w}} \cdot \mathbf{n} = \tau_{\mathrm{o}} \cdot \mathbf{n} + 2\sigma_{\mathrm{ow}}\kappa\mathbf{n} \quad \text{at } \partial_{\mathrm{ow}}(t). \tag{7}$$

Here $\sigma_{\mathrm{ow}}$ denotes the water-oil interfacial tension, $\kappa$ is the curvature of the oil-water interface and $\mathbf{n}$ is a unit normal to it. The stress tensor $\tau(\mathbf{x}, t)$ for the two fluids is given in terms of $\mathbf{v}$ and $p$ as

$$\tau = -p\,\mathbf{1} + \mu\mathcal{S}\,\nabla\mathbf{v}^{T} \tag{8}$$

where the symmetrization operator S acts as

$$\mathcal{S}\mathbf{A} = \tfrac{1}{2}(\mathbf{A} + \mathbf{A}^{T} - 2/3\,\mathrm{tr}\,\mathbf{A}\,\mathbf{1}) \tag{9}$$

on the matrix $\mathbf{A}$ and $\mathbf{1}$ is the identity matrix.

The pore space boundary $\partial_{\mathrm{r}}$ is given and fixed while the fluid-fluid interface $\partial_{\mathrm{wo}}(t)$ has to be determined selfconsistently as part of the solution. For $G_{\mathrm{w}} = \emptyset$ or $G_{\mathrm{o}} = \emptyset$ the above formulation of two phase flow at the pore scale reduces to the standard formulation of single phase flow of water or oil at the pore scale. The pore scale equations of motion require an additional slipping model describing the slipping of contact lines. This necessity is well known [14], and it relates to the microscopic wetting properties of the system. As we are interested in the more macroscopic processes we refrain from further discussing this point.

## 3. Pore Scale Dimensional Analysis

Given a microscopic model for contact line slipping the next step is to evaluate the relative importance of the different terms in the equations of motion at the pore scale. This is done by casting them into dimensionless form using the definitions

$$\mathbf{x} = l\widehat{\mathbf{x}}, \tag{10}$$

$$\nabla = \frac{\widehat{\nabla}}{l}, \tag{11}$$

$$\mathbf{v} = u\widehat{\mathbf{v}}, \tag{12}$$

$$t = \frac{l\widehat{t}}{u}, \tag{13}$$

$$\kappa = \frac{\widehat{\kappa}}{l}, \tag{14}$$

$$p = \frac{\sigma_{\mathrm{ow}}}{l}\widehat{p}, \tag{15}$$

where $l$ is a microscopic length, $u$ is a microscopic velocity and $\widehat{A}$ denotes the dimensionless equivalent of the quantity $A$.

With these definitions the dimensionless equations of motion on the pore scale can be written as

$$\begin{aligned} \frac{\partial \widehat{\mathbf{v}}_{\mathrm{w}}}{\partial \widehat{t}} + (\widehat{\mathbf{v}}_{\mathrm{w}}^{T} \cdot \widehat{\nabla})\widehat{\mathbf{v}}_{\mathrm{w}} &= \frac{1}{\mathrm{Re}_{\mathrm{w}}}\widehat{\Delta}\widehat{\mathbf{v}}_{\mathrm{w}} + \frac{1}{\mathrm{Fr}^2}\widehat{\nabla}\widehat{z} - \frac{1}{\mathrm{We}_{\mathrm{w}}}\widehat{\nabla}\widehat{p}_{\mathrm{w}}, \\ \frac{\partial \widehat{\mathbf{v}}_{\mathrm{o}}}{\partial \widehat{t}} + (\widehat{\mathbf{v}}_{\mathrm{o}}^{T} \cdot \widehat{\nabla})\widehat{\mathbf{v}}_{\mathrm{o}} &= \frac{1}{\mathrm{Re}_{\mathrm{o}}}\widehat{\Delta}\widehat{\mathbf{v}}_{\mathrm{w}} + \frac{1}{\mathrm{Fr}^2}\widehat{\nabla}\widehat{z} - \frac{1}{\mathrm{We}_{\mathrm{o}}}\widehat{\nabla}\widehat{p}_{\mathrm{o}}, \end{aligned} \tag{16}$$

$$\widehat{\nabla}^{T} \cdot \widehat{\mathbf{v}}_{\mathrm{w}} = 0, \qquad \widehat{\nabla}^{T} \cdot \widehat{\mathbf{v}}_{\mathrm{o}} = 0, \tag{17}$$

with dimensionless boundary conditions

$$\widehat{\mathbf{v}}_{\mathrm{w}} = \widehat{\mathbf{v}}_{\mathrm{o}} = 0 \quad \text{at}\, \partial_{\mathrm{r}}, \tag{18}$$

$$\widehat{\mathbf{v}}_{\mathrm{w}} = \widehat{\mathbf{v}}_{\mathrm{o}} \quad \text{at}\, \partial_{\mathrm{ow}}(t), \tag{19}$$

$$\begin{aligned} (\widehat{p}_{\mathrm{o}} - \widehat{p}_{\mathrm{w}})\mathbf{n} &= \left(\frac{\mathrm{We}_{\mathrm{w}}}{\mathrm{Re}_{\mathrm{w}}}\mathcal{S}\widehat{\nabla}\widehat{\mathbf{v}}_{\mathrm{w}} - \frac{\mathrm{We}_{\mathrm{o}}}{\mathrm{Re}_{\mathrm{o}}}\mathcal{S}\widehat{\nabla}\widehat{\mathbf{v}}_{\mathrm{o}}\right) \cdot \mathbf{n} + \\ &\quad + 2\widehat{\kappa}\mathbf{n} \quad \text{at}\, \partial_{\mathrm{ow}}(t). \end{aligned} \tag{20}$$

In these equations, the microscopic dimensionless ratio

$$\mathrm{Re}_{\mathrm{w}} = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{\rho_{\mathrm{w}} u l}{\mu_{\mathrm{w}}} = \frac{ul}{\nu_{\mathrm{w}}^{*}} \tag{21}$$

is the Reynolds number, and

$$\nu_{\mathrm{w}}^{*} = \frac{\mu_{\mathrm{w}}}{\rho_{\mathrm{w}}} \tag{22}$$

is the kinematic viscosity which may be interpreted as a specific action or a specific momentum transfer. The other fluid dynamic numbers are defined as

$$\mathrm{Fr} = \sqrt{\frac{u^2}{gl}} = \sqrt{\frac{\text{inertial forces}}{\text{gravitational forces}}} \tag{23}$$

for the Froude number, and

$$\mathrm{We}_{\mathrm{w}} = \frac{\rho_{\mathrm{w}} u^2 l}{\sigma_{\mathrm{ow}}} = \frac{\text{inertial forces}}{\text{capillary forces}} \tag{24}$$

TABLE I. Order of magnitude estimates for densities, viscosities and surface tension of oil and water under reservoir conditions.

| $\rho_o$ | $\rho_w$ | $\mu_o$ | $\mu_w$ | $\sigma_{ow}$ |
|---|---|---|---|---|
| 800 kg m$^{-3}$ | 1000 kg m$^{-3}$ | 0.0018 N m$^{-2}$s | 0.0009 N m$^{-2}$s | 0.035 N m$^{-1}$ |

for the Weber number. The corresponding dimensionless ratios for the oil phase are related to those for the water phase as $\mathrm{Re}_o = \mathrm{Re}_w(\rho_o/\rho_w)(\mu_w/\mu_o)$ and $\mathrm{We}_o = \mathrm{We}_w(\rho_o/\rho_w)$ by viscosity and density ratios.

Table I gives approximate values for densities, viscosities and surface tensions under reservoir conditions. In the following these values will be used to make order of magnitude estimates. Typical pore sizes in an oil reservoir are of order $l \approx 10^{-4}$ m and microscopic fluid velocities for reservoir floods range around $u \approx 3 \times 10^{-6}$ m s$^{-1}$. Combining these estimates with those of Table I shows that the dimensionless ratios obey $\mathrm{Re}_o, \mathrm{Re}_w, \mathrm{Fr}^2, \mathrm{We}_o, \mathrm{We}_w \ll 1$. Therefore, the pore scale equations (16) reduce to the simpler Stokes form

$$\begin{aligned} 0 &= \widehat{\Delta}\widehat{\mathbf{v}}_w + \frac{1}{\mathrm{Gr}_w}\widehat{\nabla}\widehat{z} - \frac{1}{\mathrm{Ca}_w}\widehat{\nabla}\widehat{p}_w, \\ 0 &= \widehat{\Delta}\widehat{\mathbf{v}}_o + \frac{1}{\mathrm{Gr}_o}\widehat{\nabla}\widehat{z} - \frac{1}{\mathrm{Ca}_o}\widehat{\nabla}\widehat{p}_o, \end{aligned} \tag{25}$$

where

$$\mathrm{Ca}_w = \frac{\mathrm{We}_w}{\mathrm{Re}_w} = \frac{\text{viscous forces}}{\text{capillary forces}} = \frac{\mu_w u}{\sigma_{ow}} = \frac{u}{u_w^*} \tag{26}$$

is the microscopic capillary number of water defined above in Equation (1), and

$$\mathrm{Gr}_w = \frac{\mathrm{Fr}^2}{\mathrm{Re}_w} = \frac{\text{viscous forces}}{\text{gravity forces}} = \frac{\mu_w u}{\rho_w g l^2} \tag{27}$$

is the microscopic ‘gravity number’ of water. The capillary number is a measure of velocity in units of

$$u_w^* = \frac{\sigma_{ow}}{\mu_w} \tag{28}$$

a characteristic velocity at which the coherence of the oil-water interface is destroyed by viscous forces. The capillary and gravity numbers for the oil phase can again be obtained from multiplication with density and viscosity ratios as $\mathrm{Ca}_o = \mathrm{Ca}_w(\mu_o/\mu_w)$ and $\mathrm{Gr}_o = \mathrm{Gr}_w(\rho_w/\rho_o)(\mu_o/\mu_w)$.

Many other dimensionless ratios may be defined. Of general interest are dimensionless space and time variables. Such ratios are formed as

$$\mathrm{Gl}_{\mathrm{w}} = \frac{\mathrm{Ca}_{\mathrm{w}}}{\mathrm{Gr}_{\mathrm{w}}} = \frac{\mathrm{We}_{\mathrm{w}}}{\mathrm{Fr}^2} = \frac{\text{gravity forces}}{\text{capillary forces}} = \frac{\rho_{\mathrm{w}} g l^2}{\sigma_{\mathrm{ow}}} = \frac{l^2}{l_{\mathrm{w}}^{*2}} \tag{29}$$

which we shall call the 'gravillary number'. The gravillary number becomes the bond number if the density $\rho_{\mathrm{w}}$ is replaced with the density difference $\rho_{\mathrm{w}} - \rho_{\mathrm{o}}$. The corresponding length

$$l_{\mathrm{w}}^* = \sqrt{\frac{\sigma_{\mathrm{ow}}}{\rho_{\mathrm{w}} g}} \tag{30}$$

separates capillary waves with wavelengths below $l_{\mathrm{w}}^*$ from gravity waves with wavelengths above $l_{\mathrm{w}}^*$. A dimensionless time variable is formed from the gravillary and capillary numbers as

$$\begin{aligned}\frac{\sqrt{\mathrm{Gl}_{\mathrm{w}}}}{\mathrm{Ca}_{\mathrm{w}}} &= \frac{\mathrm{Re}_{\mathrm{w}}}{\mathrm{Fr}\sqrt{\mathrm{We}_{\mathrm{w}}}} = \frac{(\text{capillary f.} \times \text{gravity f.})^{1/2}}{\text{viscous f.}} \\ &= \frac{t\sqrt{\rho_{\mathrm{w}}\sigma_{\mathrm{ow}} g}}{\mu_{\mathrm{w}}} = \frac{t}{t_{\mathrm{w}}^*},\end{aligned} \tag{31}$$

where

$$t_{\mathrm{w}}^* = \frac{l_{\mathrm{w}}^*}{u_{\mathrm{w}}^*} = \frac{\mu_{\mathrm{w}}}{\sqrt{\sigma_{\mathrm{ow}}\rho_{\mathrm{w}} g}} \tag{32}$$

is a characteristic time after which the influence of capillary and gravity forces dominates viscous effects.

Table II collects definitions and estimates for the dimensionless groups and the numbers $l^*$, $u^*$ and $\nu^*$ characterizing the oil-water system. For these estimates the values in Table I together with the above estimates of $l$ and $u$ have been used. Table II shows the familiar pore scale result that

$$\text{viscous forces} \ll \text{gravity forces} \ll \text{capillary forces}, \tag{33}$$

and hence capillary forces dominate on the microscopic scale.

From the Stokes equation (25) it follows immediately that for low capillary number floods (Ca $\ll$ 1) the viscous term and the shear term in the boundary condition (20) become negligible. Therefore the velocity field drops out, and the problem reduces to finding the equilibrium capillary pressure field. The equilibrium configuration of the oil-water interface then defines timeindependent pathways for the flow of oil and water. This shows that for flows with microscopic capillary

TABLE II. Overview of definitions and estimates for characteristic microscopic numbers describing oil and water flow under reservoir conditions.

| Quantity | Definition | Estimate |
|---|---|---|
| $\mathrm{Re}_\mathrm{w}$ | $\frac{\rho_\mathrm{w} u l}{\mu_\mathrm{w}}$ | $3.3\times10^{-4}$ |
| $\mathrm{Ca}_\mathrm{w}$ | $\frac{\mu_\mathrm{w} u}{\sigma_\mathrm{ow}}$ | $7.7\times10^{-8}$ |
| $\mathrm{Gr}_\mathrm{w}$ | $\frac{\mu_\mathrm{w} u}{\rho_\mathrm{w} g l^2}$ | $2.8\times10^{-5}$ |
| $\mathrm{Gl}_\mathrm{w}$ | $\frac{\rho_\mathrm{w} g l^2}{\sigma_\mathrm{ow}}$ | $2.8\times10^{-3}$ |
| $\nu_\mathrm{w}^*$ | $\frac{\mu_\mathrm{w}}{\rho_\mathrm{w}}$ | $9\times10^{-7}\,\mathrm{m}^2\,\mathrm{s}^{-1}$ |
| $u_\mathrm{w}^*$ | $\frac{\sigma_\mathrm{ow}}{\mu_\mathrm{w}}$ | $38.9\,\mathrm{m\,s}^{-1}$ |
| $l_\mathrm{w}^*$ | $\sqrt{\frac{\sigma_\mathrm{ow}}{\rho_\mathrm{w} g}}$ | 1.9 cm |
| $t_\mathrm{w}^*$ | $\frac{\mu_\mathrm{w}}{\sqrt{\sigma_\mathrm{ow}\rho_\mathrm{w} g}}$ | $4.9\times10^{-4}$ s |

numbers $\mathrm{Ca} \ll 1$ an improved methodology for a quantitative description of immiscible dislplacement from pore scale physics involves first and foremost an improved methodology for calculating capillary pressures from the pore scale.

## 4. Large Scale Equations of Motion

It is impractical to specify the microstructure of the rock given by the set $G_\mathrm{r}$ for either a laboratory core or a whole reservoir. For this reason most discussions of multiphase flow begin with equations of motion appropriate at larger scales. The analogous procedure for single phase flow involves establishing Darcy's law by homogenization techniques [15] and calculating effective absolute permeabilities using the local generalization of effective medium theory [16–18]. For multiphase flow the generalization of Darcy's law is well known, while the systematic calculation of relative permeabilities from pore scale equations of motion remains an unsolved problem.

The accepted large scale equations of motions involve a generalization of Darcy's law to relative permeabilities including offdiagonal viscous coupling terms.

These equations are believed to describe multiphase flow on the reservoir scale as well as on the laboratory scale, and they may be written as [19–22]

$$\overline{\phi}\frac{\partial \overline{S}_{\mathrm{w}}}{\partial t} = \overline{\nabla}\cdot\overline{\mathbf{v}}_{\mathrm{w}}, \qquad \overline{\phi}\frac{\partial \overline{S}_{\mathrm{o}}}{\partial \overline{t}} = \overline{\nabla}\cdot\overline{\mathbf{v}}_{\mathrm{o}}, \tag{34}$$

$$\overline{\mathbf{v}}_{\mathrm{w}} = -\left[\mathbf{K}^{r}_{\mathrm{ww}}\frac{\mathbf{K}}{\mu_{\mathrm{w}}}(\overline{\nabla}\overline{p}_{\mathrm{w}} - \rho_{\mathrm{w}}g\overline{\nabla}\overline{z}) + \mathbf{K}^{r}_{\mathrm{wo}}\frac{\mathbf{K}}{\mu_{\mathrm{o}}}(\overline{\nabla}\overline{p}_{\mathrm{o}} - \rho_{\mathrm{o}}g\overline{\nabla}\overline{z})\right],$$

$$\overline{\mathbf{v}}_{\mathrm{o}} = -\left[\mathbf{K}^{r}_{\mathrm{ow}}\frac{\mathbf{K}}{\mu_{\mathrm{w}}}(\overline{\nabla}\overline{p}_{\mathrm{w}} - \rho_{\mathrm{w}}g\overline{\nabla}\overline{z}) + \mathbf{K}^{r}_{\mathrm{oo}}\frac{\mathbf{K}}{\mu_{\mathrm{o}}}(\overline{\nabla}\overline{p}_{\mathrm{o}} - \rho_{\mathrm{o}}g\overline{\nabla}\overline{z})\right], \tag{35}$$

$$\overline{S}_{\mathrm{w}} + \overline{S}_{\mathrm{o}} = 1, \tag{36}$$

$$\overline{p}_{\mathrm{o}} - \overline{p}_{\mathrm{w}} = \overline{p}_{c}(\overline{S}_{\mathrm{w}}), \tag{37}$$

where $\bar{A}$ denotes the macroscopic equivalent of the pore scale quantity $A$. In the equations above $\mathbf{K}$ stands for the absolute (single phase flow) permeability tensor, $\mathbf{K}^{r}_{\mathrm{ww}}$ is the relative permeability tensor for water, $\mathbf{K}^{r}_{\mathrm{oo}}$ the oil relative permeability tensor, and $\mathbf{K}^{r}_{\mathrm{wo}}, \mathbf{K}^{r}_{\mathrm{ow}}$ denote the possibly anisotropic coupling terms. The relative permeabilities are matrix valued functions of saturation. The saturations are denoted as $\overline{S}_{\mathrm{w}}, \overline{S}_{\mathrm{o}}$ and they depend on the macroscopic space and time variables $(\overline{\mathbf{x}}, \overline{t})$. The capillary pressure curve $\overline{p}_{c}(\overline{S}_{\mathrm{w}})$ and the relative permeability tensors $\mathbf{K}^{r}_{ij}(\overline{S}_{\mathrm{w}}), i, j = \mathrm{w}, \mathrm{o}$ must be known either from solving the pore scale equations of motion, or from experiment. $\mathbf{K}^{r}_{ij}(\overline{S}_{\mathrm{w}})$ and $\overline{p}_{c}(\overline{S}_{\mathrm{w}})$ are conventionally assumed to be independent of $\overline{\mathbf{v}}$ and $\overline{p}$ and we follow this convention here, although it is conceivable that this is not generally correct.

Eliminating $\overline{\mathbf{v}}$ and choosing $\overline{p}_{\mathrm{w}}(\overline{\mathbf{x}}, \overline{t})$ and $\overline{S}_{\mathrm{w}}(\overline{\mathbf{x}}, \overline{t})$ as the principal unknowns one arrives at the large scale two-phase flow equations

$$\begin{aligned}\overline{\phi}\frac{\partial \overline{S}_{\mathrm{w}}}{\partial \overline{t}} = \overline{\nabla}\cdot\Big\{&\mathbf{K}^{r}_{\mathrm{ww}}(\overline{S}_{\mathrm{w}})\frac{\mathbf{K}}{\mu_{\mathrm{w}}}(\overline{\nabla}\overline{p}_{\mathrm{w}} - \rho_{\mathrm{w}}g\overline{\nabla}\overline{z}) + \\ &+ \mathbf{K}^{r}_{\mathrm{wo}}(\overline{S}_{\mathrm{w}})\frac{\mathbf{K}}{\mu_{\mathrm{o}}}[(\overline{\nabla}\overline{p}_{\mathrm{w}} - \rho_{\mathrm{w}}g\overline{\nabla}\overline{z}) + \\ &+ \overline{\nabla}\overline{p}_{c}(\overline{S}_{\mathrm{w}}) + (\rho_{\mathrm{w}} - \rho_{\mathrm{o}})g\overline{\nabla}\overline{z}]\Big\},\end{aligned} \tag{38}$$

$$\begin{aligned}\overline{\phi}\frac{\partial(1 - \overline{S}_{\mathrm{w}})}{\partial \overline{t}} = \overline{\nabla}\cdot\Big\{&\mathbf{K}^{r}_{\mathrm{ow}}(\overline{S}_{\mathrm{w}})\frac{\mathbf{K}}{\mu_{\mathrm{w}}}(\overline{\nabla}\overline{p}_{\mathrm{w}} - \rho_{\mathrm{w}}g\overline{\nabla}\overline{z}) + \\ &+ \mathbf{K}^{r}_{\mathrm{oo}}(\overline{S}_{\mathrm{w}})\frac{\mathbf{K}}{\mu_{\mathrm{o}}}[(\overline{\nabla}\overline{p}_{\mathrm{w}} - \rho_{\mathrm{w}}g\overline{\nabla}\overline{z}) + \\ &+ \overline{\nabla}\overline{p}_{c}(\overline{S}_{\mathrm{w}}) + (\rho_{\mathrm{w}} - \rho_{\mathrm{o}})g\overline{\nabla}\overline{z}]\Big\}\end{aligned} \tag{39}$$

for these two unknowns. Equations (38) and (39) are coupled nonlinear partial differential equations for the large scale pressure and saturation field of the water phase.

These equations must be complemented with large scale boundary conditions. For core experiments these are typically given by a surface source on one side of the core, a surface sink on the opposite face, and impermeable walls on the other faces. For a reservoir the boundary conditions depend upon the drive configuration and the geological modeling of the reservoir environment, so that Dirichlet as well as von Neumann problems arise in practice.

## 5. Large-Scale Dimensional Analysis

The large-scale equations of motion can be cast in dimensionless form using the definitions

$$\overline{\mathbf{x}} = \overline{l}\,\widehat{\mathbf{x}}, \tag{40}$$

$$\overline{\nabla} = \frac{\widehat{\nabla}}{\overline{l}}, \tag{41}$$

$$\overline{\mathbf{v}} = \overline{u}\,\widehat{\mathbf{v}}, \tag{42}$$

$$\overline{t} = \frac{\overline{l}\,\widehat{t}}{\overline{u}}, \tag{43}$$

$$\overline{p} = \overline{p}_b\,\widehat{p}, \tag{44}$$

where as before $\widehat{A}$ denotes the dimensionless equivalent of the macroscopic quantity $\overline{A}$. The length $\overline{l}$ is now a macroscopic length, and $\overline{u}$ a macroscopic (Seepage or Darcy) velocity. The pressure $\overline{p}_b$ denotes the 'breakthrough' pressure from the capillary pressure curve $\overline{p}_c(\overline{S}_\mathrm{w})$. It is defined as

$$\overline{p}_b = \overline{p}_c(\overline{S}_b), \tag{45}$$

where $\overline{S}_b$ is the breakthrough saturation defined as the solution of the equation

$$\frac{\mathrm{d}^2\,\overline{p}_c(\overline{S}_\mathrm{w})}{\mathrm{d}\,\overline{S}_\mathrm{w}^2} = 0. \tag{46}$$

Thus, the dimensionless pressure is defined in terms of the inflection point $(\overline{p}_b, \overline{S}_b)$ on the capillary pressure curve, and it gives a measure of the macroscopic capillary pressure. Note that $\overline{p}_b$ is process dependent, i.e. it will in general differ between imbibition and drainage. This dependence reflects the importance of microscopic wetting properties and flow mechanisms on the macroscale. Other normalizations could also be employed instead of (45) provided they are based on the capillary pressure curve $\overline{p}_c(\overline{S}_{\mathrm{w}})$ which measures the macroscopic capillary forces.

Our definition (44) differs from the traditional analysis [10–13]. In the traditional analysis the normalized pressure field is defined as

$$\overline{p} = \frac{\mu_{\mathrm{w}}\overline{u}\overline{l}}{k}\widehat{\overline{p}}, \tag{47}$$

which immediately gives rise to three problems. Firstly the permeability is a tensor, and thus a certain nonuniqueness results in anisotropic situations [4]. Secondly equation (47) neglects the importance of microscopic wetting and saturation history dependence. The main problem however is that Equation (47) is not based on macroscopic capillary pressures but on Darcy's law which describes macroscopic viscous pressure effects. On the other hand our normalization is free from these problems and it includes macroscopic capillarity in the same way as the microscopic normalization (15) includes microscopic capillarity.

With the normalizations introduced above the dimensionless form of the macroscopic two-phase flow Equations (38), (39) becomes

$$\begin{aligned}\overline{\phi}\frac{\partial\overline{S}_{\mathrm{w}}}{\partial\widehat{\overline{t}}} = {}& \widehat{\overline{\nabla}}\cdot\Big\{\mathbf{K}^r_{\mathrm{ww}}(\overline{S}_{\mathrm{w}})\left(\overline{\mathbf{Ca}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{p}}_{\mathrm{w}} - \overline{\mathbf{Gr}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{z}}\right) + \\ & + \mathbf{K}^r_{\mathrm{wo}}(\overline{S}_{\mathrm{w}})\frac{\mu_{\mathrm{w}}}{\mu_{\mathrm{o}}}\Big[\left(\overline{\mathbf{Ca}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{p}}_{\mathrm{w}} - \overline{\mathbf{Gr}}_{\mathrm{w}}^{-1}\overline{\nabla}\widehat{\overline{z}}\right) + \\ & + \overline{\mathbf{Ca}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{p}}_c(\overline{S}_{\mathrm{w}}) + \left(1-\frac{\rho_{\mathrm{o}}}{\rho_{\mathrm{w}}}\right)\overline{\mathbf{Gr}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{z}}\Big]\Big\}\end{aligned} \tag{48}$$

$$\begin{aligned}\overline{\phi}\frac{\partial(1-\overline{S}_{\mathrm{w}})}{\partial\widehat{\overline{t}}} = {}& \widehat{\overline{\nabla}}\cdot\Big\{\mathbf{K}^r_{\mathrm{ow}}(\overline{S}_{\mathrm{w}})\left(\overline{\mathbf{Ca}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{p}}_{\mathrm{w}} - \overline{\mathbf{Gr}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{z}}\right) + \\ & + \mathbf{K}^r_{\mathrm{oo}}(\overline{S}_{\mathrm{w}})\frac{\mu_{\mathrm{w}}}{\mu_{\mathrm{o}}}\Big[\left(\overline{\mathbf{Ca}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{p}}_{\mathrm{w}} - \overline{\mathbf{Gr}}_{\mathrm{w}}^{-1}\overline{\nabla}\widehat{\overline{z}}\right) + \\ & + \overline{\mathbf{Ca}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{p}}_c(\overline{S}_{\mathrm{w}}) + \left(1-\frac{\rho_{\mathrm{o}}}{\rho_{\mathrm{w}}}\right)\overline{\mathbf{Gr}}_{\mathrm{w}}^{-1}\,\widehat{\overline{\nabla}}\widehat{\overline{z}}\Big]\Big\}\end{aligned} \tag{49}$$

In these equations the dimensionless tensor

$$\overline{\mathbf{Ca}}_{\mathrm{w}} = \frac{\mu_{\mathrm{w}}\overline{u}\overline{l}}{\overline{p}_b}\mathbf{K}^{-1} = \frac{\text{macroscopic viscous pressure drop}}{\text{macroscopic capillary pressure}} \tag{50}$$

plays the role of a macroscopic or large scale capillary number. Similarly, the tensor

$$\overline{\mathbf{Gr}}_{\mathrm{w}} = \frac{\mu_{\mathrm{w}}\overline{u}}{\rho_{\mathrm{w}}g}\mathbf{K}^{-1} = \frac{\text{macroscopic viscous pressure drop}}{\text{macroscopic gravitational pressure}} \tag{51}$$

corresponds to the macroscopic gravity number.

If the traditional normalization (47) is used instead of our normalization (44), and isotropy is assumed then the same dimensionless equations are obtained with

$$\overline{\mathrm{Ca}}_{\mathrm{w}} = 1, \tag{52}$$

where $\overline{\mathrm{Ca}}_{\mathrm{w}}$ is the macroscopic capillary number. Thus the traditional normalization is equivalent to the assumption that the macroscopic viscous pressure drop always equals the macroscopic capillary pressure. While this assumption is not generally valid, it sometimes is a reasonable approximation as we will illustrate below. First, however, we briefly discuss the consequences of assumption (52) for the measurement of relative permeabilities.

## 6. Measurement of Relative Permeabilites

For simplicity we consider only the isotropic case from now on, i.e. we set $\mathbf{K} = k\mathbf{1}$ where $\mathbf{1}$ is the identity matrix. The tensors $\overline{\mathbf{Ca}}_{\mathrm{w}}$ and $\overline{\mathbf{Gr}}_{\mathrm{w}}$ then become $\overline{\mathbf{Ca}}_{\mathrm{w}} = \overline{\mathrm{Ca}}_{\mathrm{w}}\mathbf{1}$ and $\overline{\mathbf{Gr}}_{\mathrm{w}} = \overline{\mathrm{Gr}}_{\mathrm{w}}\mathbf{1}$ where $\overline{\mathrm{Ca}}_{\mathrm{w}}$ and $\overline{\mathrm{Gr}}_{\mathrm{w}}$ are the macroscopic capillary and gravity numbers.

The unsteady state or displacement method of measuring relative permeabilities consists of monitoring the production history and pressure drop across the sample during a laboratory displacement process [1,23]. The relative permeability is obtained as the solution of an inverse problem. The inverse problem consists in matching the measured production history and pressure drop to the solutions of the multiphase flow Equations (48) and (49) using the Buckley–Leverett approximation.

In the present formulation the Buckley–Leverett approximation comprises several independent assumptions. Firstly it is assumed that gravity effects are absent, which amounts to the assumption

$$\overline{\mathrm{Ca}}_{\mathrm{w}} \ll \overline{\mathrm{Gr}}_{\mathrm{w}}. \tag{53}$$

Secondly, the viscous coupling terms are neglected, i.e.

$$k^{r}_{\mathrm{wo}}\frac{\mu_{\mathrm{w}}}{\mu_{\mathrm{o}}} \ll \overline{\mathrm{Ca}}_{\mathrm{w}} \quad \text{and} \quad k^{r}_{\mathrm{ow}} \ll \overline{\mathrm{Ca}}_{\mathrm{w}}. \tag{54}$$

Finally the resulting equations

$$\overline{\phi}\frac{\partial \overline{S}_{\mathrm{w}}}{\partial \widehat{\overline{t}}} = \widehat{\overline{\boldsymbol{\nabla}}} \cdot \left\{ k^{r}_{\mathrm{ww}}(\overline{S}_{\mathrm{w}}) \frac{\widehat{\overline{\boldsymbol{\nabla}}}\widehat{\overline{p}}_{\mathrm{w}}}{\overline{\mathrm{Ca}}_{\mathrm{w}}} \right\}, \tag{55}$$

$$\overline{\phi}\frac{\partial (1-\overline{S}_{\mathrm{w}})}{\partial \widehat{\overline{t}}} = \widehat{\overline{\boldsymbol{\nabla}}} \cdot \left\{ k^{r}_{\mathrm{oo}}(\overline{S}_{\mathrm{w}}) \frac{\mu_{\mathrm{w}}}{\mu_{\mathrm{o}}} \left[ \frac{\widehat{\overline{\boldsymbol{\nabla}}}\,\widehat{\overline{p}}_{\mathrm{w}}}{\overline{\mathrm{Ca}}_{\mathrm{w}}} + \frac{\widehat{\overline{\boldsymbol{\nabla}}}\,\widehat{\overline{p}}_{c}(\overline{S}_{\mathrm{w}})}{\overline{\mathrm{Ca}}_{\mathrm{w}}} \right] \right\} \tag{56}$$

are further simplified by assuming that the term involving $\widehat{\overline{p}}_c(\overline{S}_{\mathrm{w}})$ in Equation (56) may be neglected.

Combining (53) with the traditional normalization (52) yields the consistency condition

$$\overline{\mathrm{Gr}}_{\mathrm{w}} \gg 1 \tag{57}$$

for the application of Buckley–Leverett theory in the determination of relative permeabilities. It is now clear from the definition of the macroscopic gravity number, see Equation (51), that the consistent use of Buckley–Leverett theory for the unsteady state measurement of relative permeabilities depends strongly on the flow regime. This is valid whether or not the capillary pressure term $\widehat{\overline{p}}_c(\overline{S}_{\mathrm{w}})$ in (56) is neglected.

## 7. Pore Scale to Large Scale Comparison

The comparison between the macroscopic and the microscopic dimensional analysis is carried out by relating the microscopic and macroscopic velocities and length scales. The macroscopic velocity is taken to be a Darcy velocity defined as

$$\overline{u} = \overline{\phi} u, \tag{58}$$

where $\overline{\phi}$ is the bulk porosity and $u$ denotes the average microscopic flow velocity introduced in the microscopic analysis (Equation (12)). The length scales $l$ and $\overline{l}$ are, of course, identical ($\overline{l} = l$).

Using these relations between microscopic and macroscopic length and time scales together with the assumption of isotropy, we obtain

$$\overline{\mathrm{Ca}}_{\mathrm{w}} = \frac{\mu_{\mathrm{w}}\overline{\phi} u l}{k\overline{p}_b} = \frac{ul}{\overline{\nu^*{\mathrm{w}}}} = \frac{\sigma_{\mathrm{ow}}\overline{\phi} l}{k\overline{p}_b}\mathrm{Ca}_{\mathrm{w}}, \tag{59}$$

as the relationship between microscopic and macroscopic capillary numbers. Similarly, one obtains

$$\overline{\mathrm{Gr}}_{\mathrm{w}} = \frac{\mu_{\mathrm{w}}\overline{\phi} u}{\rho_{\mathrm{w}} g k} = \frac{u}{\overline{u^*}_{\mathrm{w}}} = \frac{\overline{\phi} l^2}{k}\mathrm{Gr}_{\mathrm{w}} \tag{60}$$

for the gravity numbers. Taking the quotient gives

$$\overline{\mathrm{Gl}}_{\mathrm{w}} = \frac{\overline{\mathrm{Ca}}_{\mathrm{w}}}{\overline{\mathrm{Gr}}_{\mathrm{w}}} = \frac{\rho_{\mathrm{w}} g l}{\overline{p}_b} = \frac{l}{\overline{l^*}_{\mathrm{w}}} = \frac{\sigma_{\mathrm{ow}}}{l\overline{p}_b}\mathrm{Gl}_{\mathrm{w}} \tag{61}$$

for the macroscopic gravillary number. Note that the ratio $\sigma_{\mathrm{ow}}/(l\overline{p}_b)$ is the ratio of the microscopic to the macroscopic capillary pressure. The characteristic numbers

$$\overline{\nu^*}_{\mathrm{w}} = \frac{k\overline{p}_b}{\overline{\phi}\mu_{\mathrm{w}}}, \tag{62}$$

$$\overline{u^*}_{\mathrm{w}} = \frac{\rho_{\mathrm{w}} g k}{\mu_{\mathrm{w}} \overline{\phi}}, \tag{63}$$

$$\overline{l^*}_{\mathrm{w}} = \frac{\overline{p}_b}{\rho_{\mathrm{w}} g} \tag{64}$$

are the macroscopic counterparts of the microscopic numbers defined in Equations (22), (28) and (30).

An interesting way of rewriting these relationships arises from interpreting the permeability as an effective microscopic cross-sectional area of flow, combined with the Leverett $J$-function. More precisely, let

$$\Lambda = \sqrt{\frac{k}{\overline{\phi}}} \tag{65}$$

denote a microscopic length which is characteristic for the pore space transport properties. Then Equations (59), (60) and (61) may be rewritten as

$$\overline{\mathrm{Ca}}_{\mathrm{w}} = \frac{\overline{l}}{\Lambda} \frac{\mathrm{Ca}_{\mathrm{w}}}{J(\overline{S}_b) \cos\theta}, \tag{66}$$

$$\overline{\mathrm{Gr}}_{\mathrm{w}} = \frac{\overline{l}^2}{\Lambda^2} \mathrm{Gr}_{\mathrm{w}}, \tag{67}$$

$$\overline{\mathrm{Gl}}_{\mathrm{w}} = \frac{\Lambda}{\overline{l}} \frac{\mathrm{Gl}_{\mathrm{w}}}{J(\overline{S}_b) \cos\theta}, \tag{68}$$

where $J(\overline{S}_b) = (\overline{p}_b \sqrt{k/\overline{\phi}})/(\sigma_{\mathrm{ow}} \cos\theta)$ is the value of the Leverett-$J$-function [3,1] at the saturation corresponding to breakthrough, and $\theta$ is the wetting angle.

Note that the capillary number scales as $(\overline{l}/\Lambda)$ while the gravity number scales as $(\overline{l}/\Lambda)^2$. Inserting (67) and (68) into (53) this implies that the Buckley–Leverett approximation (53) becomes invalid whenever $\overline{l} > \Lambda\, \mathrm{Gl}_{\mathrm{w}}/(J(\overline{S}_b) \cos\theta)$.

The significance of some of the dimensionless ratios as absolute measures of length, specific action or velocity is reversed in the transition from the microscale to the macroscale. The macroscopic capillary number in this sense is analogous to the Reynolds number. The macroscopic gravity number corresponds to the microscopic capillary number. On the other hand the gravillary number remains unchanged and it gives an absolute measure of length microscopically as well as macroscopically.

## 8. Large-Scale Estimates

In the present section we present order of magnitude estimates for the relative importance of capillary, viscous and gravity effects at different scales in representative categories of porous media. These estimates illustrate the usefulness of the

TABLE III. Representative values for porosity, permeability and breakthrough capillary pressure in unconsolidated sand, sandstone and low permeability limestone.

| Quantity | Sand | Sandstone | Limestone |
|---|---|---|---|
| $\overline{\phi}$ | 0.36 | 0.22 | 0.20 |
| $k$ | 10 000 mD | 400 mD | 3 mD |
| $\overline{p}_b$ | 2000 Pa | $10^4$ Pa | $10^5$ Pa |

newly introduced macroscopic dimensionless ratios for obtaining rough quantitative estimates from simple calculations.

Three types of porous media are considered: high permeability unconsolidated sand, intermediate permeability sandstone and low permeability limestone. Representative value for $\overline{\phi}$, $k$ and $\overline{p}_b$ are shown in Table III.

To estimate the dimensionless numbers the same microscopic velocity $u \approx 3 \times 10^{-6}$ ms$^{-1}$ as for the microscopic estimates will be used. The length scale $l$, however, differs between a laboratory displacement and a reservoir process. We use $l_{\text{lab}} \approx 0.1$ m and $l_{\text{res}} \approx 100$ m as representative values. Combining these values with those in Table I and II yields the results shown in Table IV.

The first row in Table IV can be used to check the consistency of the Buckley–Leverett approximation with the traditional normalization. We see that the consistency condition (Equation (57)) is violated for unconsolidated sand and sandstones. Such a conclusion, of course, assumes that the values given in Table III are indeed representative for these media.

The fifth row in Table IV gives the ratio between macroscopic and microscopic capillary numbers which according to Equation (59) is length scale dependent. The last row in Table IV compares this ratio to the typical critical capillary number $\text{Ca}_c$ reported for laboratory desaturation curves in the different porous media. Using the representative $\text{Ca}_c$ values cited in the introduction we find that the corresponding critical macroscopic capillary number is close to 1. This indicates that the macroscopic capillary number is indeed an appropriate measure of the relative strength of viscous and capillary forces.

As a consequence, we expect differences between residual oil saturation $S_{\text{o}r}$ in laboratory and reservoir floods. Given a laboratory measured capillary desaturation curve $S_{\text{o}r}(\text{Ca}_{\text{w}})$ as a function of the microscopic capillary number $\text{Ca}_{\text{w}}$ the foregoing analysis predicts that the residual oil saturation in a reservoir flood can be estimated from the laboratory curve as $S_{\text{o}r}(\text{Ca}_c \times \overline{\text{Ca}}_{\text{w}})$. For $\overline{\text{Ca}}_{\text{w}} > 1$ the $S_{\text{o}r}$ value based on macroscopic capillary numbers will in general be lower than the value $S_{\text{o}r}(\text{Ca}_{\text{w}})$ expected from using microscopic capillary numbers. Such differences have been frequently observed, and Morrow [24] has recently raised the question why field recoveries are sometimes so much higher than those observed in the laboratory. Our present analysis suggests a possible answer to this question.

TABLE IV. Definition and representative values for macroscopic dimensionless numbers in different porous media on laboratory ($l_{\mathrm{lab}} \approx 0.1$ m) and reservoir scale ($l_{\mathrm{res}} \approx 100$ m) under uniform flow conditions ($u \approx 3 \times 10^{-6}$ ms$^{-1}$).

| Quantity | Definition | Unconsolidated Sand | | Sandstone | | Limestone | |
|---|---|---|---|---|---|---|---|
| | | Laboratory | Reservoir | Laboratory | Reservoir | Laboratory | Reservoir |
| $\overline{\mathrm{Gr}}_{\mathrm{w}}$ | $\dfrac{\mu_{\mathrm{w}}\overline{\phi}u}{\rho_{\mathrm{w}}gk}$ | 0.01 | 0.01 | 0.13 | 0.13 | 18.6 | 18.6 |
| $\overline{\mathrm{Ca}}_{\mathrm{w}}$ | $\dfrac{\mu_{\mathrm{w}}\overline{\phi}ul}{k\overline{p}_b}$ | 0.005 | 4.9 | 0.015 | 15.0 | 0.19 | 187.5 |
| $\overline{\mathrm{Gl}}_{\mathrm{w}}$ | $\dfrac{\rho_{\mathrm{w}}gl}{\overline{p}_b}$ | 0.5 | 492 | 0.1 | 115 | 0.01 | 10 |
| $\Lambda$ | $\sqrt{\dfrac{k}{\overline{\phi}}}$ | 5.2 $\mu$m | 5.2 $\mu$m | 1.3 $\mu$m | 1.3 $\mu$m | 0.1 $\mu$m | 0.1 $\mu$m |
| $\mathrm{Ca}_{\mathrm{w}}/\overline{\mathrm{Ca}}_{\mathrm{w}}$ | $\dfrac{\Lambda}{l}J(\overline{S}_b)\cos\theta$ | $1.5 \times 10^{-4}$ | $1.5 \times 10^{-7}$ | $4.8 \times 10^{-6}$ | $4.8 \times 10^{-9}$ | $2.8 \times 10^{-7}$ | $2.8 \times 10^{-10}$ |
| $\dfrac{\mathrm{Ca}_{\mathrm{c}}\overline{\mathrm{Ca}}_{\mathrm{w}}}{\mathrm{Ca}_{\mathrm{w}}}$ | $\dfrac{\Lambda}{l}J(\overline{S}_b)\cos\theta$ | 0.67 | – – – | 0.63 | – – – | 0.71 | – – – |

TABLE V. Relative importance of viscous ($V$), gravity ($G$) and capillary ($C$) forces in unconsolidated sand, sandstone and limestone. The notation $A \ll B$ (with $A, B \in \{V, G, C\}$) indicates that $A/B < 10^{-2}$ while $A < B$ means $10^{-2} < A/B < 0.5$ and $A \approx B$ stands for $0.5 < A/B < 2$.

| | | | Sand | Sandstone | Limestone |
|---|---|---|---|---|---|
| Pore scale | | | $V \ll G \ll C$ | | |
| Large scale | Traditional analysis | | $V = C \ll G$ | $V = C < G$ | $G < V = C$ |
| | This work | Laboratory scale | $V \ll G \approx C$ | $V < G < C$ | $G < V < C$ |
| | | Field scale | $C < V \ll G$ | $C < V < G$ | $C < G < V$ |

The values of the dimensionless numbers in Table IV allow an assessment of the relative importance of the different forces for a displacement. To illustrate this consider the values $\overline{\mathrm{Gr}}_{\mathrm{w}} = 0.01, \overline{\mathrm{Ca}}_{\mathrm{w}} = 0.005$ and $\overline{\mathrm{Gl}}_{\mathrm{w}} = 0.5$ for unconsolidated sand on the laboratory scale. A moments reflection shows that this implies $V \ll G \approx C$ where $V$ stands for macroscopic viscous forces, $C$ for macroscopic capillary forces, and $G$ for gravity forces. The notation $A \ll B$ indicates that $A/B < 10^{-2}$ while $A < B$ means $10^{-2} < A/B < 0.5$ and $A \approx B$ stands for $0.5 < A/B < 2$. Repeating this for all cases in Table IV yields the results shown in Table V. In Table V we have also included the results from the microscopic dimensional analysis, as well as the results one would obtain from a traditional macroscopic dimensional analysis which assumes $\overline{\mathrm{Ca}} = 1$ (see Equation (52)).

Obviously, the relative importance of the different forces may change depending on the type of medium, the characteristic fluid velocities and the length scale. Perhaps this explains part of the general difficulty of scaling up from the laboratory to the reservoir scale.

## 9. Applications

The characteristic macroscopic velocities, length scales and kinematic viscosities defined respectively in Equations (62), (63) and (64) are intrinsic physical characteristic of the porous media and the fluid displacement processes. These characteristics can be useful in applications such as estimating the width of a gravitational segregation front, the energy input required to mobilize residual oil or gravitational relaxation times.

The microscopic gravillary number $\overline{\mathrm{Gl}}_{\mathrm{w}}$ defines an intrinsic length scale $\overline{l^*}_{\mathrm{w}}$ (see Equation (61)). Because $\overline{\mathrm{Gl}}_{\mathrm{w}}$ gives the ratio of the gravity to the capillary forces the length $\overline{l^*}_{\mathrm{w}}$ directly gives the width of a gravitational segregation front when the fluids are at rest and in gravitational equilibrium, i.e. when viscous forces are negligible or absent. Using the same estimates for $\overline{\phi}$, $k$ and $\overline{p}_b$ as those used for Table IV we obtain a characteristic front width of 20 cm for unconsolidated sand, 1 m for sandstone, and roughly 10 m for a low permeability limestone.

TABLE VI. Characteristic mascroscopic energies, velocities, length scales, time scales and volumetric flow rates for oil-water flow under reservoir conditions in unconsolidated sand, sandstone and low permeability limestone.

| Quantity | Sand | Sandstone | Limestone |
|---|---|---|---|
| $\overline{\nu^*}_{\mathrm{w}}$ | $6.1\times10^{-5}\ \mathrm{m^2s^{-1}}$ | $2.0\times10^{-5}\ \mathrm{m^2s^{-1}}$ | $1.6\times10^{-6}\ \mathrm{m^2s^{-1}}$ |
| $\overline{u^*}_{\mathrm{w}}$ | $2.99\times10^{-4}\ \mathrm{ms^{-1}}$ | $2.17\times10^{-5}\ \mathrm{ms^{-1}}$ | $1.61\times10^{-7}\ \mathrm{ms^{-1}}$ |
| $\overline{l^*}_{\mathrm{w}}$ | 0.2 m | 1.02 m | 10.2 m |
| $\overline{t^*}_{\mathrm{w}}$ | 669s | $4.7\times10^{4}$ s | $6.36\times10^{7}$ s |
| $\overline{Q^*}$ | $1.22\times10^{-6}\ \mathrm{m^3s^{-1}}$ | $2.04\times10^{-5}\ \mathrm{m^3s^{-1}}$ | $1.63\times10^{-5}\ \mathrm{m^3s^{-1}}$ |

Similarly, the macroscopic capillary number defines an intrinsic specific action (or energy input) $\overline{\nu^*}_{\mathrm{w}}$ via Equation (59) which is the energy input required to mobilize residual oil if gravity forces may be considered negligible or absent. Representatives estimates are given in Table VI.

The gravitational relaxation time is the time needed to return to gravitational equilibrium after its disturbance. This may be defined from the balance of gravitational forces versus the combined effect of viscous and capillary forces. Analogously to Equation (31) for the microscopic case we now have the dimensionless ratio

$$\frac{\overline{\mathrm{Gl}}_{\mathrm{w}}}{\overline{\mathrm{Gr}}_{\mathrm{w}}} = \frac{(\text{macr. gravitational pressure})^2}{(\text{macr. capillary pressure})\times(\text{macr. viscous pressure drop})}$$

$$= \frac{\overline{\mathrm{Ca}}_{\mathrm{w}}}{\overline{\mathrm{Gr}}^2_{\mathrm{w}}} = \frac{\rho^2_{\mathrm{w}}g^2kl}{\mu_{\mathrm{w}}\phi\overline{p}_b u} = \frac{t}{\overline{t^*}_{\mathrm{w}}} \tag{69}$$

which defines the gravitational relaxation time $\overline{t^*}_{\mathrm{w}}$ as

$$\overline{t^*}_{\mathrm{w}} = \frac{\overline{l^*}_{\mathrm{w}}}{\overline{u^*}_{\mathrm{w}}} = \frac{\mu_{\mathrm{w}}\phi\overline{p}_b}{\rho^2_{\mathrm{w}}g^2k}. \tag{70}$$

Estimated values are given in Table VI. They correspond to gravitational relaxation times of roughly 11 min for unconsolidated sand, 13 h for a sandstone and 736 days for a low permeability limestone.

Another interesting intrinsic number arises from comparing the strength of macroscopic capillary forces versus the combined effect of viscous and gravity forces

$$(\overline{\mathrm{Gl}}_{\mathrm{w}}\overline{\mathrm{Ca}}_{\mathrm{w}})^{-1} = \frac{(\text{macr. capillary pressure})^2}{(\text{macr. grav. pressure})\times(\text{macr. viscous pressure drop})}$$

$$= \frac{\overline{\mathrm{Gr}}_{\mathrm{w}}}{\overline{\mathrm{Ca}}^2_{\mathrm{w}}} = \frac{k\overline{p}^2_b}{\phi\mu_{\mathrm{w}}\rho_{\mathrm{w}}gul^2} = \frac{\overline{Q^*}_{\mathrm{w}}}{Q}, \tag{71}$$

where $Q$ denotes the volumetric flow rate. Thus $\overline{Q^*}_{\mathrm{w}}$ defined as

$$\overline{Q^*}_{\mathrm{w}} = \overline{l^*}^2_{\mathrm{w}}\overline{u^*}_{\mathrm{w}} = \frac{k\overline{p}_b^2}{\overline{\phi}\mu_{\mathrm{w}}\rho_{\mathrm{w}}g} \tag{72}$$

is an intrinsic system specific characteristic flow rate.

The estimates for $\overline{\nu^*}_{\mathrm{w}}, \overline{u^*}_{\mathrm{w}}, \overline{l^*}_{\mathrm{w}}, \overline{t^*}_{\mathrm{w}}$ and $\overline{Q^*}_{\mathrm{w}}$ are summarized in Table VI.

## 10. Conclusions

We have revisited the traditional dimensional analysis of multiphase flow equations in porous media appropriate both on the microscale and the macroscale. We propose to normalize the pressure field in a way which differs from the tradtional normalization. This gives rise to novel macroscopic dimensionless numbers. The most important of these numbers is the macroscopic capillary number $\overline{\mathrm{Ca}}$ which differs from the traditional microscopic capillary number Ca in that it depends on length scale and the breakthrough capillary pressure $\overline{p}_b$.

The traditional normalization is found to correspond to the tacit assumption that viscous and capillary forces are of equal magnitude.

With the new macroscopic capillary number $\overline{\mathrm{Ca}}$ the breakpoint $\mathrm{Ca}_c$ in capillary desaturation curves seems to occur at $\overline{\mathrm{Ca}} \approx 1$ for all types of porous media.

Representatives estimates of $\overline{\mathrm{Ca}}$ for unconsolidated sand, sandstones and limestones suggest that the residual oil saturation after a field flood will in general differ from that after a laboratory flood performed under the same conditions.

Order of a magnitude estimates of gravitational relaxation times and segregation front widths for different media appear to be consistent with experiment.

## Acknowledgements

We acknowledge Den norske stats oljeselskap a.s. (Statoil) for granting permission to publish this paper. The authors thank all members of the LoU/RES group in Trondheim for stimulating and useful discussion, and K. S. Årland and J. Alvestad for critically reading the manuscript. One of us (R. H.) is grateful to Statoil A/S for its hospitality and support during 1993 when this work was carried out.

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