Z angew Math Phys 43 (1992)
(ZAMP)

0044-2275/92/020385-12 \$ 1.50 + 0.20

A linear stability analysis of a mixed convection plume from a line source: higher order effects

By Ramesh Krishnamurthy, Dept of Mechanical Engineering, The Catholic University of America, Washington D.C. 20064, USA

Introduction

Thermal plane plumes arise in nature as well as in many industrial applications. In laboratory simulations of these flows, in order to stabilize the flow a small externally imposed stream is sometimes employed. In this situation, the buoyancy force and the free stream are in the same direction. This flow configuration is termed aiding mixed convection. Also, mixed convection plumes arise from the cylindrical sensing elements of hot-wire anemometers. In this circumstance, the flow configuration can be aiding or opposed or orthogonal depending upon the relative orientations of the sensing element and the flow. In a recent paper, Krishnamurthy [1] reported an analysis of the stability of mixed convection from a line source of heat. In [1] only first order mixed convection effects were considered. In the present paper those results have been extended to include second order mixed convection effects.

Analyses of laminar mixed convection from a horizontal line source of heat have been reported in a number of recent studies. These include the earliest by Wood [2], followed by those of Wesseling [3], Afzal [4], Haaland and Sparrow [5], Krishnamurthy and Gebhart [6] and Wilks and Hunt [7]. These studies clearly indicated that in an aiding mixed convection situation, the flow field downstream of the source can be divided into three distinct regions. Near the source, the flow field is dominated by forced convection effects. In this region the force due to buoyancy is small enough to be regarded as a perturbation on the viscous and pressure forces. Far downstream of the source, buoyancy effect dominates the flow field. In this region the presence of the free stream is a perturbation on the far-field boundary condition for the u-component of the velocity. See Fig. 1. In between these two regions lies the true mixed convection region where both the effects are important.

In this paper, the stability of such flows to small disturbances is investigated in terms of the linear stability theory. The buoyancy force and

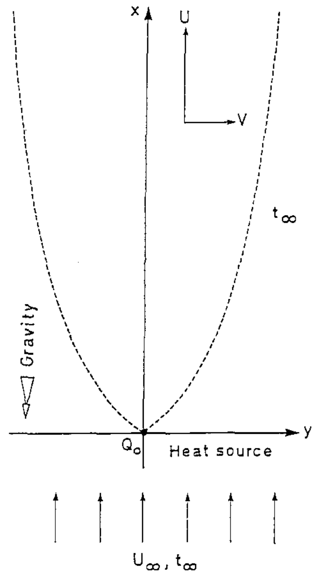

Figure 1
Flow configuration.

the free stream flow are taken to be in the same direction. The region sufficiently downstream of the source is considered, where buoyancy effects dominate. In this flow, as in other aiding mixed convection flows, the presence of the free stream enhances the stability of the flow. The physical mechanism underlying the stabilization is related to the lessening of the inflection of the velocity profile of the base flow. Details of this line of explanation can be found in [8]. The important issue addressed by this paper is the quantitative extent of the stabilization.

The present analysis includes the effects of two distinct perturbations. The first is that due to the free stream velocity on the far-field boundary condition on the u-component. This perturbation is termed the mixed convection effect and is characterized by the parameter ε_M. Also taken into account is the first order correction to the "Classical" boundary layer solution to the laminar natural convection plume. This correction results from the interaction of the plume with the irrotational flow outside the boundary layer. This perturbation is termed the higher-order effect and is characterized by ε_H. The base flow is taken to be the classical natural convection plume perturbed by ε_M and ε_H. The stability analysis is then performed by expanding the disturbance field too, in terms of these two perturbation parameters. These two perturbation parameters have been so chosen that at zero order, the governing equations reduce to that of the laminar natural convection plume. Computed results are presented and discussed for $\mathrm{Pr} = 0.7$.

Analysis

The mixed convection flow arising from an infinitely long horizontal line source of heat is considered as a two-dimensional steady flow. See Fig. 1 for a sketch of the flow configuration. With the usual Boussinesq approximations, neglecting viscous dissipation and pressure terms in the energy equation, the full two-dimensional governing equations take the form,

$$\psi_y \frac{\partial}{\partial x}(\nabla^2\psi) - \psi_x \frac{\partial}{\partial x}(\nabla^2\psi) - \nu\nabla^4\psi - g\beta\frac{\partial T}{\partial y} = 0 \tag{1}$$

$$\psi_y \frac{\partial T}{\partial x} - \psi_x \frac{\partial T}{\partial y} = \frac{\nu}{\mathrm{Pr}}(T_{xx} + T_{yy}) \tag{2}$$

where the stream function ψ has been so defined that,

$$u = \psi_y \quad \text{and} \quad v = -\psi_x$$

Boundary conditions are:

$$y = 0, \quad \psi = \psi_{yy} = T_y = 0; \quad \text{for all } x \tag{3}$$

$$y \to \infty, \quad \psi_y \to U_\infty, \quad T \to T_\infty; \quad \text{for all } x \tag{4}$$

Also for $x > 0$, the convected energy is,

$$Q(x) = \int_{-\infty}^{\infty} \varrho c_p \psi_y (T - T_\infty)\, dy = Q_0 = \text{Constant} \tag{5}$$

where Q_0 is the thermal input per unit length of the line source.

In the region $y < O(\delta)$, the base flow can be represented as,

$$\bar{\psi} = U\delta(F_1(\eta) + \varepsilon_M F_2(\eta) + \varepsilon_M^2 F_3(\eta) + \varepsilon_H F_4(\eta)) \tag{6}$$

and

$$\bar{T} - T_\infty = \Delta T(H_1(\eta) + \varepsilon_M H_2(\eta) + \varepsilon_M^2 H_3(\eta) + \varepsilon_H H_4(\eta)) \tag{7}$$

where the governing equations and corresponding boundary conditions for F_i and H_i, $i = 1, 4$, are given in the Appendix. It has been shown in [6] that the base flow as given in equations (6) and (7) satisfies the integral constraint in (5).

In the usual manner for Linear stability analyses, we superimpose on the base flow an arbitrarily small disturbance of the form,

$$\tilde{\psi} = U\delta S(\eta) \exp(i(\Lambda(x) - \omega\tau)) + \text{c.c.} \tag{8}$$

$$\tilde{T} = \Delta T\phi(\eta) \exp(i(\Lambda x) - \omega\tau)) + \text{c.c.} \tag{9}$$

where "c.c." denotes complex conjugate and S, ϕ and Λ are complex and ω is taken to be real. Also, $\tilde{u} = \tilde{\psi}_y$ and $\tilde{v} = -\tilde{\psi}_x$. In analyses of the stability of non-parallel flows, a slowly varying amplitude factor is sometimes included in the expressions (8) and (9). This is done in order to account for the steamwise variations in wavenumber and in the eigen function. An example of this approach may be found in Gaster [9]. However, in the present problem since the region far downstream of the source is considered, the Grashof number is sufficiently large to allow us to neglect the streamwise variations in the amplitude and essentially regard it to be a constant.

Each flow variable is represented by the sum of its base flow component and the disturbance component. Then by subtracting the base flow equations from the complete two-dimensional, time-dependent governing equations and combining the x- and y-momentum equations to eliminate pressure terms, the vorticity and energy equations for the disturbance components are obtained. The linearized forms of these equations are given below. Note that in these equations the base flow is not assumed to be parallel.

$$\frac{\partial \tilde{\zeta}}{\partial \tau} + \bar{u}\frac{\partial \tilde{\zeta}}{\partial x} + \tilde{u}\frac{\partial \bar{\zeta}}{\partial x} + \tilde{v}\frac{\partial \bar{\zeta}}{\partial y} + \bar{v}\frac{\partial \tilde{\zeta}}{\partial y} + \nu\nabla^2\tilde{\zeta} - g\beta\frac{\partial \tilde{T}}{\partial y} \tag{10}$$

$$\frac{\partial \tilde{T}}{\partial \tau} + \tilde{u}\frac{\partial \bar{T}}{\partial x} + \bar{u}\frac{\partial \tilde{T}}{\partial x} + \tilde{v}\frac{\partial \bar{T}}{\partial y} + \bar{v}\frac{\partial \tilde{T}}{\partial y} = \frac{\nu}{Pr}(\nabla^2\tilde{T}) \tag{11}$$

In stability analyses of natural convection boundary layers, an approach that has been successfully used in the past is to exploit the linearity of the disturbance equation by representing the disturbance field as,

$$S = \bar{S}_1 + B_2\bar{S}_2 + B_3\bar{S}_3 \tag{12}$$

$$\phi = \bar{\phi}_1 + B_2\bar{\phi}_2 + B_3\bar{\phi}_3 \tag{13}$$

where each $(\bar{S}_j, \bar{\phi}_j)$ is an integral of the coupled Orr-Sommerfeld euqations, with $j = 1$ corresponding to the inviscid limit and $j = 2, 3$, being characterized by viscous effects. This very approach has also been successfully used by Carey and Gebhart [10] in analyzing the stability of an aiding mixed convection boundary layer flow adjacent to a vertical uniform-flux surface. However, in boundary-free flows such as plumes, B_2 and B_3 have to be identically zero as pointed out by Lin [11] and discussed by Hieber and Nash [12]. A more appropriate method is to expand the disturbance field in terms of the perturbation parameters as done in [12]. Thus,

$$S = S_1(\eta) + \varepsilon_M S_2(\eta) + \varepsilon_M^2 S_3(\eta) + \varepsilon_H S_4(\eta) \tag{14}$$

$$\phi = \phi_1(\eta) + \varepsilon_M \phi_2(\eta) + \varepsilon_M^2 \phi_3(\eta) + \varepsilon_H \phi_4(\eta) \tag{15}$$

$$\Lambda = \Lambda_1(x) + \varepsilon_M \Lambda_2(x) + \varepsilon_M^2 \Lambda_3(x) + \varepsilon_H \Lambda_4(x) \tag{16}$$

Additional quantities that arise are, non-dimensional frequency Ω, complex wave number α and the complex wave speed c, given by:

$$\Omega = \delta\omega/U$$

$$\alpha = \delta \frac{d\Lambda}{dx} = \alpha_1 + \varepsilon_M \alpha_2 + \varepsilon_M^2 \alpha_3 + \varepsilon_H \alpha_4 \quad (17)$$

$$c = \Omega/\alpha = c_1 + \varepsilon_M c_2 + \varepsilon_M^2 c_3 + \varepsilon_H c_4 \quad (18)$$

Here ω the value of the frequency of the disturbance will be taken as real.

Substituting the expressions for $\bar{u}$, $\tilde{u}$, $\bar{v}$, $\tilde{v}$, $\bar{T}$, $\tilde{T}$, $\bar{\zeta}$ and $\tilde{\zeta}$ into eqns. (10) and (11) and ordering the terms in terms of ε_M and ε_H, the following equations result. The terms accounting for non-parallel nature of the base flow as well as other terms that are of the same order affect the disturbance field at $O(\varepsilon_H)$.

At zero order:

$$Ł(S_1) = (F_1'\alpha_1 - \Omega)(S_1'' - \alpha_1^2 S_1) - \alpha_1 F_1''' S_1 = 0 \quad (19)$$

$$\phi_1 = H_1 S_1 \alpha_1/(F_1'\alpha_1 - \Omega) \quad (20)$$

At $O\{\varepsilon_M\}$:

$$Ł(S_2) = R_1 + \alpha_2 R_2 \quad (21)$$

where,

$$R_1 = -F_2'\alpha_1(S_1'' - \alpha_1^2 S_1) + \alpha_1 S_1 F_2'''$$

and

$$R_2 = 2\alpha_1 S_1(F_1'\alpha_1 - \Omega)$$

$$\phi_2 = (\alpha_2(H_1' S_1 - F_1'\phi_1) + \alpha_1(S_2 H_1' + S_1 H_2' - F_2'\phi_1))/(F_1'\alpha_1 - \Omega) \quad (22)$$

At $O(\varepsilon_M^2)$:

$$Ł(S_3) = R_3 + \alpha_3 R_4 \quad (23)$$

where,

$$R_3 = -(S_1'' - \alpha_1 S_1)(F_2'\alpha_2 + \alpha_1 F_3') + \alpha_1 S_1 F_3''' + \alpha_2 S_2 F_1'''$$
$$- (F_1'\alpha_2 + F_2'\alpha_1)(S_2'' - S_2\alpha_1^2 - 2\alpha_1\alpha_2 S_1) + F_2'''(\alpha_1 S_2 + \alpha_2 S_1)$$
$$+ (F_1'\alpha_1 - \Omega)(S_1\alpha_2^2 + 2S_2\alpha_1\alpha_2)$$

and

$$R_4 = -(S_1'' - \alpha_1^2 S_1)F_1' + 2\alpha_1 S_1(F_1'\alpha_1 - \Omega) + S_1 F_1'''$$

At $O(\varepsilon_H)$:

$$Ł(S_4) = R_5 + \alpha_4 R_6 \quad (24)$$

where

$$R_5 - i(S_1^{iv} - 2\alpha_1^2 S_1'' + \alpha_1^4 S_1 + \phi_1' - i\alpha_1 F_4'(S_1'' - \alpha_1^2 S_1)$$
$$+ i\alpha_1 F_4''' S_1 - 4/5\alpha_1^2 \eta F_1' S_1' + 3/5\alpha_1^2 F_1' S_1 + 3/5\alpha_1 \gamma F_1' S_1$$
$$+ 1/5F_1' S_1''' + 1/5S_1' F_1'' - 3/5\alpha_1^2 F_1' S_1' + 3/5F_1 S_1'''$$
$$+ 3/5F_1''' S_1 + \Omega(4/5\alpha_1(\eta S_1' - S_1) - 1/5\gamma S_1))\}$$

$$R_6 = 2\alpha_1 \Omega S_1 - 3\alpha_1^2 F_1' S_1 + F_1' S_1'' - F_1''' S_1$$

The boundary conditions are that, $S_j'(0) = S_j(\infty) = 0$; $j = 1, 4$. Here, only antisymmetric modes of disturbance are considered. This choice is based on measurements in a natural convection plume reported by Pera and Gebhart [13] who found this mode of the disturbance to be more unstable than the symmetric mode. Such measurements in mixed convection plumes are not yet available. Since the homogeneous problems for S_2, S_3 and S_4 are the same as those for S_1, it is required that,

$$\int_0^\infty (R_1 + \alpha_2 R_2) W \, d\eta = 0; \quad \alpha_2 = -\int_0^\infty R_1 W \, d\eta \Big/ \int_0^\infty R_2 W \, d\eta \tag{25}$$

$$\int_0^\infty (R_3 + \alpha_3 R_4) W \, d\eta = 0; \quad \alpha_3 - -\int^\infty R_3 W \, d\eta \Big/ \int^\infty R_4 W \, d\eta \tag{26}$$

and

$$\int_0^\infty (R_5 + \alpha_4 R_6) W \, d\eta = 0; \quad \alpha_4 = -\int_0^\infty R_5 W \, d\eta \Big/ \int_0^\infty R_6 W \, d\eta \tag{27}$$

where $W(\eta)$ is a non-trivial solution of the adjoint homogeneous problem,

$$(F_1' \alpha_1 - \Omega)(W'' - \alpha_1^2 W) + 2F_1'' \alpha_1 W = 0 \tag{28}$$

with

$$W(0) = W(\infty) = 0$$

The two perturbation parameters ε_M and ε_H arise from distinct physical considerations. Yet the two can be related by,

$$\varepsilon_M = \bar{R} \varepsilon_H^{1/3} \tag{29}$$

where,

$$\bar{R} = U_\infty (v^2 k / g\beta Q_0)^{1/3} / v$$

Clearly $\bar{R}$ is independent of x. In air, for $Q_0 \sim 50 \, \mathrm{Wm}^{-1}$ and $U_\infty \sim 1 - 10 \, \mathrm{cm\,s}^{-1}$ $\bar{R}$ turns out to be of the order unity. If $\bar{R}$ is substantially different from 1, the relative ordering of terms implicit in (6) and (7) would no longer be valid. It is to be noted that in (6) and in (7), terms of

Table 1
Computed eigenvalues for the disturbance flow field

Ω	α_1	α_2	α_3	α_4
.02	.0610 − .0582i	−.0049 + .2291i	−.2601 − .4118i	−.033 + .779i
.04	.1150 − .0884i	−.1058 + .3097i	.0660 − .5797i	−.078 + .813i
.06	.1693 − .1069i	−.2250 + .3198i	.4343 − .3938i	−.123 + .842i
,08	.2213 − .1158i	−.3142 + .2743i	.5081 − .0425i	−.165 + .856i
.10	.2693 − .1181i	−.3628 + .2113i	.3763 + .1950i	−.198 + .860i
.12	.3130 − .1162i	−.3819 + .1528i	.2068 + .2903i	−.223 + .861i
.14	.3529 − .1120i	−.3843 + .1050i	.0682 + .3031i	−.242 + .863i
.16	.3897 − .1064i	−.3785 + .0674i	−.0307 + .2796i	−.258 + .866i
.18	.4238 − .1000i	−.3688 + .0382i	−.0980 + .2436i	−.271 + .870i
.20	.4558 − .0933i	−.3576 + .0155i	−.1428 + .2058i	−.282 + .875i
.24	.5147 − .0794i	−.3347 − .0164i	−.1919 + .1389i	−.300 + .888i
.28	.5685 − .0657i	−.3139 − .0366i	−.2123 + .0878i	−.316 + .904i
.32	.6187 − .0525i	−.2958 − .0496i	−.2196 + .0502i	−.329 + .925i

$O(\varepsilon_M^3)$ are omitted, although with $\bar{R} = 1$ such terms are expected to contribute at the same order as terms of $O(\varepsilon_H)$. This apparent inconsistency is resolved when the results of the base flow calculations in [6] are considered. In [6], with $\varepsilon_M = 0.4$ (an upper limit for the validity of the perturbation expansion of the base flow) the contribution of terms of $O(\varepsilon_M^3)$ to centerline velocity is seen to be less than 10% while that on centerline temperature

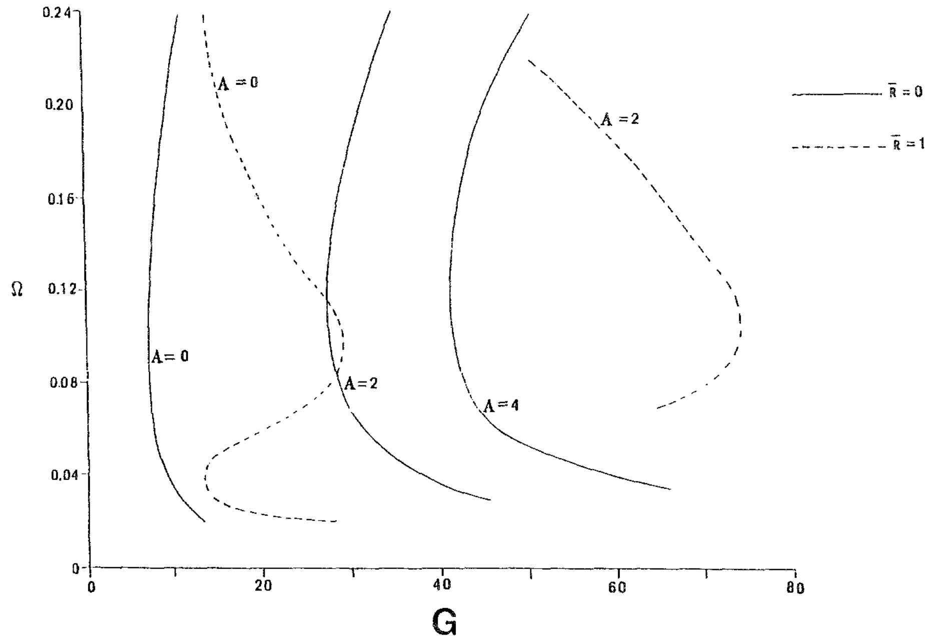

Figure 2
Contours of constant amplification in natural ($\bar{R} = 0$) and mixed convection flows ($\bar{R} = 1$).

excess is only about 3% of the corresponding contributions of term of $O(\varepsilon_H)$. Thus the effect of the terms omitted in (6) and (7) can be expected to be rather small. With $\bar{R} = 1$, the contributions of the non-linear terms such as $\varepsilon_M \varepsilon_H$ arise only at $O(\varepsilon_M^4)$. In [6] contributions of such terms on the base flow have been shown to be negligible.

Computed values of $\alpha_1, \alpha_2, \alpha_3$ and α_4 are listed in Table 1 at various values of Ω, for Pr = 0.7. Using these values neutral stability and amplification contours can be constructed. These are shown in Fig. 2.

Results and discussion

The procedure of determining $\alpha_1, \alpha_2, \alpha_3$ and α_4 is as follows. First, for a chosen value of Ω, eqn. (19) is solved to determine S_1 and α_1. (Equation (19) was integrated inwards, that is, towards the centerline of the plume from its outer edge, by making use of the asymptotic form for $S_1(\eta)$ as $\eta \to \infty$.) Then $W(\eta)$ is determined from eqn. (28). With α_1 and W known, α_2 can then be determined from eqn. (25). The procedure for obtaining α_3 and α_4 is similar. The neutral stability curve i.e. $\Omega(G)$ on which $\alpha_i \equiv 0$, is obtained by solving,

$$\alpha_i = \alpha_{1,i} + \varepsilon_M \alpha_{2,i} + \varepsilon_M^2 \alpha_{3,i} + \varepsilon_H \alpha_{4,i} \equiv 0 \tag{30}$$

Here, α_i is the imaginary part of α. The value of G at a given Ω that one obtains from eqn. (30) depends on the value of $\bar{R}$. In all the computations here, $\bar{R}$ has been taken to be unity. The neutral curve so obtained is shown in Fig. 2 along with contours of constant amplification. These latter curves represent the exponential growth of a disturbance of fixed frequency i.e. ω, as it crosses the neutral curve and propagates downstream. If A_n is the amplitude of a disturbance at a downstream location corresponding to neutral stability and A_x its amplitude further downstream, then,

$$A_x / A_n = e^A, \quad A = -\int_{x_n}^{x} \alpha_i \, dx/\delta = -5/3\left(\int_{G_n}^{G} \alpha_i \, dG\right) \tag{31}$$

with α_i being the imaginary part of α. the neutral curve is $A = 0$. Curves of constant amplification have been obtained by determining α_i at various values of G, keeping ω fixed. The integral in eqn. (31) is then evaluated by simple trapezoidal rule, with a step size in G of 2.5. α_i is a rather slowly changing function of G. This justifies the use of a step size of 2.5. Also shown for comparison, in Fig. 2 are neutral curve and contours of constant amplification for a natural convection plume.

It is clear from Fig. 2 by comparing the neutral curves and those for $A = 2$, that mixed convection effect stabilizes the flow considerably. An example of the effect of considering terms of $O(\varepsilon_M^2)$ is shown in Table 2. It

Table 2
A comparison of the neutral curves at $O(\varepsilon_M)$ and at $O(\varepsilon_M^2)$

Ω	G_n [at $O(\varepsilon_M)$]	G_n [at $O(\varepsilon_M^2)$]
.02	90.68	28.53
.04	63.18	13.33
.06	44.48	19.80
.08	29.66	28.22
.10	20.56	29.44
.12	15.55	26.57
.16	10.99	19.92
.20	10.14	16.00
.24	10.18	14.21
.28	10.80	13.76
.32	12.51	14.36

is clear that the flow is further stabilized by inclusion of the second order mixed convection terms. It is also apparent that this enhanced stabilization is not observed at lower values of Ω.

In a recent paper, Riley and Tveitereid [14] report an analysis of the stability of an *axisymmetric* plume in a uniform stream. Although the free stream was found to have a stabilizing influence, unlike here they were able to find neither a critical value of the Grashof number nor a lower branch of the neutral curve[1]. This can be attributed to the parallel flow assumption in their analysis. Pera and Gebhart [13] also employed the parallel flow assumption in an analysis of *plane natural convection* plumes. A lower branch of the neutral curve nor a critical value of the Grashof number was found. Haaland and Sparrow [15] incorporated the non-parallel effects and were able to obtain both a lower branch of the neutral curve and a critical value of the Grashof number. The analysis reported here as well as that in [12] considers non-parallel nature of the base flow in a consistent manner by inclusion of other terms in the governing equations for the disturbance field that do not represent non-parallel base flow, yet whose contributions are of the same order. In [15] these additional terms were not considered. From Fig. 11.11.1 in [16], it can be seen that the inclusion of these additional terms stabilizes the flow further.

Another interesting feature of the results reported here is the enhanced stabilization even for neutrally amplifying disturbances. However, in wall bounded mixed convection flows, an aiding free stream was found to stabilize the amplifying disturbances while for neutral disturbances destabi-

[1] An anonymous referee has pointed out that their work has been extended to include non-parallel effects. Apparently, critical values and lower neutral stability curves are exhibited by these new results which were presented in a Euromech Conference on Mixed Convection held in Poitiers, France (1986).

lization was observed. See for example, Fig. 1 in [10] and Figs. 1 and 2 in [17].

Acknowledgement

The author would like to thank the Computer Center of the Catholic University of America for providing the facilities for computations. He would also like to thank Hala Helal and Hassan Helal for their help.

Appendix

Governing equations of the base flow

At zero order:

$$F_1''' + 1/5(3F_1F_1'' - F_1'^2) + H_1 = 0$$

$$H_1' + 3/5\mathrm{Pr}\, F_1H_1 = 0$$

$$F_1(0) = F_1''(0) = F_1'(\infty) = 0; \quad \int_0^\infty F_1'H_1\, d\eta = \frac{1}{2\mathrm{Pr}}$$

At $O(\varepsilon_M)$:

$$F_2''' + 1/5(3F_1F_2'' + 2F_2F_1'' - F_1'F_2') + H_2 = 0$$

$$H_2 + 1/5\mathrm{Pr}(3F_1H_2' + 2F_2H_1' + 4H_2F_1' + 3H_1F_2') = 0$$

$$F_2(0) = F_2''(0) = F_2'(\infty) - 1 = H_2'(0) = H_2(\infty) = 0$$

At $O(\varepsilon_M^2)$:

$$F_3''' + 1/5(3F_1F_3'' + 2F_2F_2'' + F_3''F_1) + H_3 = 0$$

$$H_3'' + 1/5\mathrm{Pr}(3F_1H_3' + 5F_1'H_3 + F_3H_1' + 3F_3'H_1 + 4H_2F_2' + 2F_2H_2') = 0$$

$$F_3(0) = F_3''(\infty) = F_3'(0) = H_3'(0) = H_3(\infty) = 0$$

At $O(\varepsilon_H)$:

$$F_4''' + 1/5(3F_1F_4'' + F_1'F_4') + H_4 = 0$$

$$H_4'' + 1/5\mathrm{Pr}(3F_1H_4' + 6F_1'H_4 + 3F_4'H_1) = 0$$

$$F_4(0) = F_4''(0) = H_4'(0) = H_4(\infty) = 0$$

$$F_4'(\infty) = \left(\frac{3}{5}\cot\frac{2\pi}{5}\right)F_1(\infty)$$

The solution to these equations at zero order and at $O(\varepsilon_M)$ can be found in [4] and in [6] whereas [12] and [18] give solution at $O(\varepsilon_H)$. However, the non-dimensionalization employed here differs from that used in [4], [6] and [18]. For the sake of completeness, all the unknown boundary conditions required to numerically integrate these equations, in their present form are listed below.

i	$F_i'(0)$	$H_i(0)$
1	0.93273	0.49654
2	0.05982	−0.21831
3	0.19499	−0.00733
4	0.09969	−0.25111

References

[1] R. Krishnamurthy, *Stability of a Mixed Convection Plume: First Order Mixed Convection Effects.* Presented at 1989 National Heat Transfer Conference, Philadelphia.

[2] W. W. Wood, *Free and forced convection from fine hot-wires.* J. Fluid Mech., *55*, 419–438 (1972).

[3] P. Wesseling, *An asymptotic solution for slightly buoyant laminar plumes.* J. Fluid Mech., *70*, 81–87 (1975).

[4] N. Afzal, *Mixed convection in buoyant plume.* J. Fluid Mech., *105*, 347–368 (1981).

[5] S. E. Haaland and E. M. Sparrow, *Mixed convection plume above a horizontal line source situated in a forced convection approach flow.* Int. J. Heat Mass Transfer, *26*, 433–444 (1983).

[6] R. Krishnamurthy and B. Gebhart, *Mixed convection from a horizontal line source of heat.* Int. J. Heat Mass Transfer, *29*, No. 2, 344–347 (1986).

[7] G. Wilks and R. Hunt, *Vertical mixed convection flow about a horizontal line source of heating or cooling.* Int. J. Heat Mass Transfer, *30*, 1119–1131 (1987).

[8] R. Krishnamurthy and B. Gebhart, *An experimental study of transition to turbulence in vertical mixed convection flows.* ASME J. Heat Transfer, *111*, 121–130 (1989).

[9] M. Gaster, *On the effects of boundary-layer growth on flow stability.* J. Fluid Mech., *66*, 465–480 (1974).

[10] V. P. Carey and B. Gebhart, *The stability and disturbance amplification characteristics of vertical mixed convection flow.* J. Fluid Mech. *127*, 185–201 (1983).

[11] C. C. Lin, *The Theory of Hydrodynamic Stability.* Cambridge University Press, Cambridge (1966).

[12] C. A. Hieber and E. J. Nash, *Natural convection above a line source: Higher-order effects and stability.* Int. H. Heat Mass Transfer, *18*, 1473–1479 (1975).

[13] L. Pera and B. Gebhart, *On the stability of laminar plumes: some numerical solutions and experiments.* Int. J. Heat Mass Transfer, *14*, 975–984 (1971).

[14] D. S. Riley and M. Tveitereid, *On the stability of an axisymmetric plume in a uniform stream.* J. Fluid Mech., *142*, 171–186 (1984).

[15] S. E. Haaland and E. M. Sparrow, *Stability of buoyant boundary layers and plumes, taking account of nonparallelism of the basic flows.* ASME J. Heat Transfer, *95*, 295–301 (1973).

[16] B. Gebhart, Y. Jaluria, R. L. Mahajan and B. Sammakia, *Buoyancy-Induced Flows and Transport.* Hemisphere Publ. Co. 1988.

[17] S. L. Lee, T. S. Chen and B. F. Armaly, *Free stream effects on wave instability of buoyant flows along an isothermal vertical plate.* Int. J. Heat Mass Transfer, *30*, No. 7, 1556–1558 (1987).

[18] N. Riley, *Free convection from a horizontal line source of heat.* Z. angew. Math. Phys., *25*, 817–828 (1974).

Abstract

Aiding mixed convection flow resulting from the vertical flow of a uniform stream past a horizontal line source of heat is of importance in many practical situations such as hot-wire anemometry, etc. In this paper, the stability of such a flow to small disturbances is analyzed in terms of the linear stability theory. The analysis treats the presence of the free stream as a perturbation of a natural convection plume generated by the line source of heat. The base flow as well as the disturbance field are determined by means of a systematic perturbation expansion.

The results presented here extend the results of an earlier investigation [1], by considering second-order mixed convection effects. The results reveal that the free stream has a stabilizing effect. As expected, consideration of second-order mixed convection effects further enhances the stability of the flow. The reported results are valid at a large distance from the source where the flow field is dominated by buoyancy effects.

Résumé

Aidant la convection mixte d'un écoulement résultant d'un écoulement uniforme vertical, à l'arrière d'une source rectiligne horizontale de chaleur, est important en plusieurs situations pratiques, comme celui de l'anémomètre à fil chaud, etc. Dans cet article, la stabilité de ce genre d'écoulement aux pertubations est analysée par la théorie de la stabilité linéaire. L'analyse traits la présence de l'écoulement libre comme perturbation d'une convection naturelle du panache crée par la source rectiligne de chaleur. L'écoulement de base ainsi que le champ exposé aux perturbations sont déterminés au moyen d'une série de perturbations systématiques. L'analyse prend en considération la nature non-parallèle de l'écoulement de base.

Les résultats présentés ici, élargissent les résultats d'une étude précédente [1], en considérant le second ordre des effets de la convection mixte. La consideration du second ordre des effets de la convection mixte augmente la stabilité de l'écoulement. De même, l'effet de la convection mixte sur la courbe neutre semble différent de celle-ci pour une paroi limitée des écoulements.

(Received: February 12, 1990; revised: December 28, 1990 and June 12, 1991)