

Marine gravity surveying line system adjustment

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Received 25 January 1993; Accepted 31 March 1995

Abstract. The general theories and methods of marine surveying line system adjustment were introduced in К о у г и я (1979) and Tang (1991). According to the characteristics of marine gravity measurement, this paper presents a new method of combined adjustment which takes into account both direct and indirect influence of position errors. The method is particularly suitable to be used in the post-processing of marine gravity observation data. With some practical applications, it is proved to be effective in improving the quality of marine gravity data.

error is the major error source of marine gravity measurement. To obtain the accuracy better than 1 milligal, the accuracy of speed and heading must be higher than 0.2 knots and one degree respectively (Dehlinger, 1978; Chen, 1983; Zhao, 1983; Bell, 1986). Such characteristics of marine gravity measurement require adjusting the position of the observation point while filtering the observation noise of gravity.

Marine gravity surveying line system usually consists of main lines and cross lines. They are always designed to intersect to make up a network. This means that there exist redundant observation of gravity at the crossing points. The true position of the crossing point on the main line is rarely coincident with that on the cross line because of the influence of the location errors. The combined influence of the location and observation errors will cause the difference of gravity at the crossing points. These differences are the basic data for adjusting marine gravity surveying line system.

1. Introduction

Compared to the land gravity measurement, marine gravity measurement is mainly characterized by the fact that its foundation on which the surveying instrument is installed is not so stable as that on land. As a result of this difference, the Eötvös effect correction and the influence caused by the position errors of the observed point are increased. Position error

affects marine gravity data in two ways: first, it causes the difference between the observed position and the true position, and as a result, there exists a gravity discrepancy which makes up the observed errors directly; second, it causes the heading and speed errors which will be taken into the Eötvös correction, and thus causes the observed errors indirectly. Many researches have shown that the Eötvös correction

2. The Recovery of Surveying Traces

Generally speaking, the marine gravity surveying ship is required to be kept on the planned surveying line at a fixed speed. The position of the ship is fixed by navigator at the constant intervals. Based on the present positioning results, the operator can decide the future trend of the ship at any time. Because of the random factors such as irregular wind,

current, wave, swell and the power of the ship, etc., the process of fix-adjustment-fix again-adjustment again will result in a zigzag trace. At present in China, most of the surveying ships have been equipped with the integrated navigation systems which usually consist of satellite (NNSS OR GPS) and radio system. In order to eliminate the random disturbance, the data processing program in the navigation system always has the function for filtering noise. The Kalman filter is used frequently, which can provide an optimum estimate of the ship position in real time. The random disturbance in the location data will mostly be eliminated after the above pre-processing. And the accuracy of the observed data of gravity can be roughly estimated at that time according to the results thus obtained. The purpose of this step is to assure that the quality of the observed results meets the requirements of the survey specifications. If it does, the ship will make a return voyage; otherwise a revision observation or reobservation will be necessary.

Marine gravimetry is a continuous survey along lines. The number of the observed points is large. The data obtained from field observation mainly consist of the point coordinates and the observed gravity. They are the initial data for post-processing. However, the model of dependent function among the points must be first constructed if all of the points are to be used in the adjustment. The process of making up the function model is usually called surveying trace recovery or trace fit. In order to keep the initial feature of the data, the minimum degree of fitting is required in practical uses.

It is supposed that any regular or irregular curve can be arbitrarily approximated through the repeated addition of some simple lines. In this paper, each marine gravity surveying trace is expressed as a linear combination of some base functions which take time as argument:

$$\begin{cases} x(t) = a_0 F_0(t) + a_1 F_1(t) + \dots + a_{n-1} F_{n-1}(t) \\ y(t) = b_0 F_0(t) + b_1 F_1(t) + \dots + b_{n-1} F_{n-1}(t) \end{cases} \quad (1)$$

Where X-Y is a local plane coordinate system in Mercator projection, with its X-axis directed towards the north pole, its Y-axis directed towards the east. And the error equations corresponding to the observed coordinates of the discrete points can be made up as follows:

$$V = F \cdot A - L; \quad P_L \quad (2)$$

$2m \quad 1 \quad 2m \quad 2n \quad 2n \quad 1 \quad 2m \quad 1 \quad 2m \quad 2m$

Where

$$L = (x_1, x_2, \dots, x_m, y_1, y_2, \dots, y_m)^T$$

$$A = (a_0, a_1, \dots, a_{n-1}, b_0, b_1, \dots, b_{n-1})^T$$

$$F = \begin{bmatrix} F_x(t_1) & 0 \\ 1 & n & 1 & n \\ \dots & \dots \\ F_x(t_m) & 0 \\ 1 & n & 1 & n \\ 0 & F_y(t_1) \\ 1 & n & 1 & n \\ \dots & \dots \\ 0 & F_y(t_m) \\ 1 & n & 1 & n \end{bmatrix}$$

P_L represents the weight matrix of the observed values vector; m is the number of the observed points on a certain line; $2n$ is the number of unknown parameters.

Define the degree function of fitting: $\Phi = V^T P_L V$. While letting $\Phi = \min$, we have

$$A = (F^T P_L F)^{-1} F^T P_L L \quad (3)$$

$$P_a = (F^T P_L F) \quad (4)$$

The fitting coordinates, speed and heading can also be obtained from:

$$\begin{cases} x(p) = a_0 F_0(t_p) + a_1 F_1(t_p) + \dots + a_{n-1} F_{n-1}(t_p) \\ y(p) = b_0 F_0(t_p) + b_1 F_1(t_p) + \dots + b_{n-1} F_{n-1}(t_p) \end{cases} \quad (5)$$

$$\begin{cases} v_x(p) = a_0 F_0'(t_p) + a_1 F_1'(t_p) + \dots + a_{n-1} F_{n-1}'(t_p) \\ v_y(p) = b_0 F_0'(t_p) + b_1 F_1'(t_p) + \dots + b_{n-1} F_{n-1}'(t_p) \end{cases} \quad (6)$$

$$v(p) = \sqrt{v_x^2(p) + v_y^2(p)} \quad (7)$$

$$\alpha(p) = \arctg \frac{v_y(p)}{v_x(p)} \quad (8)$$

Where $F'_j(t)$ is the derivation of $F_j(t)$ and $p=1, 2, \dots, m$.

At this stage, the (main and cross) lines are independent of each other, so we can treat each line independently. Any of general polynomial, orthogonal polynomial, or trigonometric polynomial can be used as the base function of fitting trace. It may be some other function. The conclusions of Huang (1987) and Huang (1988) have shown that the optimal results of recovery can be obtained if spline function is chosen as the base function. The spline function used in this paper is (Huang, 1987)

$$F_{i-4}(t) = \sum_{j=i-4}^i q_{i-4,j} (t - T_j)^3 \quad (9)$$

Where

$$q_{i-4,j} = \frac{4!}{U_{i-4}(T_j)}$$

$$U_{i-4}(T_j) = \prod_{\substack{k=i-4 \\ k \neq j}}^i (T_j - T_k)$$

$$(t - T_j)^3 = \begin{cases} 0 & ; \quad t \leq T_j \\ (t - T_j)^3 & ; \quad t > T_j \end{cases}$$

T_j is the spline node. Using the function above as fitting function can bring about a good result of recovery and a high numerical stability as well (Huang, 1987).

3. Surveying Line System Adjustment

In order to evaluate the observed data, the marine gravity measurement is normally designed with main lines and cross lines. At the crossing, there exist two observed values of gravity corresponding to the main line and the cross line respectively. The condition equation of the

cross-point is thus formed. How to use the condition equations to adjust the fitting traces again is the second stage of the post-processing of data in the marine gravimetry.

According to the analyses above, the error of the marine gravity measurement mainly comes from three sources: the direct and indirect influences of the position errors and the observation noises of gravity. Now the gravity difference is divided into three parts corresponding to the above three aspects, and therefore, the condition equation can be established as below:

$$B_1 \cdot \Delta A + B_2 \cdot \Delta A + C \cdot V_g - W = 0 \quad (10)$$

Where the first and second terms represent the direct and indirect influences of the location errors respectively; the third term denotes the observation noises of gravity; ΔA indicates the correction vector of unknown parameters of fitting function; V_g is the correction vector of the observed gravity; W is the gravity difference vector; B_1 , B_2 and C are the coefficient matrixes.

Regarding ΔA as a random vector with a prior weight matrix P_A and solving equation (10) with the condition adjustment, the followings can be obtained (К о у г и я, 1979):

$$\Delta A = P_A^{-1} B^T K \quad (11)$$

$$V_g = P_g^{-1} C^T K \quad (12)$$

$$K = (B P_A^{-1} B^T + C P_g^{-1} C^T)^{-1} W \quad (13)$$

$$B = B_1 + B_2 \quad (14)$$

Where P_g represents the weight matrix of the observed gravity vector. The result of formula (11) plus formula (3) is the final trace model after the surveying line system adjustment. According to the new model, the adjusted coordinates, speeds and headings can be calculated easily. These adjusted values can be used further to adjust the Eötvös corrections.

Compared to the model used in К о у г и я

(1979) and Tang(1991), the model (equation (10)) of surveying line system adjustment provided in this paper has the following special features: (1) in the new model there exists the term of the indirect influence of the location errors, which can meet the special requirements of marine gravimetry; (2) in К о у р и я , the coordinates of the crossings are used as the unknown parameters, so the observed points could be entered into a connection with each other only through the covariance matrix of the crossings and the observed points. In the new model, the regression parameters of fitting traces are used to replace the coordinates of the crossings. And the dependent statistical model is replaced by the dependent function model. This not only shows the overall feature of surveying line system adjustment, but also simplifies the computation of the adjustment.

4. The Concrete Form of the

Condition Equation

At each crossing of the surveying line system, the condition equation corresponding to formula (10) can be expressed in detail as follows:

$$g_{x_{ij}} \cdot \Delta x_{p_{ij}} + g_{y_{ij}} \cdot \Delta y_{p_{ij}} - g_{x_{ji}} \cdot \Delta x_{p_{ji}} - g_{y_{ji}} \cdot \Delta y_{p_{ji}} + (\Delta E_{ji} - \Delta E_{ij}) + (V_{g_{ji}} - V_{g_{ij}}) - (g_{ij} - g_{ji}) = 0 \quad (15)$$

Where $g_{x_{ij}}$ and $g_{y_{ij}}$ represent the projections of the horizontal gravity gradients in the directions of X and Y axis near the crossing P(i,j) respectively, and their values can be calculated with (see К о у р и я , 1979)

$$\begin{cases} g_{x_{ij}} = (g_i \cdot \sin \alpha_{ji} - g_j \cdot \sin \alpha_{ij}) / \sin(\alpha_{ji} - \alpha_{ij}) \\ g_{y_{ij}} = (g_j \cdot \cos \alpha_{ij} - g_i \cdot \cos \alpha_{ji}) / \sin(\alpha_{ji} - \alpha_{ij}) \end{cases} \quad (16)$$

Where g_i and g_j denote the gravity gradients in the directions of the main line i and the cross line j respectively; α_{ij} and α_{ji} are the headings of the i-th main line and the j-th cross line at the (i,j) cross-over point.

In equation (15), $\Delta x_{p_{ij}}$, $\Delta y_{p_{ij}}$ and $\Delta x_{p_{ji}}$, $\Delta y_{p_{ji}}$ represent, respectively, the coordinate corrections of the crossing on line i and line j. They can be expressed with the correction vector of the unknowns as follows:

$$\begin{cases} \Delta x_{p_{ij}} \\ \Delta y_{p_{ij}} \end{cases} = D_{ij} \begin{cases} \Delta a_{xi} \\ \Delta a_{yi} \end{cases} \quad (17)$$

$$\begin{cases} \Delta x_{p_{ji}} \\ \Delta y_{p_{ji}} \end{cases} = D_{ji} \begin{cases} \Delta a_{xj} \\ \Delta a_{yj} \end{cases} \quad (18)$$

The coefficient matrixes D_{ij} and D_{ji} are related to F's in equation (5). Δa 's are the correction vectors of the unknowns in (5) and (6).

In equation (15), ΔE_{ij} and ΔE_{ji} indicate the corrections of the Eötvös effect values. The computation formula of the Eötvös effect is

(Dehlinger, 1978; Bell, 1986)

$$E = 7.503 \cdot v \cdot \sin \alpha \cdot \cos \varphi + 0.004 \cdot v^2 \quad (19)$$

Where v — the speed computed by formula (7), with the unit of knot.

α — the heading computed by formula (8).

φ — the geographical latitude of the point.

Differentiating equation (19), ignoring the second term (when $v=15$ knots and $\Delta v=0.2$ knots, $\Delta E=0.024$ mGal) and substituting the increments for the differential elements, the Eötvös effect correction can be expressed as

$$\Delta E = 7.503 \cdot \sin \alpha \cdot \cos \varphi \cdot \Delta v + 7.503 v \cdot \cos \alpha \cdot \Delta \alpha \quad (20)$$

According to formula (7) and (8), we have

$$\begin{cases} \Delta v = (v_x \cdot \Delta v_x + v_y \cdot \Delta v_y) / v \\ \Delta \alpha = (v_x \cdot \Delta v_y - v_y \cdot \Delta v_x) / v^2 \end{cases}$$

Where

$$\Delta v_x = D' \cdot \Delta a_x$$

$$\Delta v_y = D' \cdot \Delta a_y$$

D' is the derivative of D

ship has a tonnage of about 3,000. The marine gravity navigation took place by GPS in single-receiver mode. The location errors were between about 100m and 200m. The marine gravimeter used in the work was the type of KSS-5 made in West Germany. Its observation accuracy is designed to be about 1 mGal.

The worked network consists of five main lines and four cross lines. They are about 250 KM and 160 KM long respectively. The main lines are almost orthogonal to the cross lines. There exist twenty crossings in the network. The gravity gradients along the main lines are listed in Table 1 and 2.

Tab.1 The gravity gradients along the main lines [Unit:10E]

$\begin{matrix} \backslash & i \\ j & \backslash \end{matrix}$	1	2	3	4	5
1	-0.613	-1.383	-0.218	2.244	3.333
2	0.932	-0.985	0.992	1.913	0.290
3	-0.849	-0.907	1.832	2.781	-1.691
4	-0.102	0.197	-1.403	-0.884	-6.544

Tab.2 The gravity gradients along the cross lines [Unit:10E]

$\begin{matrix} \backslash & i \\ j & \backslash \end{matrix}$	1	2	3	4	5
1	-0.044	0.835	-0.946	-1.742	-0.088
2	-0.567	-0.214	-1.098	2.197	0.219
3	0.173	0.128	0.139	-1.467	2.795
4	-0.130	-0.440	0.316	-0.445	-2.492

After the recovery of the trace, the gravity differences at the crossings are computed as in Table 3.

Tab.3 The gravity differences before system adjustment [mGal]

$\begin{matrix} \backslash & i \\ j & \backslash \end{matrix}$	1	2	3	4	5
1	-2.34	3.69	-1.54	6.08	4.11
2	4.03	-0.80	2.34	4.53	-3.50
3	-8.69	4.43	-1.42	5.79	-3.46
4	1.08	2.53	-4.20	-0.23	-4.61

While adjusting the surveying line system, the accuracy of observation was regarded as being equal. Therefore, the noise weight matrix of the gravimeter measurements P_g in (12) and (13) was an identity matrix. Then, three groups of results were computed. They respectively correspond to

$$a. \quad B = B_1 \quad (24)$$

$$b. \quad B = B_2 \quad (25)$$

$$c. \quad B = B_1 + B_2 \quad (26)$$

The equations above mean: only the direct influence of the location errors is taken into account in model (a) (used in К о у г и я (1979) and Tang(1991)); only the indirect influence of the location errors is taken into account in model (b); both direct and indirect influences of the location errors are taken into account in model (c). After having finished the surveying line system adjustment, we can compute the gravity differences at the crossings corresponding to the above three models as below:

Tab.4 The gravity differences to model (a) [mGal]

$\begin{matrix} \backslash & i \\ j & \backslash \end{matrix}$	1	2	3	4	5
1	-0.96	-0.85	-0.11	5.41	0.11
2	-0.90	-1.69	-0.51	2.58	-4.86
3	-1.55	1.51	1.26	-3.35	1.11
4	2.57	-0.21	-0.96	1.21	-0.31

Tab.5 The gravity differences to model (b) [mGal]

$\begin{matrix} \backslash & i \\ j & \backslash \end{matrix}$	1	2	3	4	5
1	-0.36	-0.03	0.39	1.61	0.18
2	-0.24	0.52	0.31	0.55	0.04
3	-0.47	0.72	0.06	-0.21	1.35
4	0.09	0.21	-0.26	0.02	0.33

Tab.6 The gravity differences to model (c) [mGal]

$\begin{matrix} \backslash & i \\ j & \backslash \end{matrix}$	1	2	3	4	5
1	-0.31	-0.02	-0.06	0.11	0.38
2	-0.15	0.35	-0.09	0.07	-0.05
3	0.07	0.14	-0.02	0.33	0.02
4	0.06	0.28	-0.09	0.04	-0.18

According to the specifications of marine gravimetry, the accuracy of the observed data of marine gravity can be estimated by (Strang Van Hees, 1983; Huang, 1990)

$$\sigma = \pm \sqrt{[\omega \omega]} / (2k) \quad (27)$$

Where ω represents the gravity difference; k is the total number of crossings. The factor of 2 is due to that ω is the difference of two observation values. The accuracies corresponding to Table 3, 4, 5 and 6 can be computed as follows:

$$\sigma_3 = \pm 2.83 \text{ (mGal)}$$

$$\sigma_4 = \pm 1.52 \text{ (mGal)}$$

$$\sigma_5 = \pm 0.40 \text{ (mGal)}$$

$$\sigma_6 = \pm 0.13 \text{ (mGal)}$$

The results above indicate that the adjusting effect of equation (25) is markedly better than that of equation (24). According to the gravity gradients in Table 1 and 2, it is obvious that the coordinates of the crossings corresponding to equation (24) must be moved for several kilometers in order to adjust the gravity differences of a few milligals. This does not match the location accuracy (about 200m). On the contrary, the response of the equation (25) to the changes of position coordinates is more sensitive than that of equation (24). A slight change of the trace shape can cause a great change of the speed and heading. Therefore, it is easy to carry out a reasonable adjustment of the differences. In reality, both direct and indirect influences of the location errors exist in marine gravimetry. They restrain and contribute to each other, and neither of them can be neglected in use. So there is reason to consider that the more reasonable model in structure is equation (26), which has been proved by the computed results above.

6. Conclusions

The following conclusions can be drawn from the above discussions and the practical applications:

- (1) The new method of marine gravity

surveying line system adjustment provided in this paper, which takes account of not only the direct but also the indirect influence of location errors, is theoretically strict and practically feasible.

(2) The years of treatment of the observed data concludes that the root mean square of the gravity differences can be decreased from $\pm 3\sim 4$ milligals to within 1 milligal after the adjustment of surveying line system. This will play an important role in improving the quality of marine gravity data in China.

References

- В. А. Коу г и я , А. И. Со ро к ц и н ,
Геоцеэические Семи На Мо
ре, Не ц р а . М о с к в а , 1979
- Tang, S. Y. : Hydrographic Surveying Line Adjustment and the System of Crossing Line Adjustment, Acta Geodaetica et Cartographica Sinica , Vol. 20, No. 2, 1991 (in Chinese)
- Dehlinger, P. : Marine Gravity, Elsevier Scientific Publishing Company Amsterdam-Oxford-New York , 1978
- Bell, R. E. and Watts A. B. : Evaluation of the BGM -3 Sea Gravity Meter System onboard R/V Conrad, Geophysics, Vol. 51, No. 7, 1986
- Strang Van Hees G. L. : Gravity Survey on North Sea , Marine Geodesy, Vol. 6, No. 2, 1983
- Chen, Z. J. : Discussions on the Error of Eötvös Correction , Hydrographic Surveying and Charting, No. 1, 1983 (in Chinese)
- Zhao, M. C. : On the Eötvös Correction in Marine Gravimetry, Hydrographic Surveying and Charting, No. 1, 1983 (in Chinese)
- Huang, M. T. : Study on the Methods of Processing Location Data in Marine Gravimetry, Hydrographic Surveying and Charting, No. 2, 1987 (in Chinese)
- Huang, M. T. : The Application of Trigonometric Polynomial in Processing the Location Data of Marine Gravimetry, Hydrographic Surveying and Charting, No. 1, 1988 (in Chinese)
- Huang, M. T. : Examination, Adjustment and Precision Estimation of Half-systematic Error in Marine Gravity Surveying, Marine Science Bulletin, Vol. 9, No. 4, 1990 (in Chinese)