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Stress-intensity factors for semi-elliptical surface cracks in welded joints

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Abstract. A method for calculating stress intensity factors for edge and surface cracks in weldments has been presented. The weight function method was applied and appropriate weight functions have been derived using the Petroski–Achenbach crack opening displacement expression. The derived weight functions account for both the global weldment geometry and the weld profile characterised by the weld angle.

Finally analysis of several parameters such as the type of loading, crack aspect ratio, weld angle and weld toe radius have been carried out assessing their effect on the stress intensity factor.

The calculated stress intensity factors were verified against available finite element data.

Very close agreement was achieved between the finite element data and the weight function based calculations.

Notation

a	crack length for an edge crack or depth for a semi-elliptical surface crack
c	half crack length for semi-elliptical surface crack
E	modulus of elasticity
F, $F(a/t)$	geometric stress intensity correction factor
F_r, $F_r(a/t)$	geometric correction factor for the reference stress intensity factor
F_{sf}^{α}	geometric stress intensity factor for an edge crack emanating from an angular corner α in a semi-finite plate with a step
F_{sf}^{90}	geometric stress intensity correction factor for a crack emanating from the right angle corner ($\alpha = 90°$) in a semi-finite plate with a step
$G(a/t)$	parameter of the crack opening displacement function
H	generalised modulus of elasticity $H = E$ – for plane stress $H = E/(1 - \nu^2)$ – for plane strain
h	weld leg length (or the step thickness)
$I_1(a)$, $I_2(a)$, $I_3(a)$	parameters of the crack opening displacement function
K	stress intensity factor
K_r	reference stress intensity factor corresponding to the local reference stress $\sigma_r(x)$ and nominal stress S_r
K_e^p	stress intensity factor for an edge crack in a flat plate
K_e^w	stress intensity factor for an edge crack in a weldment
K_s^p	stress intensity factor for a semi-elliptical surface crack in a flat plate
K_s^w	stress intensity factor for a semi-elliptical surface crack in a weldment
$m(x, a)$	weight function
$m(x, a, \alpha)$	weight function for an edge crack emanating from an angular corner α in a finite thickness plate with a step or weight function for an edge crack in a T-butt welded joint
$m_B(x, a)$	Bueckner's weight function for an edge crack in a flat plate
$m_s(x, a, c)$	weight function for a semi-elliptical surface crack in a flat plate
$\sqrt{Q} = \pi/2$	elliptical integral of second kind for a circular crack

S	characteristic stress or the nominal stress at the weld toe
S_r	characteristic stress or the nominal stress at the weld toe for the reference stress intensity factor
t	plate thickness
x	distance from the surface or from the corner tip
Y	geometric stress intensity correction factor for a surface semi-elliptical crack in a T-butt joint
$u(x, a)$, $u(x, a, c)$	crack opening displacement for an edge and semi-elliptical crack respectively
$u_r(x, a)$, $u_r(x, a, c)$	crack opening displacement function corresponding to the reference stress system $\sigma_r(x)$ for an edge and semi-elliptical crack respectively
W	width of the welded T-butt joint
α	weld angle or the corner angle
φ	angular coordinate for description of the elliptical crack front
Φ	elliptical integral of second kind for an elliptical crack
ϱ	weld toe radius
$\sigma(x)$	stress component normal to the crack surface, i.e., normal stress in plane x–z

1. Introduction

It has been recognised that a large proportion of the fatigue life of a welded structure can be spent on propagation of a crack [1] initiated at existing welding defects or stress concentration sites such as the weld toe. Thus prediction of the fatigue life of a welded joint has to be based on the accurate analysis of fatigue crack growth in the highly stressed region. This consequently requires development of fracture mechanics approaches to calculate stress intensity factors.

These calculations are usually complicated by the fact that semi-elliptical cracks have to be analysed in a three-dimensional stress state. It is known that the stress intensity factor for a crack in a welded joint depends on the global geometry of the joint, the weld profile, crack geometry and type of loading and it can be written in the form

$$K = S\sqrt{\pi a}F. \tag{1}$$

Calculation of the stress intensity factor even for one type of weldment such as a T-butt joint requires detailed analysis of several parameters including plate thickness, weld thickness, weld angle, weld toe radius and the loading system. For this reason it is necessary to derive appropriate formulae which can be used to calculate the stress intensity factors for a variety of loading systems and geometric parameters. Derivation of such formulae can be approached by using the weight function method [2, 3].

2. Derivation of weight functions using Petroski–Achenbach crack opening displacement function

The weight function $m(x, a)$ for a crack under mode I loading is a property of geometry and boundary conditions. In the case of prescribed tractions only the weight function becomes a unique property of geometry and it enables stress intensity factor to be calculated [4–6] for any stress/load system applied to a cracked body.

$$K = \int_0^a \sigma(x)m(x, a)\,dx. \tag{2}$$

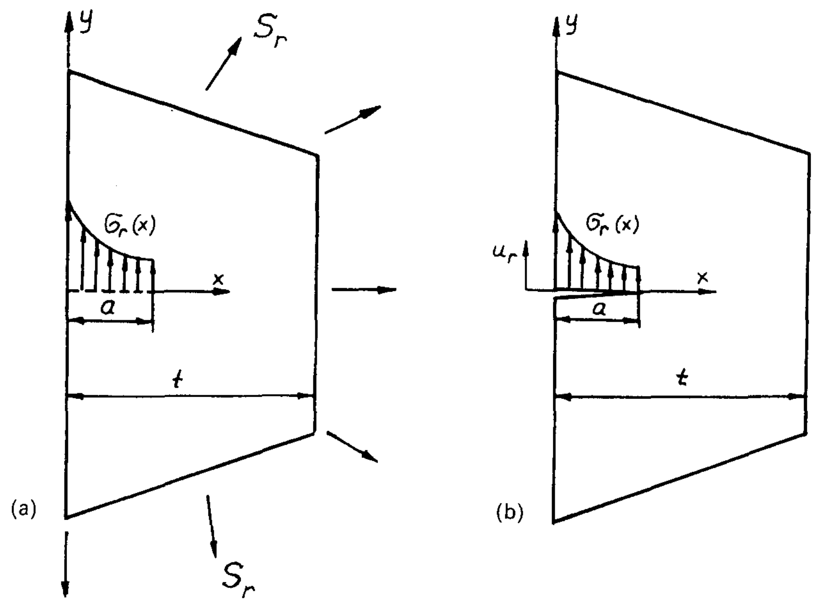

Fig. 1. Stress systems S_r, $\sigma_r(x)$ and displacements u_r necessary for the derivation of the weight function $m(x, a)$ for an edge crack. (a) Stress $\sigma_r(x)$ in uncracked body. (b) Notation and quantities necessary for calculating stress intensity factor.

It was shown by Bueckner [2] and Rice [3] that the weight function $m(x, a)$ for a two-dimensional cracked body (Fig. 1) can be written in the form

$$m(x, a) = \frac{H}{K_r} \cdot \frac{\partial u_r}{\partial a}. \tag{3}$$

In order to derive the weight function $m(x, a)$ as given above, a reference stress intensity factor K_r for a given geometry under any mode I stress system S_r, needs to be known together with corresponding crack opening displacements $u_r(x, a)$. An appropriate reference stress intensity factor K_r can often be found in the literature but the associated crack opening displacements $u_r(x, a)$ are difficult to find because these data very seldom accompany the stress intensity factor expressions.

To overcome this difficulty Petroski and Achenbach [7] proposed an approximate crack opening displacement function for edge cracks under mode I loading:

$$u_r = \frac{S_r}{H\sqrt{2}}\left[4F_r(a/t)\sqrt{a(a-x)} + \frac{G(a/t)(a-x)^{3/2}}{\sqrt{a}}\right], \tag{4}$$

where

$$F_r(a/t) = \frac{K_r}{S_r\sqrt{\pi a}}.$$

Thus, if the reference stress intensity factor K_r is known, the only unknown in (4) is function $G(a/t)$. Petroski and Achenbach [7] have shown that function $G(a/t)$ can be determined from the self-consistency of (2) by putting $K = K_r$, $\sigma(x) = \sigma_r(x)$ and substituting (3) for the weight function $m(x, a)$:

$$K_r = \frac{H}{K_r} \int_0^a \sigma_r(x) \cdot \frac{\partial u_r}{\partial a} \, dx. \tag{6}$$

After substituting (4) in (6) and solving it for $G(a/t)$ Petroski and Achenbach obtained the following expressions:

$$G(a/t) = \frac{[I_1(a) - 4F_r(a/t)\sqrt{a}I_2(a)]\sqrt{a}}{I_3(a)}, \tag{7}$$

where

$$I_1(a) = S_r\pi\sqrt{2} \int_0^a F_r^2(a/t)a \, da \tag{8}$$

$$I_2(a) = \int_0^a \sigma_r(x)(a - x)^{1/2} dx \tag{9}$$

$$I_3(a) = \int_0^a \sigma_r(x)(a - x)^{3/2} dx. \tag{10}$$

Further details about the practical application of this method can be found in [5–9].

3. Derivation of the weight function for an edge crack emanating from the weld toe of a T-butt welded joint

The displacement function (4) described in [7] was used for deriving the weight function $m(x, a, \alpha)$. It should also be pointed out that the problem illustrated in Fig. 2 is in general non-symmetric, leading to a mixed mode crack loading. However, it was discussed in [6] that the mode II contribution is not significant for the most frequent crack path shown on Fig. 2. The highest mode II contribution $K_{II}/K_I = 0.15$ can be found only for unrealistically high angles $\alpha = \pi/2$ or large weld toe radii $\varrho = 0.4t$. For small weld toe radii $1/15 < \varrho/t < 1/50$ and weld angles $\pi/3 < \alpha < \pi/6$ which usually occur in welded joints this effect is small and can be neglected.

The weight function $m(x, a, \alpha)$ for an edge crack emanating from the weld toe in a T-butt joint (Fig. 2) was finally given [6] in the polynomial form (11)

$$m(x, a, \alpha) = \frac{F_{sf}^{\alpha}}{F_{sf}^{90}} M(x, a, 90), \tag{11}$$

where

$$M(x, a, 90) = \frac{2}{\sqrt{2\pi(a - x)}} \left[1 + M_1\left(\frac{a - x}{a}\right) + M_2\left(\frac{a - x}{a}\right)^2\right] \tag{12}$$

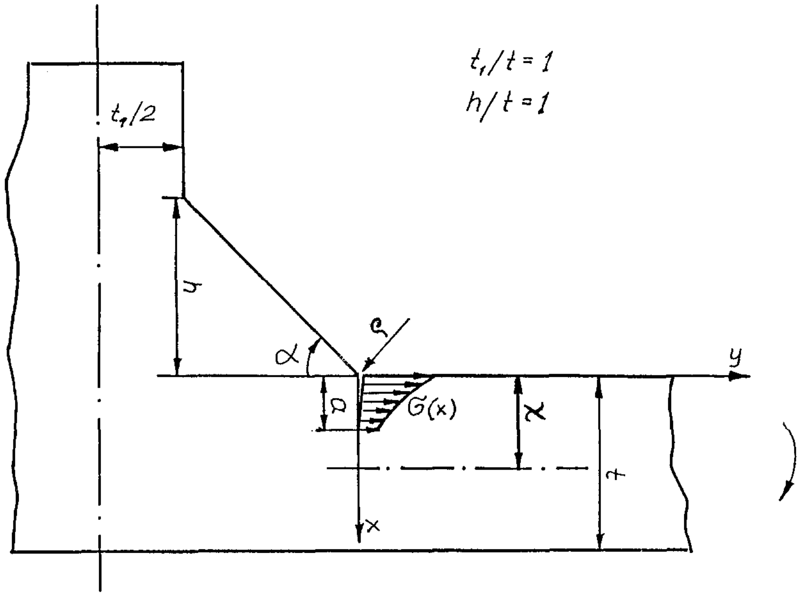

Fig. 2. Geometry of the T-butt joint with an edge crack.

and

$$M_1 = 0.6643 - 12.7438(a/t)^{1.5} + 397.8081(a/t)^{3.0} - 3285.1810(a/t)^{4.5} + 14162.5870(a/t)^{6.0} - 30127.1580(a/t)^{7.5} + 25119.5350(a/t)^{9.0}$$

$$M_2 = 0.1117 + 3.8570(a/t)^{1.5} - 47.1626(a/t)^{3.0} + 285.4393(a/t)^{4.5} - 646.6118(a/t)^{6.0} + 934.4538(a/t)^{7.5} - 596.8319(a/t)^{9.0}$$

$$\frac{F_{sf}^{\alpha}}{F_{sf}^{90}} = 1 + \left(\frac{6}{\pi}\alpha - 2\right)[1 - f(a/h)] \; for \; \frac{\pi}{6} \leqslant \alpha \leqslant \frac{\pi}{3}$$

$$f(a/h) = 1.0355 - 3.3324(a/h)^{0.5} + 21.5999(a/h)^{1.0} - 58.8513(a/h)^{1.5} + 81.6246(a/h)^{2} - 56.9396(a/h)^{2.5} + 15.8784\,(a/h)^{3.0}.$$

The weight fraction (11) makes it possible to calculate stress intensity factors for edge cracks [6] in T-butt joints subjected to any applied mode I stress system $\sigma(x)$. It also enables account to be made of the effect of the global weldment geometry and the effects of the weld profile characterised by the weld angle α. For short cracks the weight function (11) tends to Bueckner's weight function [10] derived for edge cracks in flat plates. The effect of the finite weld toe radius or the corner tip radius can be evaluated by using appropriate stress field $\sigma(x)$ in (2) corresponding to given radius ϱ. Some characteristic through the thickness stress distributions $\sigma(x)$ accounting for weld toe radius ϱ were presented in [6].

4. Weight function and stress intensity factors for a semi-elliptical surface crack emanating from the weld toe of a T-butt welded joint

Mattheck et al. [8, 9] have shown that the method proposed by Petroski and Achenbach [7] can also be used for the elliptical surface cracks in flat plates. A similar method was also used in the present work, related to cracks in welded joints (Fig. 3).

The crack opening displacement field $u_r(x, a, c)$ along axis x and normal to the plane x–z for a semi-elliptical surface crack in a flat plate was determined first in order to derive the weight function and to calculate the stress intensity factor at the deepest crack point. It should be mentioned that in general crack opening displacements depend on both coordinates x and z (Fig. 3). However, in order to calculate the stress intensity factor at the deepest point of the crack front the displacements along axis x only are sufficient. The displacements $u_r(x, a, c)$ were calculated from (4) in which unknown parameters were determined from (6) and the reference stress intensity factor (14) derived by Newman

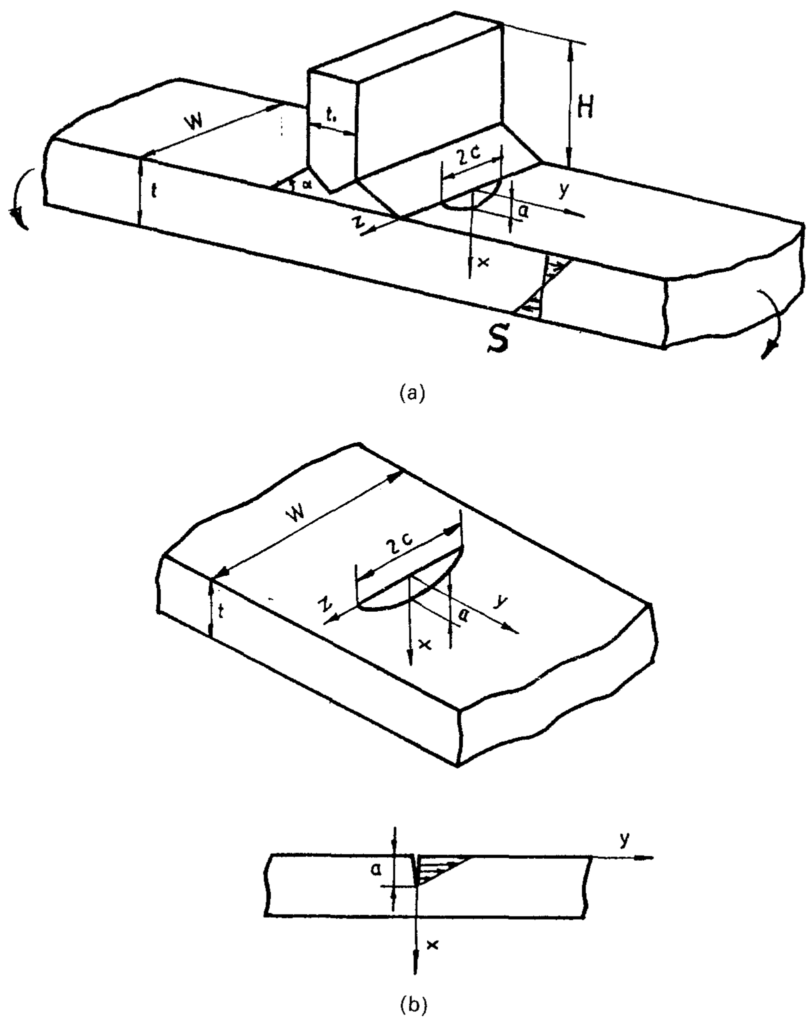

Fig. 3. Notation for semi-elliptical surface cracks. (a) Semi-elliptical surface crack emanating from the weld toe of a T-butt welded joint, (b) Semi-elliptical surface crack in a flat plate under varying surface pressure.

and Raju [11] for semi-elliptical crack under uniform tensile stress:

$$K_r = S_r\sqrt{\pi a}\,F_r(a/t,\ a/c,\ c/W,\ \varphi), \tag{14}$$

where

$$F_r = [A_1 + A_2(a/t)^2 + A_3(a/t)^4]\cdot g(a/t,\ \varphi)\cdot f(a/c,\ \varphi)\cdot h(a/t,\ c/W)/\sqrt{\Phi}$$

$$A_1 = 1.13 - 0.09(a/c)$$

$$A_2 = -0.54 + \frac{0.89}{0.2 + a/c}$$

$$A_3 = 0.5 - \frac{1}{0.65 + a/c} + 14(1 - a/c)^{24}$$

$$g(a/t,\ \varphi) = 1 + [0.1 + 0.35(a/t)^2](1 - \sin\varphi)^2$$

$$f(a/c,\ \varphi) = [(a/c)^2\cos^2\varphi + \sin^2\varphi]^{1/4}$$

$$h(a/t,\ c/W) = \sqrt{\sec\left(\frac{\pi c}{W}\sqrt{a/t}\right)}$$

$$\Phi = 1 + 1.464(a/c)^{1.65}.$$

Having determined (Fig. 4) the crack opening displacement field $u_r(x,\ a,\ c)$ and consequently the weight function $m_s(x,\ a,\ c)$ for the deepest point of the semi-elliptical crack, it is possible to calculate (15) stress intensity factors for any stress field $\sigma(x)$ normal to the crack surface and different from $\sigma_r(x)$:

$$K_s^p = \int_0^a \sigma(x)m_s(x,\ a,\ c)\mathrm{d}x. \tag{15}$$

However, the weight function $m_s(x,\ a,\ c)$ for a semi-elliptical crack in a flat plate had to be calculated for each individual set of parameters a/t and a/c. The possibility of obtaining closed forms of appropriate weight functions has been discussed in [12].

It should be noted that the calculations were based on the assumption that the crack opening displacement function (4), derived for an edge crack, was also applicable for the crack opening displacement along axis x for a surface semi-elliptical crack. The results presented by Mattheck et al. [8, 9] seem to justify this assumption.

In addition to the above assumption it was also assumed in this paper that the contributions of the weldment geometry to the stress intensity factor for edge cracks were the same as in the case of semi-elliptical cracks (Fig. 3). In other words the ratio of the stress intensity factor for an edge crack in a flat plate to that of an edge crack in a weldment was the same

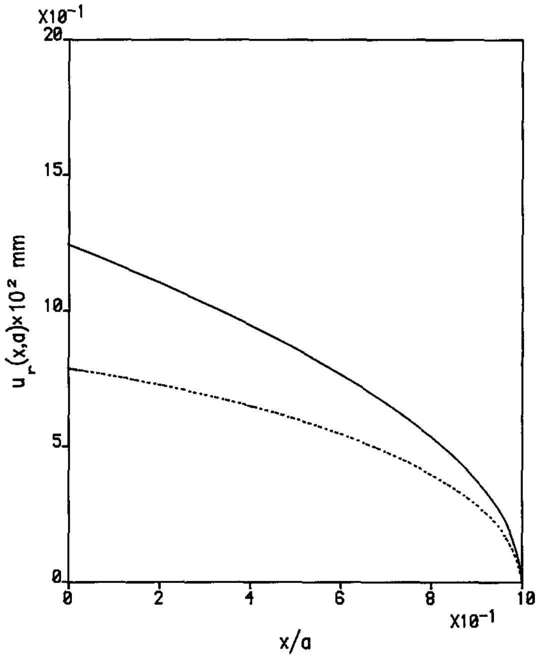

Fig. 4. Crack opening displacement distributions along axis x and normal to the plane x–z for surface semi-elliptical cracks in a flat plate under tension ($a/c = 0.2$, $a = 5\,\mathrm{mm}$, $S = 100\,\mathrm{MPa}$, $E = 2 \times 10^5\,\mathrm{MPa}$). ---- $a/t = 0.5$. —— $a/t = 0.8$.

as analogous ratio for semi-elliptical cracks of the same depth and under the same stress system $\sigma(x)$:

$$\frac{K_e^p}{K_e^w} = \frac{K_s^p}{K_s^w}. \tag{16}$$

Equation (16) can be re-arranged in the form

$$K_s^w = \frac{K_e^w}{K_e^p} K_s^p. \tag{17}$$

Thus, the stress intensity factor K_s^w for the deepest point of a semi-elliptical crack in a weldment can be calculated if the stress intensity factor K_s^p for a similar crack in a flat plate is known together with appropriate stress intensity factors K_e^w and K_e^p for edge cracks of the same depth a.

In order to calculate the stress intensity factor K_s^w for the deepest point of a semi-elliptical crack in a welded joint the following procedure is recommended.

(a) Calculate the local stress distribution $\sigma(x)$ normal to the plane x–z in the uncracked welded joint under consideration. Some information regarding stress distributions in T-butt joints was given in [6].

(b) Calculate the stress intensity factor K_e^p for an edge crack in a flat plate under the local stress system $\sigma(x)$ determined for the uncracked welded joint. This requires integration of Bueckner's weight function (18) derived for an edge crack in a flat plate [10]:

$$m_B(x, a) = \frac{2}{\sqrt{2\pi(a - x)}}\left[1 + m_1\left(\frac{a - x}{a}\right) + m_2\left(\frac{a - x}{a}\right)^2\right] \tag{18}$$

$$m_1 = A_1 + B_1(a/t)^2 + C_1(a/t)^6; \quad m_2 = A_2 + B_2(a/t)^2 + C_2(a/t)^6;$$

$$A_1 = 0.6147; \quad B_1 = 17.1844; \quad C_1 = 8.7822$$

$$A_2 = 0.2502; \quad B_2 = 3.2899; \quad C_2 = 70.0444.$$

The stress intensity factor K_e^p can finally be written in the form

$$K_e^p = \int_0^a \sigma(x) m_B(x, a) \mathrm{d}x. \tag{19}$$

(c) Calculate the stress intensity factor K_e^w for an edge crack in the weldment using the weight function (11):

$$K_e^w = \int_0^a \sigma(x) m(x, a, \alpha) \mathrm{d}x. \tag{20}$$

(d) Calculate the stress intensity factor K_s^p for the deepest point of the semi-elliptical crack in a flat plate under the same stress system $\sigma(x)$ using relation (15).

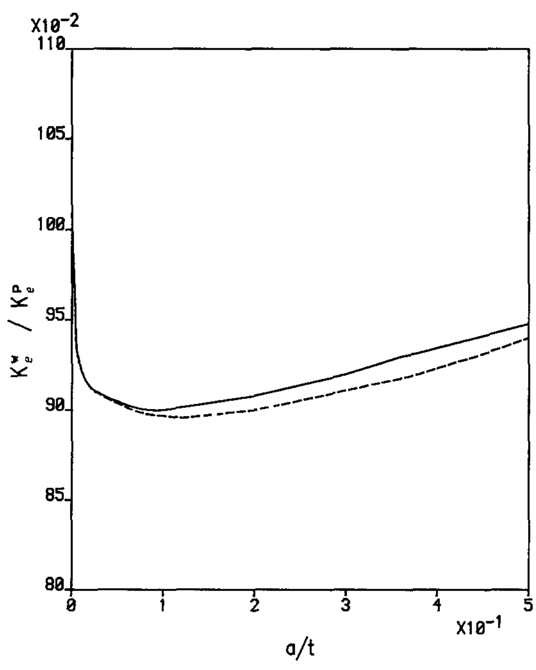

Fig. 5. An effect of the loading stress system $\sigma(x)$ on the ratio of stress intensity factors K_e^w/K_e^p. —— tension. ---- bending.

(e) Calculate the stress intensity factor K_s^w at the deepest point of the semi-elliptical crack in the weldment using (17). The stress intensity factor can be finally written in the form of expression (21):

$$K_s^w = S\sqrt{\pi a}Y. \tag{21}$$

In practice only geometric factor Y has to be calculated from the weight function and therefore all data given below are discussed in terms of factor Y.

Thus, the calculations require derivation of the weight functions for an edge crack in the weldment and for a surface semi-elliptical crack in a flat plate. It may be worth noting that the stress intensity factor ratio K_e^w/K_e^p was found almost to be independent of the stress system $\sigma(x)$, as illustrated in Fig. 5. Therefore the ratio K_e^w/K_e^p calculated once for one simple stress distribution e.g., ($\sigma(x)$ = const.) can be used in (17) for other more complex local systems $\sigma(x)$. However, this has to be calculated for each individual weld angle α.

It is also important to note that the geometry effect represented by the ratio K_e^w/K_e^p was not strongly dependent on the crack length. Its value was (Fig. 5) varying within the range $0.9 \leqslant K_e^w/K_e^p \leqslant 0.95$ over the crack length range $0.03 \leqslant a/t \leqslant 0.5$, for the weld angle $\alpha = 45°$.

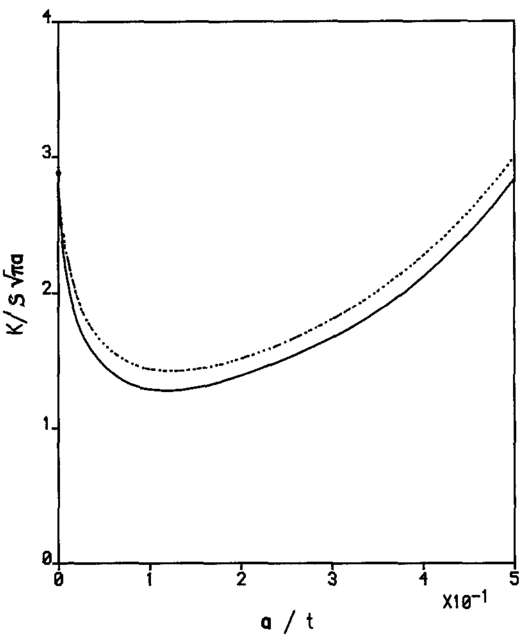

Fig. 6. Weight function effect on stress intensity factor for an edge crack in a T-butt joint under tension ($\alpha = 45°$, $\varrho/t = 1/25$). ---- Bueckner's weight function (18). —— present weight function (11).

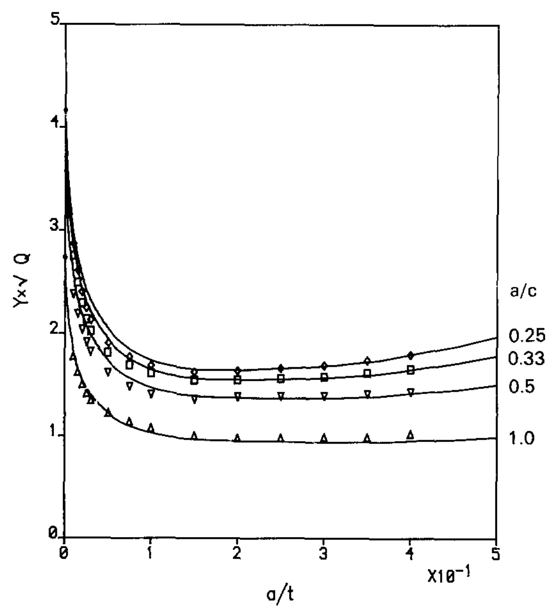

Fig. 7. Comparison of the weight function based stress intensity factors with finite element calculations for the deepest point of a semi-elliptical crack in a T-butt joint under tension ($\alpha = 45°$, $\varrho/t = 1/25$). △, ▽, □, ◇ finite element calculations [13]. —— weight function method (17).

5. Validation of calculated stress intensity factors against finite element data

Stress intensity factors for edge cracks

The stress intensity factors for an edge crack in a T-butt welded joint under bending were analysed in [6] and it was found that the results obtained by using the method described above were in excellent agreement with available finite element data. It was also shown that stress intensity factors calculated on the basis of the weight function (11) derived for a crack in a T-butt weldment were lower than those calculated on the basis of Beuckner's function (18). A similar situation was also found in the case of edge cracks under tensile loading (Fig. 6). The difference between these two solutions varied and depended on weld angle α.

Stress intensity factors at the deepest point of a semi-elliptical crack in a T-butt joint under tensile loading

The method described above was verified against available literature data. Both pure bending and pure tension loads were analysed. Figure 7 shows the comparison of stress intensity factors calculated from (17) with the finite element results obtained by Bell [13]. It can be seen that, the agreement is very good for wide ranges of crack depth a/t and crack aspect ratios a/c. Some discrepancy occurred for long shallow cracks with aspect ratios

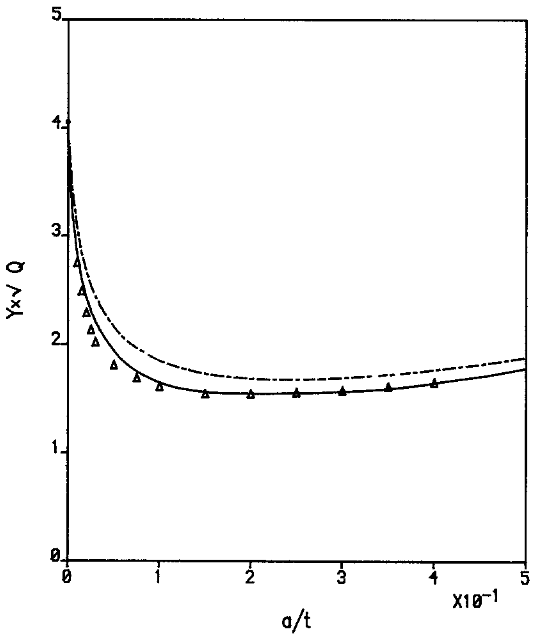

Fig. 8. Effect of weight function on calculated stress intensity factor for the deepest point of a semi-elliptical crack in a T-butt joint under tension (α = 45°, ϱ/t = 1/25, a/c = 0.33). ---- weight function for a semi-elliptical crack in a flat plate (15). —— weight function for a semi-elliptical crack in a T-butt joint (17). △ finite element calculations [13].

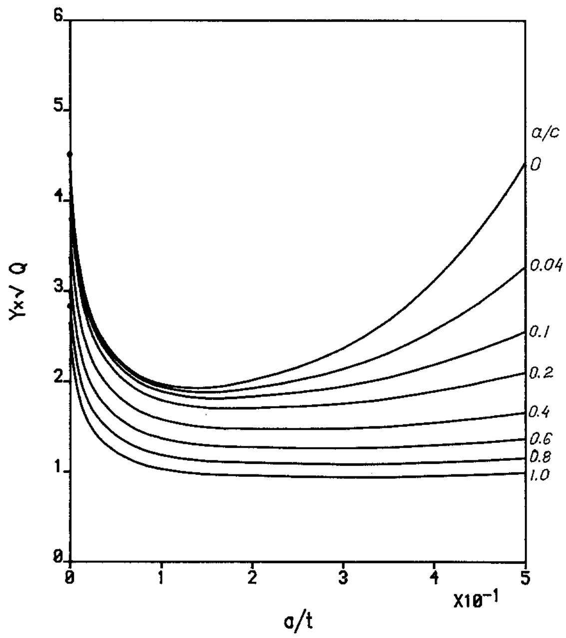

Fig. 9. Aspect ratio a/c effect on stress intensity factor for the deepest point of a semi-elliptical crack in a T-butt weldment under tension, α = 45°, ϱ/t = 1/25.

$a/c < 1/3$ and depths $a/t < 0.1$. However this could be due to numerical errors introduced by finite element calculation.

It is also worth mentioning that the stress intensity factors calculated on the basis of the weight function for a semi-elliptical crack in a flat plate were higher (Fig. 8) than those calculated from (17) or by using the finite element method. This suggests that by using the flat plate weight function one will overestimate the stress intensity factor in the weldment. Figure 9 illustrates the effect of the aspect ratio a/c on the stress intensity factor at the deepest point of a semi-elliptical crack in a T-butt weldment under tensile loading. It was found that changes in the aspect ratio a/c greatly affected the stress intensity factor especially for values $0 < a/c < 0.3$. For cracks with aspect ratio $a/c > 0.3$ and depth $a/t > 0.2$ the geometry factor $Yx\sqrt{Q}$ for given aspect ratio a/c was almost constant up to half of the thickness. The constant $\sqrt{Q} = \pi/2$ was used in order to compare the calculated Y factors with Bell's [12] finite element data. Bell normalised his results with respect to semi-circular surface crack stress intensity factors.

Stress intensity factors at the deepest point of a semi-elliptical crack in a T-butt joint under bending load

The same procedure as above and (17) were used for calculating stress intensity factors under bending load. Figure 10 shows the results of the calculation against finite element data [13]. Again, good agreement has been achieved for both weld angles $\alpha = 30°$ and $\alpha = 45°$.

The effect of the weight function on calculated stress intensity factors is illustrated in Fig. 11. Application of the weight function derived for a crack in a flat plate resulted in overestimation of the stress intensity factor in the weldment. Better agreement with the finite element data was achieved for results obtained from (17). The differences between the results

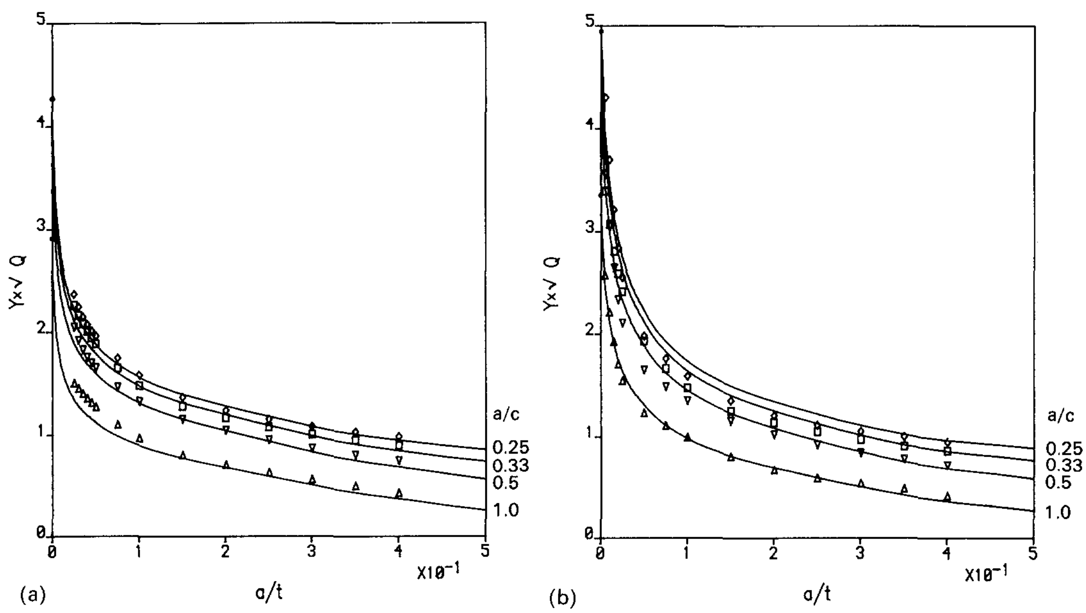

Fig. 10. Comparison of the weight function based stress intensity factors with finite element calculations for the deepest point of a semi-elliptical crack in a T-butt joint under bending, $\varrho/t = 1/25$. △, ▽, □, ◇ finite element calculations [13]. —— weight function (17). (a) $\alpha = 30°$. (b) $\alpha = 45°$.

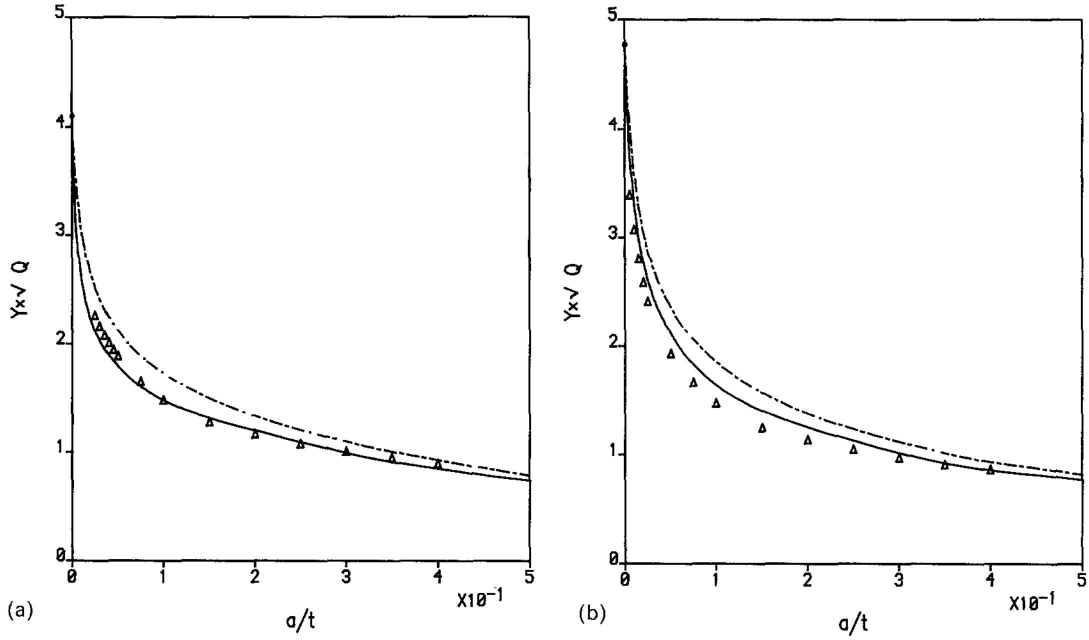

Fig. 11. Effect of weight function on calculated stress intensity factor for the deepest point of a semi-elliptical crack in a T-butt joint under bending; $a/c = 0.33$, $\varrho/t = 1/25$ ------ weight function for a semi-elliptical crack in a flat plate (15). —— weight function for a semi-elliptical crack in a T-butt joint (17). △ finite element calculations [13]. (a) $-\alpha = 30°$. (b) $-\alpha = 45°$.

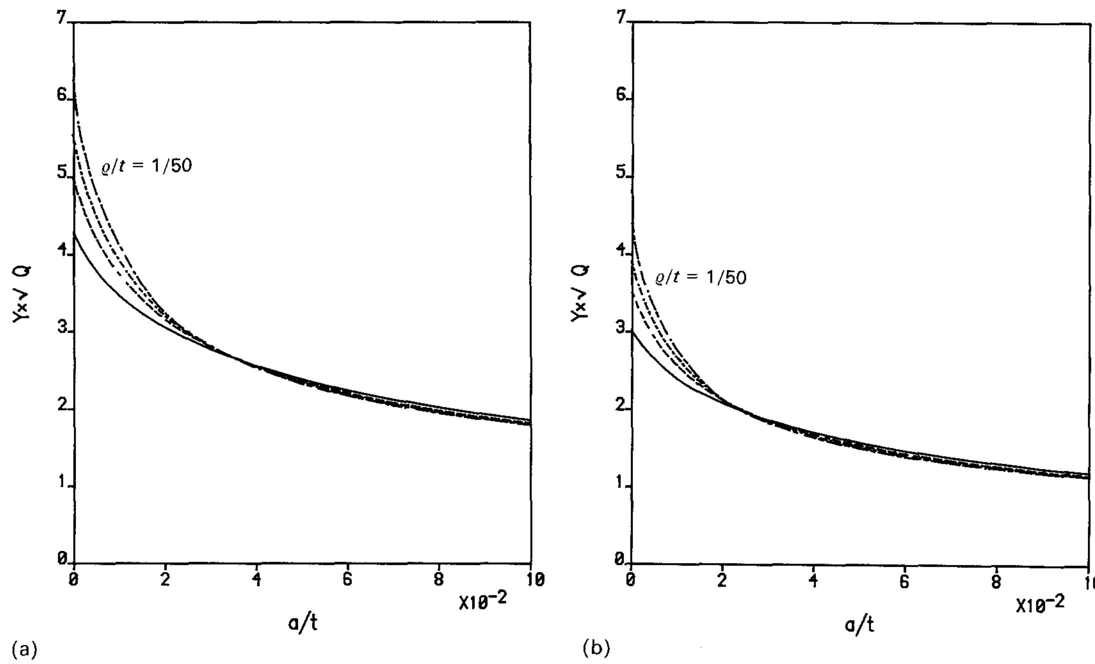

Fig. 12. Weld toe radius ϱ effect on stress intensity factor at the deepest point of a semi-elliptical crack in a T-butt joint under bending loading. (a) $a/c = 0.2$; $\varrho/t = 1/15, 1/25, 1/35, 1/50$; $\alpha = 45°$. (b) $a/c = 0.8$; $\varrho/t = 1/15, 1/25, 1/35, 1/50$; $\alpha = 45°$.

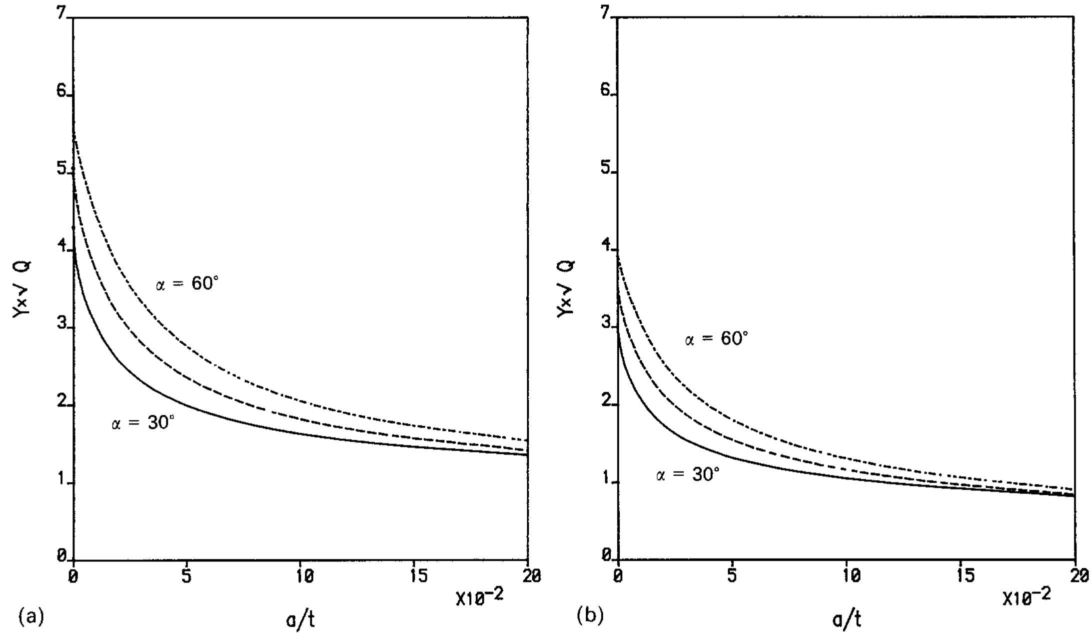

Fig. 13. Weld angle α effect on stress intensity factor at the deepest point of a semi-elliptical crack in a T-butt joint under bending. (a) $\alpha = 30°, 45°, 60°$; $a/c = 0.2$; $\varrho = 1/25$. (b) $\alpha = 30°, 45°, 60°$; $a/c = 0.8$; $\varrho = 1/25$.

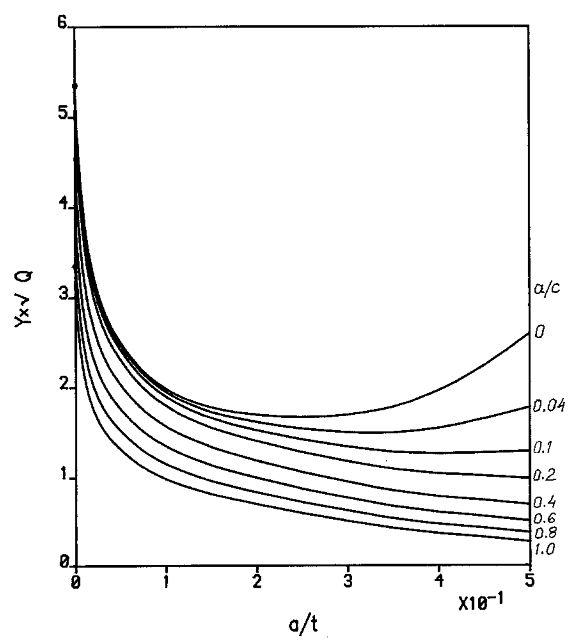

Fig. 14. Aspect ratio a/c effect on stress intensity factor at the deepest point of a semi-elliptical crack in a T-butt joint under bending. $\alpha = 45°$, $\varrho/t = 1/25$.

obtained from the two weight functions were smaller under bending than in the case of analogous cracks under tension loading. The differences between the two weight functions were almost negligible for deep cracks $a/t > 0.5$ where the crack tip was far away from the weld toe stress concentration region.

It is also apparent that the effect of the weld toe radius ϱ (Fig. 12) was noticeable for shallow cracks only with $a/t < 0.03$. The weld toe radius effect on the stress intensity factor was negligible for deep cracks and it can be neglected for the whole range of weld toe radii analysed in this paper. The weld toe radius effect was calculated by substituting for $\sigma(x)$ in (15), (19) and (20), where the stress field $\sigma(x)$ is associated with given weld toe radius ϱ.

It was apparent that the weld angle α was more influential (Fig. 13) than the weld toe radius ϱ. The effect of weld angle α was more significant for cracks with aspect ratios $a/c < 0.2$ but it was almost negligible for deep cracks $a/t > 0.5$ for all tested aspect ratios a/c.

Finally the effect of the aspect ratio a/c on stress intensity factor was studied using (17). The results shown in Fig. 14 revealed that the most sensitive to aspect ratio changes are deep cracks for $a/t > 0.25$. The deeper the crack is, the more significant the aspect ratio effect will be. It is also found that for long and shallow cracks the stress intensity factor tends to the edge crack solution multiplied by the stress concentration factor K_t. In the case of crack depth, small in comparison to plate thickness, the solution tends to the stress intensity factor for a surface crack emanating from a corner in a semi-infinite body.

6. Discussion

It was shown above that the weight function method leads to reasonable estimates of stress intensity factors, making it possible to analyse the effect of geometry and loading conditions. The whole analysis was addressed to mode I cracking and one should bear in mind this limitation because a weld toe crack might be under the influence of mixed-mode cracking. The available experimental data shows, however, that the mode II contribution is usually small and it is very often sufficient in engineering practice to consider mode I cracking only.

It is also known that the weight function method requires calculation of the stress distribution in the potential crack plane of an uncracked body. The weight functions discussed above are limited to one-dimensional stress distributions $\sigma(x) = f(x)$.

The method presented above relates to calculating stress intensity factors for the deepest point of a semi-elliptical crack. However, the stress intensity factor at the end of a crack on the surface needs to be calculated if fatigue crack growth analysis is to be performed. It was shown by Mattheck et al. [9] that a similar method can be applied to both the deepest crack point and to the crack ends on the surface.

7. Conclusions

It was shown that the weight function method can be successfully used for calculating stress intensity factors for both edge and semi-elliptical surface cracks in T-butt weldments. The calculation of stress intensity factors derived for semi-elliptical cracks can be simplified by using weight functions derived for edge cracks, as the relative weldment geometry effect on the stress intensity factor is the same for both edge and semi-elliptical cracks and it is almost independent of the loading system.

The derived weight functions made it possible to analyse the effect of several parameters on the stress intensity factor such as the weld toe, weld angle and loading system. It was found that the effect of weld toe radius on the stress intensity factor was significant for short cracks of $a/t < 0.03$ deep. The weld angle effect was apparent over a larger region covering cracks of depths $a/t < 0.3$. It was also shown that the effect of weld geometry was dependent on the crack aspect ratio a/c and the type of loading (i.e., bending or tension). The weld geometry effect was insignificant for deep cracks i.e., $a/t > 0.5$. Therefore, for cracks $a/t > 0.5$ deep, the weight functions derived for cracks in flat plates might be used in engineering practice.

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Résumé. On présente une méthode de calcul des facteurs d'intensité de contraintes dans le cas de fissures de bord et de surface dans des soudures. On applique la méthode des fonctions pondérales et, en utilisant l'expression du COD proposée par Petroski et Achenbach, on tire les fonctions pondérales appropriées, qui tiennent compte de la géométrie générale de la soudure et du profil du joint, caractérisé par l'angle à la racine. Enfin, on a procédé à l'analyse de divers paramètres tels que le type de sollicitation, l'aspect de la fissure, l'angle et le rayon du congé à la racine, en vue de déterminer leur effet sur le facteur d'intensité des contraintes.

On vérifie les facteurs d'intensité de contraintes par rapport aux données par éléments finis qui sont disponibles. On obtient un accord très proche entre les données numériques venant des éléments finis et les calculs basés sur les fonctions pondérales.