ASYMPTOTIC EXPANSIONS FOR THE PROBABILITIES OF LARGE DEVIATIONS. NORMAL APPROXIMATION. III

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1. INTRODUCTION

Many classical problems in the theory of probabilities and mathematical statistics reduce to the study of the asymptotic behavior of the probability distribution, depending on the infinite growth of the parameter. Therefore, henceforward we set $F\{A\} = F_\Delta\{A\}$; in addition $F_\Delta\{A\}$ converges weakly to an unbounded divisible law G as $\Delta \to \infty$. Here A is a Borel set.

We shall look for an approach to the general problem of constructing the asymptotic Kramer-type expansion for $F_\Delta\{A\}$, by means of two important special cases, when the limiting laws are normal and Poisson. Following Kramer, we introduce the conjugate random variable ξ_h with distribution $F_h\{dx\}$ (see [4, Part II]). From [4, Part II] we have the formal equation

$$F\{A\}=R(h)\Big[\int\limits_A e^{-hx}Q\{dx\}+\int\limits_A e^{-hx}(F_h-Q)\{dx\}\Big], \tag{1}$$

connecting the distributions $F_\Delta\{dx\}$ and $F_h\{dx\}$ (see [6, Part II]). Henceforward, instead of $F_\Delta\{A\}$ we shall use the notation $F\{A\}$.

The conditions we adopt below ensure that the integrals in (1) converge. We first require that

$$0<R(h)=\int\limits_{-\infty}^{\infty} e^{hx}F\{dx\}<\infty \tag{K_{h_0}}$$

for $|h| < h_0$. However, the convergence of the integral R(h) is not a sufficient condition for the construction of asymptotic Kramer-type formulas for $F_\Delta\{dx\}$ as $\Delta \to \infty$.

In fact, for small values of the parameter λ, the Poisson distribution and the normal distribution with parameters (m, σ^2) are essentially different, although they both satisfy condition (K_{h_0}), i.e.,

$$R_N(h)=\frac{1}{\sigma\sqrt{2\pi}}\int\limits_{-\infty}^{\infty} e^{-hu-\frac{1}{2}\frac{(u-m)^2}{\sigma^2}}du=e^{mh+\frac{\sigma^2h^2}{2}}$$

and

$$R_P(h)=\sum_{k=0}^{\infty} e^{hk}\frac{\lambda^k e^{-\lambda}}{k!}=e^{\lambda(e^h-1)}.$$

We shall formulate conditions under which we shall study $F\{A\} = P\{\xi\in A\}$.

In both cases, when G is either the Poisson or the normal law, we shall assume that ξ has all finite moments, i.e., $M|\xi|^k < \infty$ for $k = 2, 3, \ldots$, and has positive dispersion σ^2.

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In the normal approximation we shall require that the semivariants $\varkappa_k$ of the random variable ξ satisfy the condition of V. Statulyavichus

$$|\varkappa_k| \leqslant \frac{H(\Delta)\, k!\, \sigma^k}{\Delta^{k-2}}, \qquad k=3,\ 4,\ \ldots. \tag{S}$$

Here $H(\Delta)$ is some positive and bounded function for $0 < \Delta < \infty$.

If the limit law G is the Poisson law, we shall suppose that the factorial semivariants γ_k, $k = 1, 2, \ldots$, of the random variable ξ satisfy the inequality

$$|\gamma_k| \leqslant \frac{\pi(\Delta)\, k!\, \gamma_1^k}{\Delta^{k-1}}, \qquad k=2,\ 3,\ \ldots,$$

and $\gamma_1 > 0$. Here $\pi(\Delta)$ is some positive and bounded function for $0 < \Delta < \infty$. We shall define the functions $H(\Delta)$ and $\pi(\Delta)$ below in more detail.

In (1) h is any number in the interval $(-h_0, h_0)$. In the second part (in the "smoothing inequalities") we chose $h = h(a)$ to be the solution of the equation

$$a = \frac{d}{dh} \ln R(h), \tag{2}$$

where $a = \inf\{y : y \in A\}$ for $A \in (0, \infty)$ and $a = \sup\{y : y \in A\}$ for $A \in (-\infty, 0)$.

Further study showed that h could be defined differently; viz., in the normal approximation

$$h = h_N = \frac{a-m}{\sigma^2},$$

where $m = M\xi$, and in the approximation $F\{A\}$ to the Poisson distribution, $h = h_P$ is defined by the equation

$$\gamma_1 e^{h_P} = a,$$

i.e., h_N is the solution of Eq. (2), when we substitute the integral $R_N(h)$ for $R(h)$, and h_P is the solution of (2) when the sum $R_P(h)$ replaces $R(h)$.

Such a choice of h simplifies many calculations; e.g., in the normal approximation we obtain the Kramer series $\lambda(t)$ from the equation

$$\ln R(h_N) - mh_N - \frac{1}{2}\left(\frac{a-m}{\sigma}\right)^2 - \frac{1}{2}\left(\frac{a\sigma + m - m(h_N)}{\sigma(h_N)}\right)^2 = \frac{1}{\Delta}\left(\frac{a-m}{\sigma}\right)^3 \lambda\left(\frac{a-m}{\Delta\sigma}\right),$$

but we require a more precise estimate of the integral

$$\int_A e^{-h_N x} (F_{h_N} - Q)\{dx\}.$$

Here $m(h) = (d/dh)\ln R(h)$ and $\sigma^2(h) = (d^2/dh^2)\ln R(h)$.

So in the normal approximation we choose $h = h(a)$ to be the solution of Eq. (2), and in the Poisson approximation $h = h_P$ is the solution of the equation $\gamma_1 e^h = a$.

Furthermore, the question arises as to a suitable selection of a generalized measure $Q\{A\}$ in (1). It follows from the lemmas of Part II that we must choose $Q\{A\}$ such that the integral

$$\int_{-\infty}^{\infty} g(x)(F_h - Q)\{dx\}$$

for Borel functions $g(x)$, belonging to some class, is small. For $Q\{A\}$ we may choose asymptotic expansions of a classical type, of the conjugate distribution $F_h\{A\}$. For example, the asymptotic expansions of Chebyshev and Kramer [6, p. 173], V. P. Zolotorev [12], and Grigelionis and Franken [13, 14] can be used.

2. THE FORMAL CONSTRUCTION OF EXPANSIONS. NORMAL APPROXIMATION

We continue our study of the asymptotic expansion of $F\{A\}$ using the characteristics of the normal law, started in the first part of this work. It remains to consider the cases when $F\{A\}$ is a latticed probability distribution, and when A belongs to some class of Borel sets.

Much is known about approximation to step probability distributions (see [15, 16]). We shall take the following point of view: we approximate the step function $F_h\{dx\}$, with jumps at the points $c + d\nu$, where $d > 0$ and $\nu = 0, \pm 1, \pm 2, \ldots$, by sums of the form

$$\sum_{c+d\nu \in A} q(c+d\nu).$$

To some extent the multidimensional limit theorems of [17] justify this approach.

Suppose we have the formal Chebyshev–Kramer asymptotic expansions:

If $F\{A\}$ is a nonlatticed distribution,

$$F_h\{A\} = \frac{1}{\sigma(h)} \sum_{j=0}^{\infty} \frac{1}{\Delta^j} \int_A q_{jh}\left(\frac{y-m(h)}{\sigma(h)}\right) \varphi\left(\frac{y-m(h)}{\sigma(h)}\right) dy. \tag{3}$$

If, however, ξ takes values in the arithmetic progression $\{c + d\nu\}$, where d is the maximal step in the distribution, then

$$F_h\{A\} = \frac{d}{\sigma(h)} \sum_{j=0}^{\infty} \frac{1}{\Delta^j} \sum_{c+d\nu \in A} q_{jh}\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right) \varphi\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right). \tag{3'}$$

Here $m(h) = M\xi_h$, $\sigma^2(h) = D\xi_h$, and

$$q_{jh}(x) = \sum{}^{*} H_{j+2s}(x) \prod_{m=1}^{j} \frac{1}{k_m!} \left(\frac{\varkappa_{m+2}(h)\Delta^m}{(m+2)!\,\sigma^{m+2}(h)}\right)^{k_m} \tag{4}$$

for $j = 1, 2, \ldots$, $q_{0h} = 1$, $\varphi(x) = (1/\sqrt{2\pi})e^{-x^2/2}$, $H_m(x)$ is the Chebyshev–Hermite polynomial, and Σ^* is the sum defined in Part I of this work.

Consequently, it is approporate to choose

$$Q\{A\} = Q_{sh}\{A\} = \begin{cases} \dfrac{1}{\sigma(h)} \displaystyle\sum_{j=0}^{s} \frac{1}{\Delta^j} \int_A q_{jh}\left(\frac{y-m(h)}{\sigma(h)}\right) \varphi\left(\frac{y-m(h)}{\sigma(h)}\right) dy & \text{in the nonlatticed case,} \\ \dfrac{d}{\sigma(h)} \displaystyle\sum_{j=0}^{s} \frac{1}{\Delta^j} \sum_{c+d\nu \in A} q_{jh}\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right) \varphi\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right) & \text{in the latticed case.} \end{cases} \tag{5}$$

The generalized measure $Q_{sh}\{A\}$ depends on the auxiliary parameter h, i.e., on the solution $h = h[(a-m)/\sigma\Delta]$ of Eq. (2).

If condition (S) is satisfied,

$$a - m = \sum_{l=2}^{\infty} \frac{\varkappa_l h^{l-1}}{(l-1)!} \tag{6}$$

and for $|(a-m)/\sigma| < \bar{\delta}_H \Delta$

$$h = h\left(\frac{a-m}{\sigma\Delta}\right) = \sum_{l=1}^{\infty} a_l \left(\frac{a-m}{\sigma\Delta}\right)^{l-1}, \tag{7}$$

where a_l, $l = 1, 2, \ldots$, are defined in Part I by formula (20). The series (7) converges for $|(a - m)/\sigma| < \bar{\delta}_H \Delta$, where $\bar{\delta}_H$ is as in Lemma 1.

Using (7), we expand the semivariants $\varkappa_m(h)$, $m = 1, 2, \ldots$, in a series in $(a - m)/\sigma\Delta$:

$$\varkappa_\nu(h) = \varkappa_\nu + \sum_{l=1}^{\infty} p_{l\nu} \left(\frac{a-m}{\sigma\Delta}\right)^l, \qquad \nu = 1, 2, \ldots, \tag{8}$$

where

$$p_{l\nu} = \sum\nolimits^* \varkappa_{\nu+r} \prod_{j=1}^{l} \frac{a_j^{k_j}}{k_j!}.$$

Consequently,

$$\left(\frac{y-m(h)}{\sigma^2(h)}\right)^2 = \left(\frac{y-m}{\sigma}\right)^2 + \left(\frac{y-m}{\sigma}\right)^2 \sum_{r=1}^{\infty} c_r \left(\frac{a-m}{\sigma\Delta}\right)^r - 2\,\frac{y-m}{\sigma} \sum_{r=0}^{\infty} c_r \left(\frac{a-m}{\sigma\Delta}\right)^{r+1} + \sum_{r=0}^{\infty} c_r \left(\frac{a-m}{\sigma\Delta}\right)^{r+2} \tag{9}$$

and

$$h(y-a) = -\left(\frac{a-m}{\sigma}\right)^2 + \frac{(y-m)(a-m)}{\sigma^2} + (y-a) \sum_{l=2}^{\infty} a_l \left(\frac{a-m}{\sigma\Delta}\right)^l. \tag{10}$$

Here

$$c_r = \sum\nolimits^* (-1)^l\, l! \prod_{m=1}^{r} \frac{p_{m_2}^{k_m}}{k_m!}, \qquad l = k_1 + k_2 + \ldots + k_r.$$

Moreover,

$$R(h) \exp\left\{-ah - h(y-a) - \frac{1}{2}\left(\frac{y-m(h)}{\sigma(h)}\right)^2\right\} =$$

$$= \exp\left\{\frac{1}{\Delta}\left(\frac{a-m}{\sigma}\right)^3 \lambda\left(\frac{a-m}{\sigma\Delta}\right) - \frac{1}{2}\left(\frac{y-m}{\sigma}\right)^2\right\} \exp\left\{-\frac{1}{2}\left(\frac{y-m}{\sigma}\right)^2 \sum_{r=1}^{\infty} c_r \left(\frac{a-m}{\sigma\Delta}\right)^r\right\} \times$$

$$\times \exp\left\{\frac{y-m}{\sigma} \sum_{r=1}^{\infty} c_r \left(\frac{a-m}{\sigma\Delta}\right)^{r+1}\right\} \exp\left\{-\frac{1}{2} \sum_{r=1}^{\infty} c_r \left(\frac{a-m}{\sigma\Delta}\right)^{r+2}\right\}. \tag{11}$$

Since

$$\exp\left\{-\frac{1}{2}\left(\frac{y-m}{\sigma}\right)^2 \sum_{r=1}^{\infty} c_r \left(\frac{a-m}{\sigma\Delta}\right)^r\right\} = 1 + \sum_{j=1}^{\infty} \left(\frac{a-m}{\sigma\Delta}\right)^j \sum\nolimits^* \prod_{l=1}^{j} \frac{1}{k_l!} \left\{-\frac{c_l}{2}\left(\frac{y-m}{\sigma}\right)^2\right\}^{k_l},$$

$$\exp\left\{\sum_{r=1}^{\infty} c_r \left(\frac{y-m}{\sigma}\right)\left(\frac{a-m}{\Delta\sigma}\right)^{r+1}\right\} = 1 + \sum_{j=1}^{\infty} \left(\frac{a-m}{\sigma\Delta}\right)^j \sum\nolimits^* \prod_{l=1}^{j} \frac{1}{k_l!} \left\{\bar{c}_l \left(\frac{y-m}{\sigma}\right)\right\}^{k_l},$$

where $\bar{c}_1 = c_0 = 0$ and $\bar{c}_l = c_{l-1}$ for $l = 2, 3, \ldots$, and

$$\exp\left\{\sum_{r=1}^{\infty} c_r \left(-\frac{1}{2}\right)\left(\frac{a-m}{\sigma\Delta}\right)^{r+2}\right\} = 1 + \sum_{j=1}^{\infty} \sum\nolimits^* \prod_{l=1}^{j} \frac{1}{k_l!} \left\{-\frac{\bar{\bar{c}}_l}{2}\right\}^{k_l} \left(\frac{a-m}{\sigma\Delta}\right)^j,$$

where $\bar{\bar{c}}_l = 0$ for $l = 1, 2$ and $\bar{\bar{c}}_l = c_{l-2}$ for $l = 3, 4, \ldots$, then

$$R(h)\exp\left\{-ah-h(y-a)-\frac{1}{2}\left(\frac{y-m(h)}{\sigma(h)}\right)^2\right\} = e^{\frac{1}{\Delta}\left(\frac{a-m}{\sigma}\right)^3\lambda\left(\frac{a-m}{\sigma\Delta}\right)-\frac{1}{2}\left(\frac{y-m}{\sigma}\right)^2}\left(1+\sum_{r=1}^{\infty} b_r\left(\frac{y-m}{\sigma}\right)\left(\frac{a-m}{\sigma\Delta}\right)^r\right), \tag{12}$$

where

$$b_r\left(\frac{y-m}{\sigma}\right)=\sum_{j=0}^{r}\sum_{k=0}^{j}\left[\sum{}^*\prod_{l=0}^{j}\frac{1}{k_l!}\left\{-\frac{c_l}{2}\right\}^{k_l}\right]\left[\sum{}^*\prod_{l=0}^{j-k}\frac{1}{k_l!}\left\{\bar{c}_l\left(\frac{y-m}{\sigma}\right)\right\}^{k_l}\right]\left[\sum{}^*\prod_{l=0}^{r-j}\frac{1}{k_l!}\left\{-\frac{c_l}{2}\left(\frac{y-m}{\sigma}\right)^2\right\}^{k_l}\right].$$

From (3),

$$R(h)\,e^{-ah}\int_A e^{-h(y-a)}Q_{\infty h}\{dy\}=\begin{cases}\dfrac{1}{\sigma(h)}\displaystyle\sum_{j=0}^{\infty}\frac{1}{\Delta^j}\int_A R(h)\,e^{-ah-y(h-a)}\varphi\left(\frac{y-m(h)}{\sigma(h)}\right)q_{jh}\left(\frac{y-m(h)}{\sigma(h)}\right)dy & \text{in the nonlatticed case,}\\[2ex] \dfrac{d}{\sigma(h)}\displaystyle\sum_{j=0}^{\infty}\frac{1}{\Delta^j}\sum_{y+d\nu\in A}R(h)\,e^{-ah-h(c+d\nu-a)}\varphi\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right)q_{jh}\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right) & \text{in the latticed case.}\end{cases} \tag{13}$$

Hence, and from [6, Part II], it follows that there exist polynomials $d'_j[(a-m)/\sigma,\,(y-m)/\sigma]$, satisfying the following formal equations:

$$F_\Delta\{A\}=e^{\frac{1}{\Delta}\left(\frac{a-m}{\sigma}\right)^3\lambda\left(\frac{a-m}{\sigma\Delta}\right)}\sum_{j=0}^{\infty}\frac{1}{\Delta^j}\begin{cases}\dfrac{1}{\sigma}\displaystyle\int_A\varphi\left(\frac{y-m}{\sigma}\right)d'_j\left(\frac{y-m}{\sigma},\,\frac{a-m}{\sigma}\right)dy & \text{in the nonlatticed case,}\\[2ex] \dfrac{d}{\sigma}\displaystyle\sum_{c+d\nu\in A}\varphi\left(\frac{c+d\nu-a}{\sigma}\right)d'_j\left(\frac{c+d\nu-m}{\sigma},\,\frac{a-m}{\sigma}\right) & \text{in the latticed case.}\end{cases} \tag{14}$$

They may be calculated in two ways. Let $x=(a-m)/\sigma$,

$$h(\varepsilon x)=\sum_{l=1}^{\infty}a_l(\varepsilon x)^l,$$

$$\varkappa_m\left(h(\varepsilon x)\right)=\varkappa_m+\sum_{l=1}^{\infty}p_{lm}(\varepsilon x)^l.$$

Then $h(x/\Delta)=h$ and $\varkappa_m(h(x/\Delta))=\varkappa_m(h)$.

The function

$$\omega_\infty\left(\frac{y-a}{\sigma\left(h(\varepsilon x)\right)},\,\varepsilon\right)=\sum_{j=0}^{\infty}\frac{\varepsilon^j}{\sigma\left(h(\varepsilon x)\right)}e^{-yh(\varepsilon x)}\varphi\left(\frac{y-a}{\sigma\left(h(\varepsilon x)\right)}\right)\cdot q_{jh(\varepsilon x)}\left(\frac{y-a}{\sigma\left(h(\varepsilon x)\right)}\right) \tag{15}$$

may be formally expanded in a Maclaurin series in powers of ε,

$$\omega_\infty\left(\frac{y-a}{\sigma\left(h(\varepsilon x)\right)},\,\varepsilon\right)=\frac{1}{\sigma}\sum_{j=1}^{\infty}\varepsilon^j\varphi\left(\frac{y-m}{\sigma}\right)d'_j\left(\frac{y-m}{\sigma},\,x\right). \tag{16}$$

Here

$$d'_j\left(\frac{y-m}{\sigma},\,x\right)=\frac{d^j}{d\varepsilon^j}\,\omega_\infty\left(\frac{y-a}{\sigma\left(h(\varepsilon x)\right)},\,\varepsilon\right)\Bigg|_{\varepsilon=0}. \tag{17}$$

It is very complicated to calculate the derivatives of the function $\omega_\infty[(y-a)/\sigma(h(\varepsilon x)),\varepsilon]$. Therefore, using this method we only estimate the remainder term, and in order to calculate the polynomials $d'_j[(y-m)/\sigma, x]$ we use the classical asymptotic Chebyshev–Kramer expansion of the probability distribution $F_\Delta\{A\}$:

$$F_\Delta\{A\}=\sum_{j=0}^{\infty}\frac{1}{\Delta^j}\begin{cases}\frac{1}{\sigma}\int_A q_j\left(\frac{y-m}{\sigma}\right)\varphi\left(\frac{y-m}{\sigma}\right)dy & \text{in the nonlatticed case,}\\ \frac{d}{\sigma}\sum_{c+d\nu\in A} q_j\left(\frac{c+d\nu-m}{\sigma}\right)\varphi\left(\frac{c+d\nu-m}{\sigma}\right) & \text{in the latticed case.}\end{cases} \tag{18}$$

We substitute ε for $1/\Delta$ in Eqs. (14) and (18) and compare their right-hand sides: in the nonlatticed case we obtain the formal equation

$$\sum_{j=0}^{\infty}\varepsilon^j\frac{1}{\sigma}\int_A q_j\left(\frac{y-m}{\sigma}\right)\varphi\left(\frac{y-m}{\sigma}\right)dy=e^{\varepsilon x^3\lambda(\varepsilon x)}\sum_{j=0}^{\infty}\varepsilon^j\frac{1}{\sigma}\int_A\varphi\left(\frac{y-m}{\sigma}\right)d_j'\left(\frac{y-m}{\sigma},\ x\right)dy,$$

and in the latticed case

$$\sum_{j=0}^{\infty}\varepsilon^j\frac{d}{\sigma}\sum_{c+d\nu\in A}q_j\left(\frac{c+d\nu-m}{\sigma}\right)\varphi\left(\frac{c+d\nu-m}{\sigma}\right)=e^{\varepsilon x^3\lambda(\varepsilon x)}\sum_{j=0}^{\infty}\varepsilon^j\frac{d}{\sigma}\sum_{c+d\nu\in A}\varphi\left(\frac{c+d\nu-m}{\sigma}\right)d_j'\left(\frac{c+d\nu-m}{\sigma}\right).$$

We now differentiate formally with respect to ε and obtain

$$d_j'(y,\ x)=\sum_{k=0}^{j}\bar{P}_k(x)\,q_{j-k}(y),$$

where

$$\bar{P}_k(x)=\sum{}^{*}\prod_{m=1}^{k}\frac{1}{k_m!}\left(-\Delta^m\lambda_{m+2}x^{m+2}\right)^{k_m},$$

and λ_{m+2} is the coefficient of the Kramer series of order m.

3. STUDY OF THE REMAINDER TERMS

We turn to the estimation of the remainder terms, when the random variable ξ has bounded density and take values in the arithmetic progression $c + d\nu$, where $\nu = 0, \pm 1, \pm 2, \ldots$.

We write

$$\rho_s\left(A,\ \int\right)=\left|F_\Delta\{A\}-e^{\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right)}\frac{1}{\sigma}\sum_{j=0}^{s}\frac{1}{\Delta^j}\int_A\varphi\left(\frac{y-m}{\sigma}\right)d_j'\left(\frac{y-m}{\sigma},\ x\right)dy\right|$$

and

$$\rho_s(A,\ \Sigma)=\left|F_\Delta\{A\}-e^{\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right)}\frac{d}{\sigma}\sum_{j=0}^{s}\frac{1}{\Delta^j}\sum_{c+d\nu\in A}\varphi\left(\frac{c+d\nu}{\sigma}\right)d_j'\left(\frac{c+d\nu-m}{\sigma},\ x\right)\right|.$$

If $F\{A\}$ has bounded density, then by Lemma 3, Part II,

$$\rho_s\left(A,\ \int\right)\leqslant e^{\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right)}\left[\rho_{1s}\left(A,\ \int\right)+\rho_{2s}\left(A,\int\right]\right. \tag{19}$$

Here

$$\rho_{1s}\left(A,\ \int\right)=\left|\sum_{j=0}^{s}\frac{1}{\Delta^j}\int_A\left[\frac{1}{\sigma}\varphi\left(\frac{y-m}{\sigma}\right)d_j'\left(\frac{y-m}{\sigma},\ x\right)-\frac{R(h)}{\sigma(h)}\varphi\left(\frac{y-m(h)}{\sigma(h)}\right)q_{jh}\left(\frac{y-m(h)}{\sigma(h)}\right)e^{-hy-\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right)}\right]dy\right| \tag{20}$$

and

$$\rho_{2s}\left(A, \int\right)=e^{-\frac{x^2}{2}}\left[\frac{1}{4\pi h} \int\limits_{-\infty}^{\infty} |f_h(t)-g_s(t)|^2 dt\right]^{\frac{1}{2}}, \tag{21}$$

where $f_h(t)$ is the characteristic function of the random variable ξ_h,

$$g_s(t)=\sum_{j=0}^{s} \frac{1}{\Delta^j} P_{jh}\left(t\sigma(h)\right) \exp\left\{itm(h)-\frac{t^2\sigma^2(h)}{2}\right\},$$

$$P_{jh}(v)=\sum{}^{*} \prod_{l=1}^{j} \frac{1}{k_l!}\left(\frac{\varkappa_{l+2}(h)\Delta^l (iv)^{l+2}}{(l+2)!\,\sigma^{l+2}(h)}\right)^{k_l}.$$

In the latticed case we use Lemma 6, Part II, and obtain

$$\rho_s(A, \Sigma)\leqslant e^{\frac{x^2}{\Delta}\lambda\left(\frac{x}{\Delta}\right)}\left[\rho_{1s}(A, \Sigma)+\rho_{2s}(A, \Sigma)\right]. \tag{22}$$

Here

$$\rho_{1s}(A, \Sigma)=\left|\sum_{j=0}^{s}\frac{1}{\Delta^j}\sum_{c+d\nu\in A}\left[\frac{d}{\sigma}\varphi\left(\frac{c+d\nu-m}{\sigma}\right)d_j'\left(\frac{c+d\nu-m}{\sigma}, x\right)-\frac{d\cdot R(h)}{\sigma(h)}\varphi\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right)q_{jh}\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right)e^{-h(c+d\nu)-\frac{x^2}{\Delta}\lambda\left(\frac{x}{\Delta}\right)}\right]\right| \tag{23}$$

and

$$\rho_{2s}(A, \Sigma)=e^{-\frac{x^2}{2}}\left[\frac{d\exp\left\{-2h\left(c-a+d\left[\frac{a-c}{d}\right]\right)\right\}}{2\pi(1-e^{-dh})}\int\limits_{-\frac{\pi}{d}}^{\frac{\pi}{d}}|f_h(t)-\bar{g}_s(t)|^2 dt\right]^{\frac{1}{2}},$$

where

$$\bar{g}_s(t)=\sum_{\nu=-\infty}^{\infty} e^{it(c+d\nu)}\frac{d}{\sigma(h)}\sum_{j=0}^{s}\frac{1}{\Delta^j}\varphi\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right)q_{jh}\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right). \tag{24}$$

The functions $g_s(t)$ and $\bar{g}_s(t)$ are connected by the equation

$$\bar{g}_s(t)=\sum_{\lambda=-\infty}^{\infty} g_s\left(t+\frac{2\pi\lambda}{d}\right)e^{-\frac{2\pi i\lambda c}{d}}. \tag{25}$$

In fact,

$$Q_s'\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right)=\frac{d}{\sigma(h)}\sum_{j=0}^{s}\frac{1}{\Delta^j}q_{jh}\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right)\varphi\left(\frac{c+d\nu-m(h)}{\sigma(h)}\right)=\frac{d}{\sigma(h)}\frac{1}{2\pi}\int\limits_{-\infty}^{\infty}e^{-it\frac{c+d\nu}{\sigma(h)}}g_s\left(\frac{t}{\sigma(h)}\right)dt. \tag{26}$$

We make a change of variable $t = v\sigma(h)$ in the integral. We then split the interval of integration $(-\infty, \infty)$ into intervals

$$\left[\frac{2\lambda-1}{d}\pi, \frac{2\lambda+1}{d}\pi\right), \qquad \lambda=0, \pm 1, \pm 2, \ldots,$$

and obtain

$$Q_s'\left(\frac{c+dv-m(h)}{\sigma(h)}\right)=\frac{d}{2\pi}\sum_{\lambda=-\infty}^{\infty}\int_{\frac{2\lambda-1}{d}\pi}^{\frac{2\lambda+1}{d}\pi} e^{-iv(c+dv)}g_s(v)\,dv.$$

Let $v = u + 2\pi\lambda/d$, and then

$$Q_s'\left(\frac{c+dv-m(h)}{\sigma(h)}\right)=\frac{d}{2\pi}\sum_{\lambda=-\infty}^{\infty}e^{-\frac{2\pi i\lambda}{d}}\int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} e^{-iu(c+dv)}g_s\left(u+\frac{2\pi\lambda}{d}\right)du.$$

Hence and from (25)-(26) it follows that

$$\frac{d}{2\pi}\int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} e^{-it\left(c+dv-m(h)\right)}\bar{g}_s(t)\,e^{-itm(h)}\,dt=\frac{d}{2\pi}\int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} e^{-it(c+dv)}\sum_{\lambda=-\infty}^{\infty}e^{-\frac{2\pi i\lambda c}{d}}g_s\left(t+\frac{2\pi\lambda}{d}\right)dt.$$

Equation (25) is proved.

We first consider the remainders $\rho_{1s}(A, \int)$ and $\rho_{1s}(A, \Sigma)$. It is easily seen that their asymptotic behavior as $\Delta \to \infty$ depends on the remainder

$$U_\Delta(x, u)=e^{\frac{x^2}{2}}\sum_{j=0}^{s}\frac{1}{\Delta^j}\varphi(u+x)\,d_j'(u+x, x)-\frac{\sigma}{\sigma(h)}\sum_{j=0}^{s}\frac{1}{\Delta^j}e^{-hu\sigma}\varphi\left(\frac{u\sigma}{\sigma(h)}\right)q_{jh}\left(\frac{u\sigma}{\sigma(h)}\right)$$

for sign u = sign x. Here $h = h(x/\Delta)$ is the solution of Eq. (2), where

$$x=\frac{u-m}{\sigma};\quad u=\frac{y-a}{\sigma}.$$

We define the set X_Δ of arguments x for which the estimates for $U_\Delta(x, u)$ are correct as follows:

$$X_\Delta=\left\{x: 1<|x|<\frac{\delta(1+\delta)}{2}\Delta,\ \text{where}\ \delta<\beta_H\right\}.$$

Here β_H is the real root of the equation $6H\delta = (1-\delta)^3$, $H = H(\Delta)$. We note that $\beta_H < 1$.

Denote by D . . . (. . .) the polynomials whose coefficients depend only on s, the indicated variables being in the brackets.

LEMMA 1. Let condition (S) be satisfied. Then

$$|U_\Delta(x, u)|\leqslant D_{1s}(|ux|, V, L, \bar{H}, \sqrt{1+\rho})\frac{|x|^{s+1}}{\Delta^{s+1}}e^{-\frac{ux}{1+\frac{\rho}{2}(1-\delta)}}\tag{27}$$

for $x \in X_\Delta$ and sign u = sign x.

Here

$$\rho=\frac{6H\delta}{(1-\delta)^3},\quad V=\frac{\bar{\delta}\beta_H}{\bar{\delta}_H},\quad L=(\bar{\delta}_H-\bar{\delta})^{-1},$$

$$\bar{H}=H(\Delta)\left(1+\frac{\delta(s+3)}{(1-\delta)^{s+3}}\right),\quad \bar{\delta}=\frac{\delta(1+\delta)}{2},\quad \bar{\delta}_H=\frac{\beta_H(1+\beta_H)}{2}.$$

Proof of Lemma 1. By (S),

$$|\varkappa_{m+2}(h)| \leqslant \frac{\bar{H}(m+2)!\,\sigma^{m+2}(h)}{\Delta^m} \tag{28}$$

for m = 1, 2, . . . , s.

Using Cauchy's inequality, we have

$$h\left(\frac{x}{\Delta}\right) = \frac{x}{\sigma} \cdot \frac{1}{1 + \frac{\Theta\rho}{2}(1-\delta)} \tag{29}$$

for $x \in X_\Delta$, where $|\Theta| < 1$.

For $\varepsilon \in (0, 1/\Delta)$ we have

$$\left|\frac{d^m}{d\varepsilon^m} h(\varepsilon x)\right| \leqslant VL^{m+1} \left|\frac{x}{\sigma}\right|^{m+1}, \qquad m = 0, 1, \ldots,$$

for $x \in X_\Delta$ and $\bar{\delta} < \bar{\delta}_H$.

It follows from the definition of the polynomials $d_j^{\prime}(x + u, x)$ that the derivatives with respect to ε of orders l = 0, 1, . . . , s of the remainder

$$\omega_s\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}, \varepsilon\right) = \sum_{j=0}^{s} \varepsilon^j \left[\frac{e^{-u\sigma(h(\varepsilon x))}}{\sigma(h(\varepsilon x))} \varphi\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}\right) q_{jh(\varepsilon x)}\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}\right) - \frac{e^{-\frac{x^2}{2}}}{\sigma} \varphi(x+u)\, d_j^{\prime}(u+x,\ x)\right]$$

at the point $\varepsilon = 0$ are equal to zero, i.e.

$$\frac{d^l}{d\varepsilon^l} \omega_s\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}, \varepsilon\right)\Bigg|_{\varepsilon=0} = 0 \tag{30}$$

for l = 0, 1, . . . , s.

Consequently,

$$\omega_s\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}, \varepsilon\right) = \frac{\varepsilon^{s+1}}{(s+1)!} \frac{d^{s+1}}{d\varepsilon^{s+1}} \omega_s\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}, \varepsilon\right)\Bigg|_{\varepsilon=\Theta\varepsilon}, \ 0 < \Theta < 1. \tag{31}$$

Let

$$G_j\left(h(\varepsilon x)\right) = \frac{\Delta^{-j}}{\sigma(h(\varepsilon x))} e^{-u\sigma(h(\varepsilon x))} \varphi\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}\right) q_{jh(\varepsilon x)}\left(\frac{u\sigma}{\sigma(h(\varepsilon x))}\right).$$

Then

$$\frac{d^{s+1}}{d\varepsilon^{s+1}} \omega_s\left(\frac{u\sigma}{\sigma(h(\varepsilon h))}, \varepsilon\right) = \sum_{j=0}^{s} \frac{d^{s+1}}{d\varepsilon^{s+1}}\left[(\Delta\varepsilon)^j G_j\left(h(\varepsilon x)\right)\right] = \sum_{j=0}^{s} \sum_{l=s+1-j}^{s+1} \frac{(s+1)!\,j!\,\Delta^{-j}\varepsilon^{j+l-s-1}}{l!\,(s+1-l)!\,(j+l-s)!} \frac{d^l}{d\varepsilon^l} G_j\left(h(\varepsilon x)\right), \tag{32}$$

where

$$\frac{d^l}{d\varepsilon^l} G_j\left(h(\varepsilon x)\right) = l! \sum{}^{*} \frac{d^R G_j(h)}{dh^R} \prod_{m_1=1}^{l} \frac{1}{k_{m_1}!} \left(\frac{1}{m_1!} \frac{d^{m_1} h(\varepsilon x)}{d\varepsilon^{m_1}}\right)^{k_{m_1}} \tag{33}$$

and $R = k_1 + k_2 + \ldots + k_l$.

It is easily seen that

$$\frac{d^R}{dh^R} G_j(h) = \sum_{p=0}^{R} C_R^p \frac{d^{R-p} e^{-uh\sigma}}{dh^{R-p}} \cdot \frac{d^p}{dh^p} \left(\frac{\Delta^{-j}}{\sigma(h)} \varphi\left(\frac{u\sigma}{\sigma(h)}\right) q_{jh}\left(\frac{u\sigma}{\sigma(h)}\right)\right) =$$

$$= \sum_{p=0}^{R} C_R^p (-u\sigma)^{R-p} e^{-u\sigma h} \frac{d^p}{dh^p} \left[\frac{\Delta^{-j}}{\sigma(h)} \varphi\left(\frac{u\sigma}{\sigma(h)}\right) q_{jh}\left(\frac{u\sigma}{\sigma(h)}\right)\right]. \quad (34)$$

Turning to the study of the derivatives of the functions

$$W_j(h) = \frac{\Delta^{-j}}{\sigma(h)} \varphi\left(\frac{u\sigma}{\sigma(h)}\right) q_{jh}\left(\frac{u\sigma}{\sigma(h)}\right), \qquad j = 0, 1, \ldots, s, \quad (35)$$

we express them in terms of Fourier integrals of the functions

$$\Delta^{-j} P_{jh}\left(t\sigma(h)\right) e^{-\frac{t^2\sigma^2(h)}{2}} = \sum{}^{*} \prod_{m_s=1}^{j} \frac{1}{k_{m_s}!} \left(\frac{\varkappa_{m_s+2}(h)(it)^{m_s+2}}{(m_s+2)!}\right)^{k_{m_s}} e^{-\frac{t^2\sigma^2(h)}{2}},$$

i.e.,

$$W_j(h) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itu\sigma - \frac{t^2\sigma^2(h)^2}{2}} \Delta^{-j} P_{jh}\left(t\sigma(h)\right) dt.$$

Hence,

$$\frac{d^p}{dh^p} W_j(h) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-itu\sigma} \sum_{q=0}^{p} C_p^q \frac{d^{p-q}}{dh^{p-q}} e^{\frac{1}{2}(it\sigma(h))^2} \frac{d^q}{dh^q} \left(\Delta^{-j} P_{jh}\left(t\sigma(h)\right)\right) dt. \quad (36)$$

Using mathematical induction we shall show that there exists a polynomial $D_{1j}(\sigma(h)|\omega|, \bar{H})$ in $|\omega|\sigma(h)$ and $\bar{H}$, whose coefficients depend only on j and q, satisfying the inequality

$$\left|\frac{d^q}{dh^q} \Delta^{-j} P_{jh}\left(\omega\sigma(h)\right)\right| \leqslant \frac{\sigma^q(h)}{\Delta^{j+q}} D_{1j}\left(\sigma(h)|\omega|, \bar{H}\right) \quad (37)$$

for q = 0, 1, ... and j = 1, 2, ...

It is known that

$$\Delta^{-j} P_{1h}\left(\omega\sigma(h)\right) = \frac{1}{6} \varkappa_3(h) \omega^3$$

and

$$\Delta^{-j} P_{jh}\left(\omega\sigma(h)\right) = \frac{\varkappa_{j+2}(h)\omega^{j+2}}{(j+2)!} + \sum_{r=1}^{j-1} \frac{(j-r)\omega^{j+2-r}}{j(j-2-r)!} \varkappa_{j-r+2}(h) \Delta^{-r} P_{rh}\left(\omega\sigma(h)\right) \quad (38)$$

for j = 3, 4, ...

It is easily seen that

$$\left|\Delta^{-r} P_{rh}\left(\omega\sigma(h)\right)\right| \leqslant \frac{1}{\Delta^r} P_{0r}\left(|\omega|\sigma(h), \bar{H}\right) \quad (39)$$

for r = 0, 1, 2, ..., j.

Since

$$\varkappa_{m+\alpha}(h) = \frac{d^\alpha}{dh^\alpha} \varkappa_m(h),$$

then

$$\Delta^{-1} \frac{d^\alpha}{dh^\alpha} P_{1h}\left(\omega\sigma(h)\right) = \frac{1}{6} \varkappa_{3+\alpha}(h)\,\omega^3. \tag{40}$$

Hence and from (27) it follows that

$$\left| \Delta^{-j} \frac{d}{dh} P_{1h}\left(\omega\sigma(h)\right) \right| \leqslant \frac{|\omega|^3 \bar{H}\, 4! \left(\sigma(h)\right)^{1+3}}{6\Delta^{1+1}} \leqslant \frac{\sigma(h)}{\Delta^2} D_{11}\left(|\omega|\sigma(h),\ \bar{H}\right).$$

Let

$$\left| \Delta^{-r} \frac{d}{dh} P_{rh}\left(\omega\sigma(h)\right) \right| \leqslant \frac{\sigma(h)}{\Delta^{1+r}} D_{1r}\left(|\omega|\sigma(h),\ \bar{H}\right) \tag{41}$$

for r = 0, 1, ..., j − 1. We prove inequality (41) for r = j. To estimate the derivative

$$\Delta^{-j} \frac{d}{dh} P_{jh}\left(\omega\sigma(h)\right) = \frac{\varkappa_{j+3}(h)\,\omega^{j+2}}{(j+2)!} + \sum_{r=1}^{j-1} \frac{(j-r)\,\omega^{j+2-r}}{j(j-r-2)!} \left[\varkappa_{j-r+3}(h)\,\Delta^{-r} P_{rh}\left(\omega\sigma(h)\right) + \varkappa_{j-r+2}(h)\,\Delta^{-r} \frac{d}{dh} P_{rh}\left(\omega\sigma(h)\right)\right]$$

we use inequalities (27) and (41) and obtain

$$\left| \Delta^{-j} \frac{d}{dh} P_{jh}\left(\omega\sigma(h)\right) \right| \leqslant \frac{\bar{H}(j+3)!\,\sigma^{j+3}(h)\,|\omega|^{j+2}}{\Delta^{j+1}} +$$

$$+ \sum_{r=1}^{j-1} \frac{(j-r)\,|\omega|^{j+2-r}}{j(j-r+2)!} \left[\frac{\bar{H}(j-r+3)!\,\sigma^{j-r+3}(h)}{\Delta^{j-r+1}} \frac{1}{\Delta^r} D_{0r}\left(|\omega|\sigma(h),\ \bar{H}\right) + \right.$$

$$\left. + \frac{\bar{H}(j-r+2)!\,\sigma^{j-r+2}(h)}{\Delta^{j-r+1}} \frac{\sigma(h)}{\Delta^r} D_{1r}\left(|\omega|\sigma(h),\ \bar{H}\right) \right] = \frac{\sigma(h)}{\Delta^{j+1}} D_{1j}\left(|\omega|\sigma(h),\ \bar{H}\right).$$

Hence inequality (41) holds for r = 0, 1, 2, ..., j.

Let the inequality

$$\left| \Delta^{-r} \frac{d^\alpha}{dh^\alpha} P_{rh}\left(\omega\sigma(h)\right) \right| \leqslant \frac{\sigma^\alpha(h)}{\Delta^{r+\alpha}} D_{\alpha r}\left(|\omega|\sigma(h),\ \bar{H}\right) \tag{42}$$

hold for α = 0, 1, ..., q − 1. We shall prove it for α = q.

Since

$$\Delta^{-r} \frac{d^q}{dh^q} P_{jh}\left(\omega\sigma(h)\right) = \frac{\varkappa_{j+2+q}(h)\,\omega^{j+2}}{(j+2)!} + \sum_{r=1}^{j-1} \frac{(j-r)\,\omega^{j-r+2}}{j(j+2-r)!} \sum_{\alpha=0}^{q} C_q^\alpha \varkappa_{j+q-r-\alpha+2}(h) \frac{d^\alpha}{dh^\alpha} P_{rh}\left(\omega\sigma(h)\right),$$

then by (27) and (42)

$$\left| \Delta^{-j} \frac{d^q}{dh^q} P_{jh}\left(\omega\sigma(h)\right) \right| \leqslant \frac{\bar{H}(j+q+2)!\,\sigma^q(h)}{\Delta^{j+q}} \left[\frac{\left(|\omega|\sigma(h)\right)^{j+2}}{(j+2)!} + \right.$$

$$\left. + \sum_{r=1}^{j-1} \sum_{\alpha=0}^{q} \frac{(j-r)\,q!\left(|\omega|\sigma(h)\right)^{j-r+2} D_{1r}\left(|\omega|\sigma(h),\ \bar{H}\right)}{(j-r+2)!\,j\,(q-\alpha)!\,\alpha!\,(j+q+2)!} \right] = \frac{\sigma^q(h)}{\Delta^{j+q}} D_{1j}\left(|\omega|\sigma(h),\ \bar{H}\right).$$

Inequality (37) is proved.

We now consider the derivative

$$\frac{d^{p-q}}{dh^{p-q}} e^{\frac{1}{2}\left(itx\sigma(h)\right)^2} = (p-q)!\, e^{-\frac{t^2\sigma^2(h)}{2}} \sum\nolimits^{*} \prod_{m_s=1}^{p-q} \frac{1}{k_{m_s}!} \left(\frac{(itx)^2 \varkappa_{m_s+2}(h)}{m_s!} \right)^{k_{m_s}}.$$

It follows immediately from (27) that

$$\frac{d^{p-q}}{dh^{p-q}} e^{-\frac{t^2\sigma^2(h)}{2}} \leqslant \frac{(\sigma(h))^{p-q}}{\Delta^{p-q}} (p-q)!\, e^{-\frac{t^2\sigma^2(h)}{2}} \sum{}^{*} \prod_{m_3=1}^{p-q} \frac{1}{k_{m_3}!} \left((m_3+1)(m_3+2)\bar{H}\left(t\sigma(h)\right)^2\right)^{k_{m_3}} \tag{43}$$

for $p-q=0, 1, \ldots$.

We now return to integral (36). It follows from (37) and (43) that

$$\left|\frac{d^p}{dh^p} W_j(h)\right| \leqslant \frac{1}{2\pi} \sum_{q=0}^{p} C_p^q (p-q)!\, \frac{\sigma^p(h)}{\Delta^{j+p}} \int_{-\infty}^{\infty} e^{-\frac{t^2\sigma^2(h)}{2}} D_{1j}\left(|t|\sigma(h),\, \bar{H}\right) \times$$

$$\times \sum{}^{*} \prod_{m_3=1}^{p-q} \frac{1}{k_{m_3}!} \left((m_3+1)(m_3+2)\bar{H}\left(|t|\sigma(h)\right)^2\right)^{k_{m_3}} dt = \frac{\sigma^{p-1}(h)}{\Delta^{j+p}} D_{jp}(\bar{H}).$$

Hence and from (34) we have

$$\left|\frac{d^R}{dh^R} G_j(h)\right| \leqslant \sum_{p=0}^{R} C_R^p (|u|\sigma)^{R-p} e^{-u\sigma h} \frac{\sigma^{p-1}(h)}{\Delta^{j+p}} D_{jp}(\bar{H}).$$

From (29) sign x = sign h (εx). Therefore, for sign x = sign u we have the inequality

$$\left|\frac{d^R}{dh^R} G_j(h)\right| \leqslant \sum_{p=0}^{R} C_R^p (|u|\sigma)^{R-p} C^{-\frac{ux}{1+\frac{\rho}{2}(1-\delta)}} \frac{\sigma^{p-1}(h)}{\Delta^{j+p}} D_{jp}(\bar{H}).$$

Hence and from (29) and (39) it follows that

$$\left|\frac{d^l}{d\varepsilon^l} G_j\left(h(\varepsilon x)\right)\right| \leqslant l! \sum_R{}^{*} C_R^p (|u|\sigma)^{R-p} \frac{\sigma^{p-1}(h)}{\Delta^{p+j}} D_{jp}(\bar{H}) \prod_{m_1=1}^{l} \frac{1}{k_{m_1}!} \left(\frac{V(L|x|)^{m_1+1}}{m_1!\,\sigma}\right)^{k_{m_1}} \exp\left\{-\frac{ux}{1+\frac{\rho}{2}(1-\delta)}\right\} =$$

$$= l! \sum_R{}^{*} \sum_{p=0}^{R} C_R^p \frac{|u|^{R-p}}{\Delta^{j+p}\sigma(h)} \left(\frac{\sigma(h)}{\sigma}\right)^p |x|^{l+R} D_{jp}(\bar{H}) \prod_{m_1=1}^{l} \frac{1}{k_{m_1}!} \left(\frac{1}{m_1!} VL^{m_1+1}\right)^{k_{m_1}} \exp\left\{-\frac{ux}{1+\frac{\rho}{2}(1-\delta)}\right\}.$$

Since $|\varepsilon\Delta| < 1$, $1 < |x| < \Delta$, and $\sigma^2(h) = \sigma^2(1+\Theta\rho)$, where $|\Theta| < 1$, it follows from (32) and the last inequality that

$$\left|\frac{d^{s+1}}{d\varepsilon^{s+1}} \omega_s\left(\frac{u\sigma}{\sigma\left(h(\varepsilon x)\right)},\, \varepsilon\right)\right| \leqslant \frac{|x|^{s+1}}{\sigma(h)} \sum_{j=0}^{s} \sum_{l=s+1-j}^{s+1} \frac{(s+1)!\,j!\,l!\,\varepsilon^{l-s-1}}{l!(s+1-l)!(j+l-s)!} \sum_R{}^{*} \sum_{p=0}^{R} C_R^p (1+\rho)^{\frac{p}{2}} |ux|^{R-p} D_{jp}(\bar{H}) \prod_{m_1=1}^{l} \frac{1}{k_{m_1}!} \left(\frac{1}{m_1!} VL^{m_1+1}\right)^{k_{m_1}} \times$$

$$\times \exp\left\{-\frac{ux}{1+\frac{\rho}{2}(1-\delta)}\right\} = \frac{|x|^{s+1}(s+1)!}{\sigma} D_{1s}(|ux|,\, V,\, L,\, \bar{H},\, \sqrt{1+\rho}) \exp\left\{-\frac{ux}{1+\frac{\rho}{2}(1-\delta)}\right\}$$

for sign u = sign x. The statement of Lemma 1 follows from the estimate just obtained and from Eqs. (30) and (31).

Henceforward we shall write

$$\mathfrak{N}_+ = \left\{A : x \in X_\Delta \text{ and } 0 < a = \inf\{y : y \in A\}\right\}$$

and

$$\mathfrak{N}_- = \left\{A : x \in X_s \text{ and } 0 > a = \sup\{y : y \in A\}\right\}$$

for the classes of Borel sets for which the following lemma holds.

LEMMA 2. If the conditions of Lemma 1 are satisfied, then for all $A \in \mathfrak{N}_+$ or $A \in \mathfrak{N}$ we have the inequalities

$$\rho_{1s}\left(A, \int\right) \leqslant \left|\frac{x}{\Delta}\right|^{s+1} \frac{1}{|x|} e^{-\frac{x^2}{x}} D_{2s}\left(1+\frac{\rho}{2}(1-\delta), V, L, \bar{H}, \sqrt{1+\rho}\right) \tag{44}$$

and

$$\rho_{1s}(A, \Sigma) \leqslant \left|\frac{x}{\Delta}\right|^{s+1} \frac{1}{|x|} e^{-\frac{x^2}{2}} D_{4s}\left(1+\frac{\rho}{2}(1-\delta), V, L, \bar{H}, \sqrt{1+\rho}\right). \tag{45}$$

Proof of Lemma 2. Since

$$R(h) = \exp\left\{-\frac{x^2}{2} + hm(h) + \frac{x^3}{\Delta} \lambda\left(\frac{x}{\Delta}\right)\right\}, \tag{46}$$

then after a change of variables $y = \sigma u + a$ we may write integral (21) in the form

$$\rho_{1s}\left(A, \int\right) = \left|\sum_{j=0}^{s} \frac{e^{\frac{x^2}{2}}}{\Delta^j} \int_{\frac{A-a}{\sigma}} \left[\frac{1}{\sigma} \varphi\left(\frac{u\sigma+a-m}{\sigma}\right) d_j'\left(\frac{u\sigma+a-m}{\sigma}, x\right) - \frac{\sigma}{\sigma(h)} e^{-u\sigma h} \varphi\left(\frac{u\sigma}{\sigma(h)}\right) q_{jh}\left(\frac{u\sigma}{\sigma(h)}\right)\right] du\right|,$$

where $\frac{A-a}{\sigma} = \left\{\frac{v-a}{\sigma} : v \in A\right\}$, $A \in \mathfrak{N}_+$ or $A \in \mathfrak{N}_-$.

Hence and from (27) we have inequality (44).

By a similar argument it follows from (20) that

$$\rho_{1s}(A, \Sigma) = \frac{d}{\sigma} e^{-\frac{x^2}{2}} \left|\sum_{j=0}^{s} \frac{1}{\Delta^j} \sum_{u \in \frac{A-a}{\sigma}} \left[\varphi(u+x) d_j'(u+x, x) - \frac{\sigma}{\sigma(h)} e^{-u\sigma h} \varphi\left(\frac{u\sigma}{\sigma(h)}\right) q_{jh}\left(\frac{u\sigma}{\sigma(h)}\right)\right]\right|,$$

where $u = (c + d\nu - a)/\sigma$.

By Lemma 1,

$$\rho_{1s}(A, \Sigma) \leqslant \frac{2d}{\sigma} \frac{|x|^{s+1}}{\Delta^s} e^{-\frac{x^2}{2}} \sum_{\frac{c+d\nu+a}{\sigma} > 0} D_{1s}\left(\left|x \frac{c+d\nu-a}{\sigma}\right|, V, L, \bar{H}, \sqrt{1+\rho}\right) \exp\left\{-\frac{(c+d\nu-a)x}{\sigma\left(1+\frac{\rho}{2}(1-\delta)\right)}\right\}$$

for $x > 1$. In the case $x < -1$, the region of summation becomes $(c+d\nu-a)/\sigma < 0$. Since $e^{-y} \leq y^{-\alpha}\alpha^{\alpha}e^{-\alpha}$ for $y > 0$ and $\alpha > 0$,

$$\rho_{1s}(A, \Sigma) \leqslant \frac{2d}{\sigma} \left|\frac{x}{\Delta}\right|^{s+1} e^{-\frac{x^2}{2}} D_{3s}\left(1+\frac{\rho}{2}(1-\delta), V, L, \bar{H}, \sqrt{1+\rho}\right) \times$$

$$\times \sum_{c+d\nu-a>0} \exp\left\{-\frac{x(c+d\nu-a)}{\sigma\left(1+\frac{\rho}{2}(1-\delta)\right)}\right\} \leqslant \left|\frac{x}{\Delta}\right|^{s+1} e^{-\frac{x^2}{2}} \frac{1}{|x|} \left(2+\rho(1-\delta)\right) D_{3s}\left(1+\frac{\rho}{2}(1-\delta), V, L, \bar{H}, \sqrt{1+\rho}\right).$$

Lemma 2 is proved.

We now consider $\rho_{2s}(A, \int)$ and $\rho_{2s}(A, \Sigma)$, depending on the asymptotic behavior of the integrals

$$\Psi = \int_{-\infty}^{\infty} \left|f_h\left(\frac{t}{\sigma(h)}\right) - g_s\left(\frac{t}{\sigma(h)}\right)\right|^2 dt \tag{47}$$

and

$$J = \int\limits_{|t| \leqslant \frac{\pi\sigma(h)}{d}} \left| f_h\left(\frac{t}{\sigma(h)}\right) - \bar{g}_s\left(\frac{t}{\sigma(h)}\right)\right|^2 dt \tag{48}$$

as $\Delta \to \infty$.

Clearly, for the integrals Ψ and J we have the inequalities

$$\Psi \leqslant \int\limits_{-c_1\Delta}^{c_1\Delta} \left| f_h\left(\frac{t}{\sigma(h)}\right) - g_s\left(\frac{t}{\sigma(h)}\right)\right|^2 dt + 2 \int\limits_{|t| > c_1\Delta} \left| f_h\left(\frac{t}{\sigma(h)}\right)\right|^2 dt + 2 \int\limits_{|t| > c_1\Delta} \left| g_s\left(\frac{t}{\sigma(h)}\right)\right|^2 dt = \Psi_1 + \Psi_2 + \Psi_3 \tag{49}$$

and

$$J \leqslant \int\limits_{-c_1\Delta}^{c_1\Delta} \left| f_h\left(\frac{t}{\sigma(h)}\right) - \bar{g}_s\left(\frac{t}{\sigma(h)}\right)\right|^2 dt + 2 \int\limits_{c_1\Delta \leqslant |t| \leqslant \frac{\pi\sigma(h)}{d}} \left| f_h\left(\frac{t}{\sigma(h)}\right)\right|^2 dt +$$

$$+ 2 \int\limits_{c_1\Delta \leqslant |t| \leqslant \frac{\pi\sigma(h)}{d}} \left| \bar{g}_s\left(\frac{t}{\sigma(h)}\right)\right|^2 dt = J_1 + J_2 + J_3. \tag{50}$$

Here $c_1 = \frac{(\delta_2 - \delta)\sqrt{1-\rho}}{8H_2}$, $H_2 = \frac{(1-\delta)^3}{6\delta(1-\delta_2)}$, and $\delta < \delta_2 < 1$, where δ_2 satisfies the inequality $6\delta(1-\delta_2)/(1-\delta) < 1$. If $c_1\Delta > \pi\sigma(h)/d$, then $J_2 = 0$ and $J_3 = 0$.

Moreover, we write

$$c_2 = \frac{2^{3s+1}(1-\delta)\max(H_2, H_2^{s+1})}{(1-\delta_2)^{s+2}(1-\rho)^{\frac{s+1}{2}}}.$$

We suppose that in Lemmas 3-9 the random variable ξ satisfies condition (S).

LEMMA 3. The following inequality holds:

$$\Psi_1 \leqslant \frac{\sqrt{2\pi}(2s+5)!!\, c_2^2}{\Delta^{2(s+1)}}. \tag{51}$$

Proof of Lemma 3. Repeating the arguments in [11], we obtain

$$\left| f_h\left(\frac{t}{\sigma(h)}\right) - g_s\left(\frac{t}{\sigma(h)}\right)\right| \leqslant \frac{c_2 |t|^{s+3} e^{-\frac{t^2}{4}}}{\Delta^{s+1}} \tag{52'}$$

for $|t| \leq c_1\Delta$. Hence (51) follows immediately.

LEMMA 4. Let ξ have lattice distribution with maximal step $d > 0$. Then

$$J_1 \leqslant \frac{2\sqrt{2\pi}(2s+5)!!\, c_2^2}{\Delta^{2(s+1)}} + \frac{4c_1 D_S^2(d, \bar{H})\, e^{-\frac{1}{4}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2}}{\left(1 - \exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}\right)^2}. \tag{52}$$

Proof of Lemma 4. It is easily seen that

$$\left| f_h\left(\frac{t}{\sigma(h)}\right) - \bar{g}_s\left(\frac{t}{\sigma(h)}\right)\right| \leqslant \frac{c_2|t|^{s+3}}{\Delta^{s+1}} e^{-\frac{t^2}{4}} + \sum_{\lambda \neq 0}\sum_{j=0}^{s} \frac{1}{\Delta^j} \left| P_{jh}\left(t + \frac{2\pi\lambda\sigma(h)}{d}\right)\right| e^{-\frac{1}{2}\left(t + \frac{2\pi\lambda\sigma(h)}{d}\right)^2} \leqslant$$

$$\leqslant \frac{c_2|t|^{s+3} e^{-\frac{t^2}{4}}}{\Delta^{s+1}} + \sum_{\lambda \neq 0}\sum_{j=0}^{s} \frac{1}{\Delta^j} D_{1j}\left(\left|\left(t + \frac{2\pi\lambda\sigma(h)}{d}\right)\right|, \bar{H}\right) e^{-\frac{1}{2}\left(t + \frac{2\pi\lambda\sigma(h)}{d}\right)^2}.$$

Since $|t| \leq \pi\sigma(h)/d$ and $\lambda = \pm 1, \pm 2, \ldots$, there exists a polynomial $D_S(d, \bar{H})$ such that

$$\left| f_h\left(\frac{t}{\sigma(h)}\right) - \bar{g}_s\left(\frac{t}{\sigma(h)}\right) \right| \leqslant \frac{c_2 |t|^{s+3} e^{-\frac{t^2}{4}}}{\Delta^{s+1}} + \frac{D_s(d, \bar{H}) e^{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2}}{1-\exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}}. \tag{53}$$

Hence follows the statement of Lemma 4.

LEMMA 5. If ξ has bounded density, then

$$\Psi_2 \leqslant \sigma\sqrt{1+\rho} \int_{|t|>\frac{c_1\Delta}{\sigma\sqrt{1-\rho}}} |f_h(t)|^2 dt. \tag{54}$$

If also ξ satisfies the conditions of Lemma 4, then

$$J_2 \leqslant \sigma\sqrt{1+\rho} \int_{\frac{c_1\Delta}{\sigma\sqrt{1-\rho}} \leqslant |t| \leqslant \frac{\pi}{d}} |f_h(t)|^2 dt. \tag{55}$$

The proof is obvious.

LEMMA 6. If condition (S) is satisfied, then

$$\Psi_3 \leqslant D_s(\bar{H}) e^{-\frac{c_1^2\Delta^2}{2}} \tag{56}$$

and

$$J_3 \leqslant 2D_s(\bar{H}) e^{-\frac{c_1^2\Delta^2}{2}} + \frac{4\pi\sigma\sqrt{1+\rho}\, D_s^2(d, \bar{H}) \exp\left\{-\frac{1}{4}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}}{d\left(1-\exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}\right)^2}. \tag{57}$$

Proof of Lemma 6. We have

$$\left| g_s\left(\frac{t}{\sigma(h)}\right) \right| \leqslant \sum_{j=0}^{s} \frac{1}{\Delta^j} D_{1j}(|t|, \bar{H}) e^{-\frac{t^2}{2}} \tag{58}$$

[see (37)]. Consequently, there exists a polynomial $D_s(\bar{H})$ such that

$$\Psi_3 \leqslant D_s(\bar{H}) e^{-\frac{c_1^2\Delta^2}{2}}. \tag{59}$$

We showed that

$$\left| \sum_{\lambda\neq 0} g_s\left(t + \frac{2\pi\lambda\sigma(h)}{d}\right) \right| \leqslant \frac{D_s(d, \bar{H}) \exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}}{1-\exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}} \tag{60}$$

[see (53)]. Inequality (57) follows from (58) and (60).

From (49) and the estimates of the integrals Ψ_1, Ψ_2, and Ψ_3 we have the following lemma.

LEMMA 7. If ξ has bounded density, then

$$\rho_{2s}\left(A, \int\right) \leqslant e^{-\frac{x^2}{2}} \left[\frac{1+\frac{\rho}{2}(1-\delta)}{4\pi x\sqrt{1-\rho}} \left(\frac{\sqrt{2\pi}(2s-5)!!\, c_2^s}{\Delta^{2(s+1)}} + D_s(\bar{H}) e^{-\frac{c_1^2\Delta^2}{2}} + \sigma\sqrt{1+\rho} \int_{|t|>\frac{c_1\Delta}{\sigma\sqrt{1-\rho}}} |f_h(t)|^2 dt\right)\right]^{\frac{1}{2}} \tag{61}$$

for $A \in \mathfrak{N}_+$ or $A \in \mathfrak{N}_-$.

From (50) and the estimates of the integrals J_1, J_2, and J_3 we have the following lemma.

LEMMA 8. Let ξ have lattice distribution with maximal step $d > 0$. Then

$$\rho_{2s}(A, \Sigma) \leqslant e^{-\frac{x^2}{2}} \left[\frac{d \exp\left\{-2h(c-a+d)\left[\frac{a-c}{d}\right]\right\}}{2\pi(1-e^{-dh})\sigma\sqrt{1-\rho}} \left(\frac{2\sqrt{2\pi}(2s+5)!!\, c_2^s}{\Delta^{2(s+1)}} + \frac{4c_1 \Delta D_s^2(d, \bar{H}) \exp\left\{-\frac{1}{4}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}}{\left(1-\exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}\right)^2} + 2D_s(\bar{H}) e^{-\frac{c_1^2\Delta^2}{2}} + \right.\right.$$

$$\left.\left. + \frac{4\pi\sigma\sqrt{1+\rho}\, D_s^2(d, \bar{H}) \exp\left\{-\frac{1}{4}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}}{d\left(1-\exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}\right)^2} + \sigma\sqrt{1+\rho} \int\limits_{|t| > \frac{c_1\Delta}{\sigma\sqrt{1-\rho}}} |f_h(t)|^2\, dt \right)\right]^{\frac{1}{2}} \tag{62}$$

for $A \in \mathfrak{N}_+$ or $A \in \mathfrak{N}_-$.

We shall now formulate the basic results.

LEMMA 9. Let the random variable ξ satisfy condition (S) and have bounded density. Then

$$\left| F_\Delta\{A\} - e^{\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right)} \frac{1}{\sigma} \sum_{j=0}^{s} \frac{1}{\Delta^j} \int\limits_A \varphi\left(\frac{y-m}{\sigma}\right) d_j\left(\frac{y-m}{\sigma}, x\right) dy \right| \leqslant$$

$$\leqslant e^{\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right) - \frac{x^2}{2}} \left\{ \frac{|x|^s}{\Delta^{s+1}} D_{2s}\left(1 + \frac{\rho}{2}(1-\delta), V, L, \bar{H}, \sqrt{1+\rho}\right) + \right.$$

$$\left. + \frac{1 + \frac{\rho}{2}(1-\delta)}{4\pi x\sqrt{1-\rho}} \left(\frac{\sqrt{2\pi}(2s+5)!!\, c_2^s}{\Delta^{2(s+1)}} + D_s(H) e^{-\frac{c_1^2\Delta^2}{2}} + \sigma\sqrt{1+\rho} \int\limits_{|t| > \frac{c_1\Delta}{\sigma\sqrt{1-\rho}}} |f_h(t)|^2\, dt \right)^{\frac{1}{2}} \right\}$$

for $A \in \mathfrak{N}_+$ or $A \in \mathfrak{N}_-$.

LEMMA 10. Let the random variable ξ satisfy condition (S) and take values $c + d\nu$, $\nu = 0, \pm 1, \pm 2, \ldots$, with maximal step of distribution $d > 0$. Then

$$\left| F_\Delta\{A\} - e^{\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right)} \frac{d}{\sigma} \sum_{j=0}^{s} \frac{1}{\Delta^j} \sum_{c+d\nu \in A} \varphi\left(\frac{c+d\nu-m}{\sigma}\right) d_j\left(\frac{c+d\nu-m}{\sigma}, x\right) \right| \leqslant$$

$$\leqslant e^{\frac{x^3}{\Delta}\lambda\left(\frac{x}{\Delta}\right) - \frac{x^2}{2}} \left\{ \frac{|x|^s}{\Delta^{s+1}} D_{2s}\left(1 + \frac{\rho}{2}(1-\delta), V, L, \bar{H}, \sqrt{1+\rho}\right) + \frac{d \exp\left\{-2h\left(c-a+d\left[\frac{a-c}{d}\right]\right)\right\}}{2\pi(1-e^{-ah})\sigma\sqrt{1-\rho}} \left(\frac{2\sqrt{2\pi}(2s+5)!!\, c_2^s}{\Delta^{2(s+1)}} + \right.\right.$$

$$+ \frac{\left(4c_1\Delta + \frac{4\pi\sigma\sqrt{1+\rho}}{d}\right) D_s^2(d, \bar{H}) \exp\left\{-\frac{1}{4}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}}{\left(1-\exp\left\{-\frac{1}{2}\left(\frac{\pi\sigma\sqrt{1-\rho}}{d}\right)^2\right\}\right)^2} +$$

$$\left.\left. + 2D_s(H) e^{-\frac{c_1^2\Delta^2}{2}} + \sigma\sqrt{1+\rho} \int\limits_{\frac{c_1\Delta}{\sigma\sqrt{1-\rho}} \leqslant |t| \leqslant \frac{\pi}{d}} |f_h(t)|^2\, dt \right)^{\frac{1}{2}} \right\}$$

for $A \in \mathfrak{N}_+$ or $A \in \mathfrak{N}_-$. Here $x = (a - m)/\sigma$, where $a = \inf\{y : y \in A\}$ for $A \in \mathfrak{N}_+$ and $a = \sup\{y : y \in A\}$ for $A \in \mathfrak{N}_-$; h is the solution of Eq. (2).

The statements of Lemmas 9 and 10 follow from (19) and (22) and Lemmas 1, 7, and 8.

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REDUCED STOCHASTIC EQUATIONS OF THE NONLINEAR FILTRATION OF RANDOM PROCESSES

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The general stochastic equations of nonlinear filtration of random processes, which describe the evolution of a posteriori distributions of the nonobservable component of the process, are essentially nonlinear. This circumstance makes investigation of questions of the existence and uniqueness of the solution of such equations and calculational procedures difficult. However, if stochastic equations describing the evolution of a posteriori distributions multiplied by a definite positive functional of an observable process dependent on the a posteriori distribution under consideration are examined, then they turn out to be linear. By using these reduced equations, important properties of the solutions of the initial equations of nonlinear filtration are successfully investigated. This method was used in [1-6] in a number of particular cases.

The purpose of the present paper is to derive general reduced stochastic equations of nonlinear filtration when the observable component is a random process with values in the half-space of an m-dimensional Euclidean space with boundary conditions. We obtain such equations in a particular case for observable locally infinitely divisible random processes. Reduced stochastic equations of nonlinear filtration are derived analogously for observable processes with values in a finite interval (compare [7-9]).

1. REDUCED STOCHASTIC EQUATION OF NONLINEAR FILTRATION OF RANDOM PROCESSES WITH OBSERVABLE COMPONENT IN R_+^m

Henceforth, we shall use the notation and results from [8].

Let an observable random process $X = \{X_t, t \geq 0\}$ with values in R_+^m have the local characteristics (a, A, δ, γ, β, B, π) relative to the family of σ-algebras $\{\mathscr{F}_t, t \geq 0\}$ and the measure P, i.e., have the following structure:

$$X_t^1 = X_0^1 + \int_0^t \chi_G(X_s)\, a_1(s)\, ds + \int_0^t \gamma(s)\, d\varphi_s + M_t^1 + \int_0^t \int_G x_1 p(ds, dx),$$

$$X_t^i = X_0^i + \int_0^t \chi_G(X_s)\, a_i(s)\, ds + \int_0^t \beta_i(s)\, d\varphi_s + M_t^i + N_t^i +$$

$$+ \int_0^t \int_{R_+^m} \chi(x)\, x_i\, q(ds, dx) + \int_0^t \int_{R_+^m} \bigl(1 - \chi(x)\bigr)\, x_i\, p(ds, dx), \quad i = 2, \ldots m,$$

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