ENGINEERING FORMULAE FOR FATIGUE STRENGTH REDUCTION DUE TO CRACK-LIKE NOTCHES

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Among fatigue engineers, it is well known that the fatigue strength reduction factor K_f is lower than the elastic stress concentration factor K_t. This discrepancy means that the highest stress alone is no longer appropriate for characterizing the microprocess of fatigue occurring in the microstructure at the notch tip. To rectify this microstructural size effect, Neuber [1-3] has hypothesized that the controlling fracture parameter is the mean stress over the structural size ahead of the notch tip. On the other hand, Ishibashi [4] and Peterson [5] postulated that the controlling factor is the stress at the distance of the structural size ahead of the notch tip. Both Neuber [1] and Peterson [5] started with the stress distribution for deep notches and derived the following approximate formulae of the K_f-K_t relationship which were claimed to be applicable to various notches:

$$K_f = 1 + (K_t-1)/(1 + \sqrt{\rho_*/\rho}) \qquad (1)$$

$$K_f = 1 + (K_t-1)/(1 + r_*/\rho) \qquad (2)$$

where ρ is the notch-tip radius, and ρ_* and r_* are the material constant. Both ρ_* and r_* have been correlated experimentally to the ultimate tensile strength [5,6] or the yield strength [3]. McEvily and Groeger [7] proposed to use Neuber's formula (1) to explain the growth threshold of small fatigue cracks. Since $K_t = 1+2\sqrt{a/\rho}$ for an elliptic notch of length 2a in an infinite plate under tension, (1) reduces to

$$K_f = 1 + 2\sqrt{a/\rho_*} \qquad (3)$$

Peterson's formula is not applicable because it yields $K_f = 1$ for $\rho = 0$. Different formulae can be obtained if both Neuber's and Ishibashi-Peterson's hypotheses are applied to the crack stress field.

The distribution of stress σ_y on the crack plane (y=0) for a crack subjected to remote tensile stress σ as shown in Fig. 1 is

$$\sigma_y = \sigma|x|/\sqrt{(x^2-a^2)} \qquad (4)$$

By taking local coordinate, x' and y', at the crack tip, the mean stress $\overline{\sigma}_o$ over the structural size ℓ_o is determined by

$$\overline{\sigma_o} = \frac{1}{\ell_o}\int_a^{a+\ell_o} \sigma_y dx = \frac{1}{\ell_o}\int_o^{\ell_o} \sigma_y \, dx' \qquad (5)$$

and the stress σ_o at a distance ℓ_o from the notch tip is

$$\sigma_o = \sigma(x')\Big|_{x'=\ell_o} \qquad (6)$$

Since $\overline{\sigma_o}$ or σ_o is interpreted as the fatigue limit of smooth specimens σ_{wo} K_f is obtained by

$$K_f = \overline{\sigma_o}/\sigma \quad \text{or} \quad \sigma_o/\sigma \qquad (7)$$

depending on the hypothesis adopted. The stress intensity factor at the fatigue threshold is given by

$$K = \sigma\sqrt{(\pi a)} \qquad (8)$$

As shown below, σ approaches to the fatigue limit of smooth specimens as a crack becomes small; K becomes constant for long cracks. The constant value is the threshold stress intensity factor K_∞ obtained by ordinary fracture mechanics tests. Smith [8] introduced the concept of the intrinsic crack length a_o defined by

$$a_o = (K_\infty/\sigma_{wo})^2/\pi \qquad (9)$$

The equations for K_f and K/K_∞ obtained from (4) by using Neuber's formula (1), Neuber and Ishibashi-Peterson hypotheses are summarized in Table 1, together with the a_o value as a function of ρ_* or ℓ_o. For a crack subjected to antiplane shear (mode III), a similar calculation can be done with shear stress τ_{yz} on the crack plane. The stress distribution is given from (4) by changing σ_y and σ to τ_{yz} and τ, respectively. K_t for an elliptical notch is $1+\sqrt{a/\rho}$. The final expressions of K_f and K/K_∞ versus a_o are the same as in tensile (mode I) loading except $a_o = \rho_*$ in (10) and (11), where a_o is defined by

$$a_o = (K_\infty/\tau_{wo})^2/\pi \qquad (10)$$

(K_∞: the threshold stress intensity factor for mode II: τ_{wo}: the fatigue limit under antiplane shear or torsion).

Figure 2 shows the relation between $1/K_f = \sigma/\sigma_{wo}$ and a/a_o. The curves of (13) and (15) are very close, while that of (11) is far below the two curves. The dashed lines are limiting lines corresponding to constant stress and stress intensity factor. Eqn. (13) was first derived by Hadded et al. [9] from a different consideration and has been confirmed as an accurate relation to express the data of various metals [9,10]. They derived (13) by equating the stress intensity factor for the small crack of a fictitious length of a plus a_o to K_∞ for long cracks at the threshold. Eqn. (11) is far too conservative. Some other analyses [10-12] based on the micromechanisms of the fatigue process yielded the relation very close to (13). Because of simplicity, (13) is recommended in engineering applications. The material microstructure comes into the relation through a_o which is determined from the data on K_∞ and σ_{wo} by (9). When the data of a_o is not available, its estimation can be made from ρ_* where data are relatively abundant [3,6].

The same engineering hypotheses will be applied to deep notches with small tip radii. The distribution of stress σ_y near the tip of deep notches under tension (mode I) as shown in Fig. 3 is obtained by Creager [12] as

$$\sigma_y = \frac{2K_\rho}{\sqrt{\pi}} \frac{\rho+x'}{(\rho+2x')^{3/2}} \tag{17}$$

($y=0$, $x'>0$), where K_ρ is the stress intensity factor for a crack with identical dimensions to a notch except ρ. The maximum stress at the notch tip is

$$(\sigma_y)_{max} = 2K_\rho/\sqrt{(\pi\rho)} \tag{18}$$

By substituting (17) into (5) and (6), K_ρ can be determined as a function of the fatigue limit σ_{wo} $(\sigma_o, \bar{\sigma}_o)$ and ℓ_o. As shown below, K_ρ becomes a certain value at $\rho=0$, denoted by K_o, and $(\sigma_y)_{max}$ approaches σ_{wo} as ρ becomes large. ρ at the intersection of K_o and σ_{wo} constant lines is given by

$$\rho_o = (2K_o/\sigma_{wo})^2/\pi \tag{19}$$

The relations between K_ρ/K_o and ρ/ρ_o obtained by (1), (5), and (6) are summarized in Table 2, together with ρ_o. For the case of antiplane shear (mode III) loading, the distribution of stress τ_{yz} on $y=0$ is given by Creager [12] as

$$\tau_{yz} = \frac{K_\rho}{\sqrt{\pi}} \frac{1}{(\rho+2x')} \tag{20}$$

The maximum stress at the notch tip is

$$(\tau_{yz})_{max} = K_\rho/\sqrt{(\pi\rho)} \tag{21}$$

The same procedure is employed to calculate the relation between K_ρ/K_o and ρ/ρ_o, where ρ_o is determined by

$$\rho_o = (K_o/\tau_{wo})^{2/\pi} \tag{22}$$

The relations obtained are summarized in Table 2 with ρ_o.

In Fig. 4, K_ρ/K_o is plotted against $\sqrt{(\rho/\rho_o)}$. The curve of (20) obtained from Neuber's formula is above all other lines. The curves of (21), (22), (26) are close. Experimental data of the threshold of fatigue crack initiation in polymer [14] and steels [15,16] indicated that K_ρ increased proportionally to $\sqrt{\rho}$ when ρ was large. As ρ becomes small, K_ρ deviates from the proportional relation approaching to a constant value [13,15]. This transitional behavior was found to be best approximated by (21) in the experiment of low carbon steel [16]. (20) gives a dangerous estimate while (22) is conservative. A micromechanical model for fatigue crack initiation by Tanaka and Mura [12] also gives the relation close to (21). Several other proposed formulae [5, 16-18] can be proved to yield (21) for the cases of deep notches. Among (21), (22), and (26), (21) is recommended for simplicity. This equation has a simple interpretation. If the real notch is assumed to have a fictitious notch tip radius equal to ρ plus ρ_o, the fatigue limit is determined by the condition that the maximum stress at the fictitious notch tip equals the smooth specimen fatigue limit. The material parameter ρ_o is obtained from (19) or (25) and can be estimated by ρ_* which appears in Neuber's formula (1). If one assumes that K_o is identical to K_∞, it is obtained from (9), (10), (19), (25) that ρ_o is equal to $4a_o$ for tensile (mode I) loading to a_o for torsional (mode IV) loading. Experimental works are underway to check those relations and also to examine the applicability of the fictitious notch tip concept to the fracture toughness and stress corrosion tests.

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Table 1. Effect of crack length on fatigue strength reduction

	Neuber's formula	Neuber's hypothesis	Ishibashi-Peterson's hypothesis
K_f	$1+\sqrt{a/a_o}$ ...(11)	$(1+a/a_o)^{\frac{1}{2}}$...(13)	$(1+2a/a_o)/(1+4a/a_o)^{\frac{1}{2}}$ ...(15)
K/K	$[1+\sqrt{a_o/a}]^{-1}$...(12)	$(1+a_o/a)^{-\frac{1}{2}}$..(14)	$(4+a_o/a)^{\frac{1}{2}}/(2+a_o/a)$ ...(16)
a_o	$\rho_*/4$ (mode I) ρ_* (mode III)	$\ell_o/2$	$2\ell_o$

Table 2. Effect of notch-tip radius on fatigue strength reduction (deep notches)

	Neuber's formula	Neuber's hypothesis	Ishibashi-Peterson's hypothesis
Mode I			
K_ρ/K_o	$1+\sqrt{\rho/\rho_o}$ ..(20)	$(1+\rho/\rho_o)^{\frac{1}{2}}$..(21)	$(1+4\rho/\rho_o)^{3/2}/(1+8\rho/\rho_o)$...(22)
ρ_o	ρ_*	$2\ell_o$	$8\ell_o$
Mode III			
$K\rho/K_o$	$1+\sqrt{\rho/\rho_o}$ ..(20)	$(1+\rho/4\rho_o)^{\frac{1}{2}}+(\rho/4\rho_o)^{\frac{1}{2}}$..(26)	$(1+\rho/\rho_o)^{\frac{1}{2}}$ ...(21)
ρ_o	ρ_*	$\ell_o/2$	$2\ell_o$

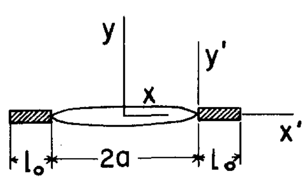

Figure 1. Crack in an infinite plate.

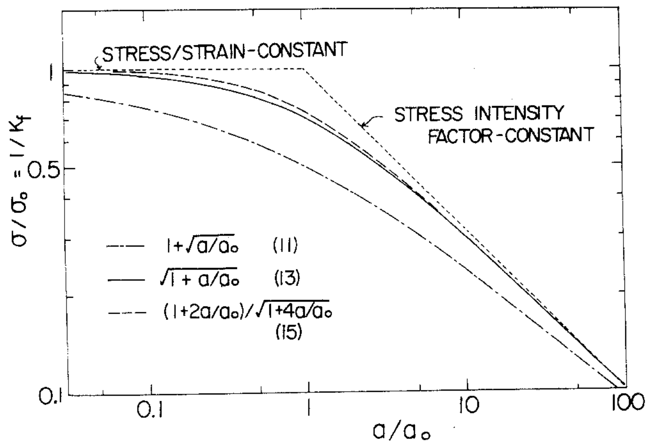

Figure 2. Effect of crack length on fatigue strength reduction.

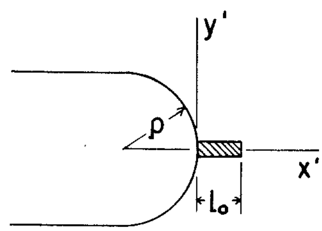

Figure 3. Deep notch tip.

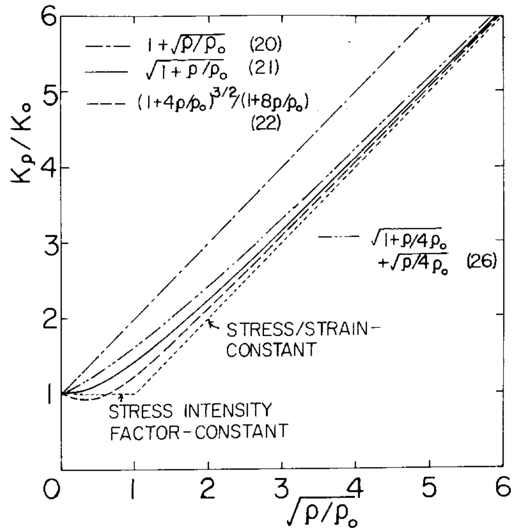

Figure 4. Effect of notch-tip radius on critical stress intensity.