Pricing Insurance and Warranties: Ambiguity and Correlated Risks

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Abstract

This paper reports on the results of a questionnaire study of pricing decisions made by professional actuaries when the probabilities of a loss are either ambiguous or nonambiguous. Theoretical hypotheses derived from the expected utility model were compared with the implications of procedures described by practicing actuaries. Actuaries were asked to act as consultants to a computer manufacturer concerning the price of a warranty. The suggested prices were considerably higher when probabilities were ambiguous than when they were well-specified. These pricing decisions were consistent with the procedures described by actuaries but inconsistent with predictions from expected utility theory when the risks are perfectly correlated. Further insight into the actual decision process is provided by interviews with actuaries and an analysis of comments written on the questionnaire forms.

Key words: Ambiguity, Pricing, Actuaries, Decision Processes.

1. Introduction

The insurance crisis in the United States has spurred considerable interest in the factors influencing decisions by insurers on whether to offer coverage and if so at what price. While considerable attention has been paid to the impact of the liability insurance crisis on the provision of insurance, the analyses to date have focused on the problems facing the industry.[1] Little attention has been paid to the actual decision processes within the insurance firm itself when dealing with situations where there is considerable ambiguity associated with specific risks.

This paper contributes to correcting this imbalance by reporting data from a study of decisions made by actuaries related to the pricing of insurance and warranties under conditions involving probabilities that are either ambiguous or known with precision. It thus continues a line of research by the authors on the effects of ambiguity on insurance decisions (Hogarth and Kunreuther, [1985, 1989]; Kunreuther, [1989]). In these experiments, we showed that economically sophisticated subjects, including professional actuaries, were influenced by ambiguity when pricing insurance in the roles of both consumers and firms in a manner incompatible with traditional economic analysis of insurance markets. For low probability-of-loss events, prices of both consumers and firms indicated aversion

to ambiguity in situations involving unique events. As probabilities increased, however, aversion to ambiguity decreased, with consumers exhibiting preference for ambiguity for high-probability-of-loss events.

The motivation behind the present work is to compare pricing decisions by firms when the events for which insurance is being considered are, alternatively, independent and perfectly correlated. The next section reviews evidence that ambiguity creates problems for private insurers and we provide explanations as to why this may be the case. Section 3 contrasts alternative models of choice for determining how actuaries price insurance under different risk conditions. Differential predictions are then tested in an experimental study in which actuaries are asked to imagine that they are advising a computer manufacturer on the price of a warranty. Our main findings are that ambiguity does increase prices for insurance. Moreover, higher premiums are quoted when risks are correlated as opposed to being independent. We conclude by discussing the implications of these results for future work on ambiguity in market settings, models of risky choice, and possible effects of ambiguity on the relative thickness of markets.

2. Impact of ambiguity on insurers' behavior

In 1921, Frank Knight alerted economists to the importance of ambiguity by drawing a distinction between risk and uncertainty. Although the expected utility model does not recognize Knight's distinction (cf. Howard [1988]), Daniel Ellsberg's [1961] celebrated experimental demonstrations showed that ambiguity can influence decisions in ways that are inconsistent with this important theory.[2]

Many experimental studies have subsequently confirmed Ellsberg's original demonstrations (Becker and Brownson, [1964]; Yates and Zukowski, [1976]; Curley and Yates, [1985]) and there has been considerable interest in recent years in incorporating possible effects of ambiguity in both psychological and theoretical models of choice under uncertainty (see, e.g., Einhorn and Hogarth, [1985, 1986]; Hogarth, [1989]; Gärdenfors and Sahlin, [1982]; Segal, [1987]; Segal and Spivak, [1988]; Hazen and Lee, [1991]; Heath and Tversky, [1991]; Camerer and Weber, [in press]).

Regarding the insurance industry, firms are reluctant to provide coverage against events where the probability of an occurrence is ambiguous either because there are limited statistical data and/or experts have different theories as to underlying causal mechanisms. Insurers' concerns are further exacerbated if the event itself can produce large losses due to either highly correlated risks or a catastrophic accident. Indeed, failures by the private-sector insurance industry due, in part, to ambiguity have resulted in private/government risk-sharing arrangements as in the case of floods (Kunreuther, et al., [1978]) and nuclear power plant accidents (U.S. Nuclear Regulatory Commission, [1983]), or the collapse of the private market as in the case of environmental pollution (National Association of Insurance Commissioners, [1986]; Kunreuther, [1987]; Katzman, [1988]).

How can one explain the reluctance of insurers to offer coverage against these and other low probability-high consequence events? The standard model of choice in economics assumes that the insurer is risk neutral and hence uses an expected profit maximization criterion in determining what premium to charge for coverage against a particular risk. Consider a case where the insurer is considering offering a policy for one time period against a risk where there is a nonambiguous probability p that a specific loss L will occur. Suppose the firm sells m different policies against this risk with at most one loss occurring for any given policy in the designated time period. Let p_j represent the probability estimate that j losses (j $\leq$ m) will occur in a given time period and A the insurer's assets prior to providing coverage. The premium, r_1, where the insurer is indifferent between maintaining the status quo or offering coverage, is determined by

$$A = A - \sum_{j=0}^{m} jp_jL + mr_1 \tag{1}$$

This implies that $r_1 = \sum_{j=0}^{m} jp_jL/m$. Note, in particular, that in the case of risk neutrality the premium is set equal to the expected loss whether or not the risks are independent.

How does ambiguity concerning probabilities affect the premium setting process? We define an ambiguous probability to be one where there are insufficient statistical data to estimate the chances of an event occurring or, because of differing underlying assumptions or theories, experts disagree. Imagine, for instance, being asked to quote premiums for insuring a businessman against risks of kidnap or hijacking during a visit to the Middle East or an accident to a satellite launched from an orbiting space vehicle.[3]

Suppose there are k different expert opinions of the probability of j losses occurring within the given time period. These are denoted p_{ij}, i = 1,...,k, and reflect the uncertainty of the data and/or disagreements among the experts. The insurer estimates the probability of j losses to be $f(p_{1j},...,p_{kj})$. If $f(p_{1j},...,p_{kj})$ is the same as p_j in (1), then a risk neutral insurer will charge the same premium r_1 whether or not the probability is ambiguous. Expected profit maximization implies that the variance does not matter in premium determination and hence uncertainty concerning estimates of the probability (as well as losses) should have no effect on insurers' pricing decisions.

Empirical data on insurers' behavior suggest that they may *not* be risk neutral. In their study of the demand for reinsurance by property/liability insurance companies, David Mayers and Clifford Smith [1990] point out that the variance associated with potential losses may be important for several reasons. The provisions of the corporate tax code, for example, imply a convex tax function for low levels of taxable income and a linear function for taxable income above $100,000: the lower the variance of pre-tax income, the lower an insurer's tax liability. The

chances of insolvency also increase with the variance in the loss. If there are transaction costs associated with bankruptcy then the expected cost associated with any risk portfolio will be lower the more certain one is of the magnitude of the outcomes.

These findings imply that insurers will charge a higher premium when there is more volatility in the probability distribution of losses, a situation equivalent to being risk averse. Bruce Greenwald and Joseph Stiglitz [1990] have shown that if professional managers in firms are rewarded with a share of the profits but suffer a large penalty in case the firm becomes insolvent, then the firm will behave as if it maximized expected utility where the utility function is characterized by decreasing absolute risk aversion. More generally, the analysis of many principal-agent problems suggests that managers have incentives to behave in a risk-averse manner even if stockholders would be better served by risk-neutral behavior. (For a review of agency issues, see Eisenhardt, [1989].)

Another reason why insurers may charge higher premiums when probabilities are ambiguous can be attributed to insurance firms acting to avoid the winner's curse phenomenon. Jeryl Mumpower [1991] has cleverly demonstrated that whenever there is uncertainty about the probability of a loss, then each insurer is concerned that the opinions of experts upon which premiums are based will err on the low side of the actual probability. Each insurance firm knows that, by charging the lowest premium, it will get all the business even though the market for insurance is far from being perfectly competitive. Thus the more ambiguous the probability of an event, the more incentive the insurer has to add a surcharge to the premium implied by (1) to protect itself against the winner's curse.[4]

3. Models for pricing insurance under ambiguity

Our interest in this paper is in examining how well different models of choice predict pricing decisions by actuaries when probabilities are ambiguous and non-ambiguous. Also embedded within this question are the effects of correlation between risks. This section contrasts the predictions from the standard model of choice in economics, expected utility theory, with a behavioral model of choice utilized by actuaries.

3.1. Expected utility theory

As noted above, there are many reasons why both insurers and their agents (e.g., actuaries) will price insurance premiums as though they are risk averse. We therefore use an assumption of risk aversion in deriving predictions for the expected utility model. Consider the case described above where the firm is considering insuring m different risks, each with the same probability p of a loss L. According

to the expected utility model, the premium recommended by the actuary is such that the expected utility from not insuring the risk is the same as that from insuring all m risks. This is given by

$$U(A) = \sum_{j=0}^{m} p_j \, U(A - jL + mr_1) \tag{2}$$

where A represents the firm's wealth, U(.) is the firm's utility function and r_1 is the premium where the actuary is indifferent between offering and not offering coverage.[5] Note that if the actuary is risk neutral then r_1 is identical to the premium given by Equation 1.

Define r_2 to be the premium for independent risks with ambiguous probability estimates. Using the notation for characterizing ambiguity developed in the previous section, Equation 2 becomes

$$U(A) = \sum_{j=0}^{m} f(p_{1j}\ldots\ldots p_{kj}) \, U(A - jL + mr_2) \tag{3}$$

To operationalize $f(p_{1j}\ldots\ldots p_{kj})$ we assume that the k experts disagree and that a linear weighting rule is utilized for combining the different probability estimates such that $p_j = \sum_{i=1}^{k} w_i \, p_{ij}$ where each expert's estimate is accorded a weight w_i with $\sum_{i=1}^{k} w_i = 1$. In other words, p_j is the ambiguous probability estimate that j losses will occur. It is important to state, however, that any other procedure that results in the same estimate of p_j for the ambiguous probability p_j will lead to the same qualitative predictions as those described below. In the Appendix, we show that the ambiguous risk has a higher variance than the nonambiguous risk. For risk-averse utility functions which depend only on mean and variance, this implies that the premium for the ambiguous risk must be higher than for the nonambiguous risk, that is $r_2 > r_1$.[6] In general, we would expect this result to hold for all risk-averse utility functions, but a general proof of the required stochastic dominance condition is not known at this point.

To illustrate that an ambiguous risk has a higher variance than a nonambiguous risk when there is more than one risk, consider the following simple example. Suppose that there are two experts with expert 1 estimating the chances of a loss (L = 1) to be .1 and expert 2 estimating it to be .3. The insurer perceives each expert to be equally credible so $w_1 = w_2 = .5$. For a single risk the equivalent nonambiguous probability is .2, which is also the weighted average of the two experts' probability estimates. Now consider the case where there are two independent identical risks (m = 2) each with the same loss distribution as given above.

The table below depicts the chances of 0, 1, and 2 losses for the nonambiguous and ambiguous cases as well as the mean (μ) and variance (σ^2) of the loss distribution.

	Probability of loss				
	No loss	1 loss	2 losses		
Nonambiguous risk (p_j)	.64	.32	.04	$\mu = .4$	$\sigma^2 = .32$
Ambiguous risk					
Expert 1 (p_{1j})	.81	.18	.01		
Expert 2 (p_{2j})	.49	.42	.09		
$f(p_{1j}, p_{2j})$	.65	.30	.05	$\mu = .4$	$\sigma^2 = .34$

Although the average loss is the same for the ambiguous and nonambiguous probabilities (μ), the variance is higher for the ambiguous case than for the non-ambiguous case (σ^2).

For perfectly correlated risks, there can be only either 0 or m losses. An implication of this is that the expected utility model predicts no difference between the indifference premiums for ambiguous and nonambiguous probabilities. To see this, define r_3 as the indifference premium for perfectly correlated risks with non-ambiguous probabilities. Equation 2 can be simplified to

$$U(A) = (1 - p_m)U(A + mr_3) + p_m U(A - m[L - r_3]) \tag{4}$$

For ambiguous probabilities, let r_4 be the analogous indifference premium and the resulting equation is

$$U(A) = \sum_{i=1}^{k} w_i[(1 - p_{im}) U(A + r_4) + p_{im} U(A - mL + mr_4)] \tag{5a}$$

Because $p_m = \sum_{i=1}^{k} w_i p_{im}$, this can be rewritten as

$$U(A) = (1 - p_m)U(A + mr_4) + p_m U (A - m[L - r_4]) \tag{5b}$$

Compare Equations 4 and 5b and note that $r_3 = r_4$. In other words, the expected utility model predicts no differences in premiums due to ambiguity when losses are perfectly correlated no matter what the shape of the utility function. Although this may seem counterintuitive, recall that the case of perfect correlation under ambiguity is equivalent to insuring a single risk with a specific probability where ambiguity does not affect premium levels if the insurer maximizes expected utility (Hogarth and Kunreuther, [1989]).

The four indifference premiums defined above can be summarized in the following simple matrix:

		Losses	
		Independent	Perfectly correlated
Probabilities	Nonambiguous	r_1	r_3
	Ambiguous	r_2	r_4

Assuming a risk-averse insurer, the expected utility model also makes a prediction concerning the difference between premiums for perfectly correlated and independent risks with the same loss per risk. This is that the indifference premium for correlated risks will be higher than the indifference premium for independent risks, since the former implies a greater probability of a large loss. Thus $r_3 > r_1$ and $r_4 > r_2$. To summarize, the expected utility model makes the following predictions concerning the relative sizes of the four cases considered above: $r_4 = r_3 > r_2 > r_1$.

3.2. Actuarial procedures

In an earlier paper (Hogarth and Kunreuther, [1989]), we established that ambiguity does affect the pricing decisions of actuaries. Moreover, those specific decisions were consistent with the predictions of the psychological model of decision making under ambiguity proposed by Einhorn and Hogarth [1985, 1986]. In the present work we have sought to deepen our understanding of the decision-making processes of actuaries by (1) interviewing actuaries concerning how they price risks involving both ambiguous and precise probabilities and (b) examining actuarial literature on this topic.

In the context of their organizational settings, one of the principal roles assumed by actuaries is to justify specific premium recommendations to underwriters who then determine whether or not to insure a given risk. There is thus likely to be a tendency for actuaries to utilize what Tetlock [1985] calls an "acceptability heuristic" so that their recommendations will be viewed as acceptable by the underwriters. Avoiding ambiguity through higher premiums appears to be one of these heuristics. Indeed, in a series of experiments, Curley, Yates, and Abrams [1986] showed that individuals preferred choices having the smallest degree of ambiguity precisely because these could be justified to others. Arguments can also be made that such behavior would be consistent with the general situation where agents are acting on behalf of principals as is likely to be the case within insurance companies (cf. Eisenhardt, [1989]).

The extensive literature actuaries are required to know for their professional examinations provides little insight into the pricing decision process when probabilities are ambiguous. Actuarial science is remarkably well developed for cases where probabilities are known with precision (see, e.g., Miccolis, [1977]). When required to base estimates of the probability of a claim on small sample sizes, the use of credibility theory is advocated.[7] This is equivalent to using Bayes's theorem to adjust prior estimates with current knowledge to obtain a posterior distribution. For risks where there is limited past experience, the actuary will assume a more diffuse prior distribution than when a large statistical data base is available.

To gain insight into how actuaries make decisions under conditions of ambiguity, we conducted a focus group with several actuaries from a large insurance company. Our discussions revealed that pricing under ambiguity depends largely on making subjective adjustments to formulae that would normally be applied to precise probability estimates. Moreover, such adjustments are judgmental in nature.

Specifically, the pricing of insurance is characterized by a two-stage process. First, since the essence of insurance depends on the law of large numbers, insurers seek to establish portfolios involving large numbers of independent risks. Thus, any new risk is initially evaluated in terms of its maximum possible loss and how its addition would affect the insurer's total portfolio of risks. In other words, new risks are evaluated within the context of a firm's existing portfolio of risks. Second, conditional on being deemed acceptable, the price reflects both estimates of the probabilities of incurring losses and ambiguity or uncertainty concerning such estimates. To a large extent, the procedures advocated are consistent with taking decisions based on expected utility theory with a risk-averse utility function and may be thought of as a way of implementing the prescriptions of expected utility theory. However, the actual premiums recommended by actuaries will vary depending on the adjustment procedures utilized. We now consider three different types of procedures.

Procedure 1—subjective ambiguity adjustment. This procedure explicitly recognizes the effects of ambiguity by inflating premiums based on expected value by a factor reflecting the amount of perceived ambiguity as well as random fluctuations. In this sense, it is viewed by actuaries as a global security loading (Lemaire, [1986]). Thus, denoting a premium calculated on the basis of expected value by μ, the premium charged is given by the formula

$$r = (1 + \alpha) \mu \tag{6}$$

where α ($\alpha > 0$) is the factor reflecting ambiguity on either probability and/or loss (although there is no discussion in the literature as to how α should be determined). The implications of this procedure are that (a) prices will be higher when probabilities are ambiguous as opposed to nonambiguous, i.e., $r_2 > r_1$ and $r_4 > r_3$, and (b) prices will be higher when risks are perfectly correlated as opposed to being independent, i.e., $r_3 > r_1$ and $r_4 > r_2$.

Procedure 2—mean-variance models. The second class of procedures recognizes the existence of variance due both to inherent variability in outcomes and the presence of ambiguity. These suggest adjusting estimates of premiums based on expected value by an amount that is a function of the estimated variance (Lemaire, [1986].) Thus,

$$r = \mu + \lambda f(\sigma) \tag{7}$$

where $f(\sigma)$ is either the variance or standard deviation of the estimated distribution and λ ($\lambda > 0$) is a constant that reflects the extent to which $f(\sigma)$ should influence r. (Once again, however, little guidance is offered on how to set λ.) The implications of this procedure are the same as for Procedure 1, i.e., effects for both ambiguity ($r_2 > r_1$ and $r_4 > r_3$), and correlation between risks ($r_3 > r_1$ and $r_4 > r_2$).

Procedure 3—constrain risk of ruin. The third procedure may be used in conjunction with others but does not address the impact of ambiguity directly. Simply stated, it suggests setting premiums such that the probability (P) of depleting the insurer's reserves is held below a certain level. It can be formalized by the rule: Select the minimum r that satisfies the condition that

$$P\{|m(L - r)| > R\} < \gamma \tag{8}$$

where R represents a specific amount of reserves, ($R \leq A$), and γ has been defined according to the insurer's policy. This approach has been discussed in some detail by Stone [1973] who indicated that insurers are interested in maximizing expected profits within certain constraints of tolerable risk and stability of operations. Specifically, insurers want to limit the risk of insolvency. One way to do this is to set premiums high enough based on a given number of policies so that the probability that their losses plus expenses exceed their income is below a prespecified level.[8] The value of γ reflects the insurer's concern for safety and will determine the rates to be set for a given sized portfolio of risks. According to this procedure, the more highly correlated are the risks, the larger the premium. In addition, prices per dollar of insurance will be higher when the size of the potential loss increases.

These different procedures leave much to the judgment of actuaries, with practice varying between different insurers or types of insurance coverage. The use of Equation 8 by actuaries is very much in the spirit of a safety-first model which postulates a threshold level of probability related to some target level of performance—in this case adequate reserves. In discussing managerial perspectives on risk taking, March and Shapira [1987] have emphasized the importance of such focal values particularly in situations where the probability of loss is poorly specified or ambiguous.

The predictions based on these procedures are summarized in Table 1 together with the corresponding predictions of the expected utility model. Note that the expected utility model and the actuarial procedures make the same qualitative

Table 1. Predictions of different models.

Predictions	Expected utility[a]	Actuarial procedures
1. Effect of ambiguity on independent risks	$r_2 > r_1$	$r_2 > r_1$
2. Effect of ambiguity on correlated risks	$r_4 = r_3$	$r_4 > r_3$
3. Effects of correlated vs. independent risks	$r_3 > r_1$ and $r_4 > r_2$	$r_3 > r_1$ and $r_4 > r_2$

Legend: r_1 = premium for independent risks with nonambiguous probabilities
r_2 = premium for independent risks with ambiguous probabilities
r_3 = premium for perfectly correlated risks with nonambiguous probabilities
r_4 = premium for perfectly correlated risks with ambiguous probabilities
[a]Assuming a risk-averse utility function

predictions except for the perfectly correlated case with ambiguous probabilities. Actuarial procedures predict that premiums will rise when ambiguity is introduced (i.e., $r_4 > r_3$) while expected utility theory suggests they will be the same (i.e., $r_4 = r_3$).

4. Experimental evidence

Whereas much suggestive market evidence attests to the effects of ambiguity on both the sale and purchase of insurance (see, e.g., Eisner and Strotz, [1961]; Kunreuther and Kleindorfer, [1983]; U.S. Nuclear Regulatory Commission, [1983]), it is problematic to obtain data on this issue at the level most acceptable to economists, namely market prices. The main difficulty lies in being able to locate data bases of transactions that are identical except for the presence and absence of ambiguity. Failing this resource, one is left with few possibilities. One involves controlled laboratory experiments where nonexpert subjects simulate market behavior (Camerer and Kunreuther, [1989]); another is the use of survey data involving expert subjects. Whereas neither method is perfect, the latter has the advantage of focusing enquiry on the relevant economic actors (cf. Hogarth and Reder, [1987] and is the approach adopted here.

4.1. Subjects

The subjects were professional actuaries who responded to a mail survey of members of the Casualty Actuarial Society residing in North America. Of the population of 1,165 actuaries, 42% provided usable responses.[9] For the study reported here, we received responses from 469 actuaries of whom 6 stated that they would refuse to insure (see below). Mean length of experience as actuaries was 13.8 years (median 12) with a range from 1 to 50 years. Responses were provided anon-

ymously. The actuarial profession is one of the smallest (in total membership), highest paid, extensively trained, and specialized in North America. Analyses made by actuaries are key inputs to pricing decisions made by insurance companies.

4.2. Survey instruments and design

Packages containing the survey instruments were mailed to the actuaries with stamped addressed envelopes provided to facilitate returns. Each package contained (a) a letter from one of the authors requesting participation in a study on risky decision making, (b) a letter from the Vice-President for Development of the Casualty Actuarial Society also urging participation, and (c) the survey questionnaire. This consisted of two or three scenarios, each of which appeared on different sheets of paper that had been stapled together in booklet form. Spaces for responses were provided and indicated on the sheets. Respondents were told that they would "find a number of questions related to the pricing of insurance and warranties in different scenarios." They were requested to answer the questions in the order in which they appeared. To standardize interpretation respondents were explicitly told that use of the words *pure premium* should be understood as meaning premiums *exclusive of all loss adjustment and underwriting expenses.* They were further asked to indicate their "length of experience as a actuary, in number of years."

   The scenarios had been pretested prior to use in the survey with two groups of actuaries in Chicago as well as by officials of the Casualty Actuarial Society. In designing the scenarios, we made an explicit trade-off between providing the respondents with the kind of detailed background information that would normally accompany real actuarial cases and making the scenarios short enough so that members of a busy profession would not be discouraged from responding. The survey required approximately 15 minutes to complete since each respondent's task was limited to at most three short scenarios. Care was exercised to constitute combinations of conditions and scenarios that would minimize possible "carry-over" effects. In addition, the order of scenarios within combinations was randomized. Finally, members of the respondent population were also randomly assigned to different combinations of scenarios and conditions.

4.3. Scenario

Respondents were asked to assume the role of an actuary called in by Computeez, a manufacturer of personal computers, to determine the price of a one-year warranty on the performance of a new line of microcomputers to be put on the market during the coming year. The warranty was to cover the failure of the XY component manufactured by Computeez. The cost of repairing a breakdown was

stated to be $100 per unit. Respondents were also informed that there could be at most one breakdown per unit during the warranty period.[10]

Experimental variations concerned (a) two levels of the number of units that Computeez expected to sell, viz., 10,000 and 100,000, (b) ambiguous versus non-ambiguous probabilities of breakdowns, (c) whether the risks of breakdowns associated with any computer were independent of other computers sold or would be common to all computers (i.e., the insured risks could either be independent across individual units or perfectly correlated), and (d) three different probability levels concerning the risk of XY component failure; these were: .001, .01, and .10.

In the ambiguous versions of the scenario, respondents were told that experts were confused by the results of tests concerning the performance of the XY component; hence there was considerable disagreement amongst the experts, and respondents should not be "at all confident in the accuracy" of their estimate of the probability of a breakdown. On the other hand, in the nonambiguous versions of the scenario experts had examined company records, conducted several independent tests of their own, and all agreed on the chances of the XY component becoming defective within a year of purchase such that the probability of this event could be confidently estimated. We further adopted the procedure of telling respondents in the ambiguous conditions that a particular number (e.g., .10) was "*your* best estimate" (emphasis added here) so that their belief in the underlying probability on which they based their responses was the same as that utilized in the corresponding nonambiguous situation.

Independence of the probability of breakdown of the XY component in different computers was noted by stating that the nature of the potential flaw was random rather than systematic across computers. Dependence was indicated by stating that the potential flaw was due to a particular aspect of the manufacturing process so that if the XY component failed in any one E-Z computer, it would fail in all others as well.[11]

Respondents were asked to state prices on a per unit basis, specifically: "What is the minimum pure premium you would recommend for the warranty (per unit sold) on the understanding that this will cover the $100 per unit cost of repairing the XY component if this fails within a year of purchase?"

4.4. Design

The study involved four between-subject variables. These were: size of potential loss with 2 levels at 10,000 and 100,000 units; ambiguous and nonambiguous probabilities; independent versus correlated risks; and three levels of probability of a breakdown at .001, .01, and .10. There were thus 24 different between-subject conditions, i.e., 2^3 x 3.

4.5. Results

Because the raw data contained several outliers and exhibited considerable variation within experimental conditions, we transformed the data to mitigate these features. Specifically, a new dependent variable was formed by dividing expected loss by the quoted price, i.e., pL/price, which we denote as z. Thus, if the quoted price were $1.10 and expected loss $1, z would be $(1/1.10) = 0.909$. Note that for prices greater than or equal to expected loss, z only varies between 1 and 0, where smaller values denote higher prices. An advantage of this measure is that it facilitates comparisons across different experimental conditions. For example, when comparing z across different probability levels (e.g., .01 vs .10), the effect of probability level can be gauged independently of other factors.[12] To eliminate remaining outliers from our data, we excluded 4 responses where quoted prices were below expected value.

The main results of the experiment are presented in Table 2 in the form of mean values of z, the ratio of expected loss to price. Before examining the detailed predictions summarized in Table 1, it is illuminating to consider the general trends

Table 2. Means of pL/price (= z) ratios for experimental conditions*

		Probabilities of breakdown		
		.001	.010	.100
Expected value per unit loss:		$0.10	$1.00	$10.00
10,000 units		$	$	$
Independent:				
Nonambiguous	(r_1)	0.79	0.87	0.93
Ambiguous	(r_2)	0.41	0.63	0.73
Correlated:				
Nonambiguous	(r_3)	0.47	0.51	0.69
Ambiguous	(r_4)	0.24	0.40	0.53
100,000 units				
Independent:				
Nonambiguous	(r_1)	0.65	0.85	0.98
Ambiguous	(r_2)	0.30	0.51	0.76
Correlated:				
Nonambiguous	(r_3)	0.66	0.60	0.73
Ambiguous	(r_4)	0.23	0.20	0.44
	(Mean)	(0.49)	(0.55)	(0.72)
Variables and interactions significant by ANOVA				
		dfs	F value	p level
Ambiguity		1,435	79.8	< 0.001
Risk type (independent vs. correlated)		1,435	52.7	< 0.001
Probability level		2,435	21.2	< 0.001

*When price is equal to expected loss, the pL/price ratio (= z) equals one. Lower values of the ratio indicate higher prices.

in the data. First, z in the ambiguous case is always smaller than its nonambiguous counterparts. Second, z for the independent risks is larger than for the correlated risks. Third, z increases as probability levels increase. In terms of quoted prices for any given expected value, these findings imply, respectively, higher premiums when risks are ambiguous as opposed to nonambiguous, when risks are correlated as opposed to independent, and disproportionately higher premiums for smaller probabilities of incurring losses. An analysis of variance, in which all possible main effects and interactions of the experimental design were tested, showed these results to be statistically significant. Details are provided at the foot of Table 2.

Results concerning the three predictions in Table 1 are summarized graphically in Figure 1. This shows that the statistically significant main effects for ambiguity and type of risk have distinctive additive effects on the z ratios, such that the data are consistent with the ordering $r_4 > r_3 = r_2 > r_1$. This differs from the predictions of expected utility theory that $r_3 = r_4$ and $r_3 > r_2$. Indeed, specific contrasts of the predictions in Table 1 (based on pL/price or z ratios) show that $r_2 > r_1$ in that pL/r_2 ($= .56$) $< pL/r_1$ ($= .85$), $F(1,224) = 59.8$, $p < .001$; $r_4 > r_3$ in that pL/r_4 ($= .36$) $< pL/r_3$ ($= .60$), $F(1,211) = 28.0$, $p < .001$; but that r_3 and r_2, as represented by pL/r_3 ($= .60$) and pL/r_2 ($= .56$) are not significantly different, $t(234) = 0.82$.

Further evidence supporting the effects of both correlated risks and ambiguity comes from examining the types of situation faced by the six respondents who replied that they would refuse to insure. Five of these six faced ambiguous, correlated risks. (The sixth person was in the ambiguous/independent cell of the research design.)

Finally, consider the significant effect of probability level on the z ratios. As shown at the foot of each column in Table 2, this increased from .49 at the .001
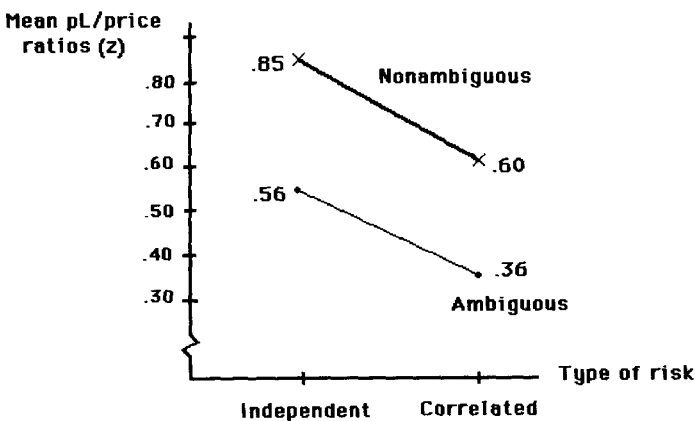

Figure 1. Mean pL/price ratios (z) in ambiguous and nonambiguous conditions by type of risk (i.e., independent versus correlated).

level, to .55 at the .01 level, to .72 at the .10 level, thereby indicating dispropor-
tionately higher prices at lower probability levels and thus an interaction between
premiums and probability levels that would not be predicted by expected utility
theory (cf. Machina, [1987]). Statistical tests (Student's t) show that whereas the
ratios at the .001 and .01 levels are not significantly different, both do differ sig-
nificantly from that at the .10 level, $p < .05$.

4.6. Qualitative analysis

Whereas it is clear that the actuaries' decisions do not adhere to all implications
of the expected utility model, it is difficult to say which combinations of the dif-
ferent actuarial procedures they might have followed. Fortunately, light can be
shed on the decision processes actually followed by some of the actuaries because
several spontaneously made notes on their questionnaires of the line of reasoning
followed in arriving at their responses. To exploit these data, the following pro-
cedures were followed.

First, questionnaires received from the survey were classified into two groups:
those which only contained responses to the questions posed, and those where
the actuaries had also written comments, e.g., steps involved in calculations. The
comments of 70 of these 89 respondents referred specifically to the reasoning
underlying their responses and could be sorted into four different groups. These
were:

	Number of responses
1. Explicit mention or calculation of expected value but then answering by giving a different (higher) price without any other indication of how the latter was determined	30
2. Explicit mention or calculation of expected value which was then the answer given	20
3. Calculation of expected value but with an adjustment factor to account explicitly for the "risk" involved. This corresponds to the actuaries' Procedure 1—subjective ambiguity adjustment.	15
4. Explicit use of the actuaries' Procedure 2—mean-variance models	5
	70

The most striking feature of these data is the important role played by expected
value, either as the method for establishing the price (the second group of re-
sponses) or as an intermediate step in reaching a final price (the three other
groups). In other words, these data support the notion that expected value forms
an "anchor" which the actuaries adjust in determining a price. Moreover, this
judgmental strategy was explicitly and independently described by actuaries in
our focus-group interviews.

Because 30 of the 70 responses (i.e., the 1st group) provided no indication as

to how the actuaries adjusted expected value in determining prices, it is instructive to examine some of the detailed responses in the 3rd and 4th groups that correspond to the 1st and 2nd actuarial procedures.

Consider the following line of reasoning given by one actuary in quoting a premium of $.12 for an ambiguous .001 probability of incurring a $100 loss: "100 × .001 = $.1 (want 20% hedge)."

Another actuary illustrated the same general method when asked to quote a premium for a potential $100,000 loss at two levels of ambiguous probabilities, .01 and .65.[13] For .01, the line of reasoning was given as:

"(.01) (100,000) = 1,000 / × (100/70)/ 1,429 => 1,450"

In other words, the actuary first calculated expected value (i.e., 1,000), and then adjusted this by a factor of 100/70 to yield 1,429 which was rounded up to 1,450.

For .65, the reasoning provided by the same actuary was:

"(.65) (100,000) = 65,000 / × (100/80) => 81,250"

The actual prices quoted for the .01 and .65 probabilities were $1,450 and $81,250, respectively. Given the result noted above that prices are disproportionately higher the smaller the probability, it is of particular interest to note that the actuary uses different coefficients to adjust expected value at the two probability levels, with the smaller probability receiving the larger adjustment.

A further example of the ambiguity adjustment method was provided by an actuary who had been asked to quote a premium for a .35 probability of a $100,000 loss. In one scenario given to this actuary, the probability was ambiguous; in another, it was nonambiguous. The notes attached to the responses for the ambiguous and nonambiguous cases were, respectively,

$$\text{"100 × .35 × 1.25 = 43,750}$$
$$\uparrow$$
$$\text{Conf factor"}$$

and

$$\text{"100,000 × .35 × 1.0"}$$

with a response of $35,000. In the nonambiguous scenario the premium was equal to the expected loss. The same scenario, however, induced a 25% upward adjustment in premium when the probability was ambiguous. (We assume that by "conf factor" the actuary was referring to a subjective "confidence factor" based on an assessment of the experimental materials.)

There were only five examples of explicit mean-variance kinds of calculations. The gist of the arguments given in these cases was to write the premium as the

sum of expected value plus a coefficient (λ) multiplied by an estimate of the standard deviation of the probable loss. In a couple of cases, comments were made as to values chosen for λ, e.g., "$\lambda = 1$," "$\lambda = 0$," or "$\lambda =$ degree of risk. Select 10%."

Of the 19 persons whose comments could not be allocated to one of the four categories provided above, it is significant that several complained that the scenarios failed to provide information concerning the amount of loss the insurance companies or manufacturers could afford to risk in the different scenarios. This may explain why no *explicit* calculations appeared to follow the 3rd actuarial procedure ("Constrain risk of ruin"). However, the presence of these comments clearly indicates that the amount of reserves companies can put at risk in any particular line of business is an important consideration in setting premiums as suggested by the insightful analysis of Stone [1973]. It also reinforces the existence of the two-state process of pricing described above (i.e., first decide if the risk is acceptable given the context of one's portfolio of risks, then set the price if it is acceptable).

To obtain further insight into the decision processes of actuaries, we obtained the cooperation of five actuaries who agreed both to respond to some scenarios and provide specific comments as to how they determined prices.[14] Analyses of these responses showed that the actuaries were particularly sensitive to both ambiguity and the difference between correlated and independent risks. For example, in commenting on a Computeez scenario involving a correlated, ambiguous risk where the best estimate of the probability of a breakdown was stated to be .10, one actuary specifically enumerated the following points: (1) perfect correlation between risks implies no spread and therefore greater exposure; (2) lack of confidence in the probability estimate greatly increases the risk; (3) adding a risk of this type to the insurer's portfolio of risk greatly increases the variance of risks faced; and (4) concerns about risk of ruin, and a "*substantial* hit to earnings" should a loss be incurred with the latter implying all kinds of ramifications involving the company's directors, stockholders, regulators, the trade press, and so on. He finished by stating that he would either be inclined to refuse to insure the risk or, at least, demand a premium that was "near 100 cents to the dollar."

Another participant indicated that actuaries approach the rate-setting process from two perspectives: (a) utilizing expected value to make the best decision but (b) having an obligation to set prices above expected value to keep the company financially sound and prevent insolvency. This line of reasoning supports the use of either procedures 1 or 2 described above.

5. General discussion

Ambiguity about probabilities is a common phenomenon. In addition to the obvious examples of insurance and warranties discussed here, consider decisions such as selling or buying new products, attempts to introduce social or technical

innovations, medical testing and the use of certain procedures on patients, the risks surrounding new technologies, and the appointment of key personnel in organizations. In all of these cases there is considerable ambiguity associated with the probability of success or failure. Indeed, it is surprising that limited empirical data exist on the role ambiguity plays in the decision-making processes of economic agents such as professional managers,[15] and that ambiguity has not been the subject of more empirical work in market settings outside experimental laboratories.

We suggest three reasons why ambiguity has not been studied in market settings. First, ambiguity effects are not likely to be found in the many studies that economists do on equilibrium behavior in markets. By definition, these involve repetitive decisions taken by "professionals," the stock market being a prototypical example, so that buyers and sellers receive quick feedback from which they can learn (for experimental evidence, see Camerer and Kunreuther, [1989]). Within these kinds of market, however, it would be interesting to examine decision making over time to assess whether ambiguity impacts on inexperienced agents at early stages of the process (cf. Arrow, [1982]).

Second, the wide-scale adoption of the expected utility paradigm in applied economics avoids the issue of ambiguity concerning probabilities. Only recently have formal models been developed that are designed to predict ambiguity effects (see Fishburn, [1986, 1988]; Hazen and Lee, [1991]; Kahn and Sarin, [1988]; Segal, [1987]; Segal and Spivak, [1988]).

Third, the quality and aggregate nature of economic data make it difficult to distinguish between "distortions" in probability due to ambiguity and genuine differences in beliefs about underlying probabilities. To the extent that trading in markets reflects differences in beliefs (Varian, [1986]), possible asymmetries in the effects of ambiguity take on added importance.

Examples of such asymmetries have recently been investigated experimentally by Hogarth [1989] within the framework of the Einhorn-Hogarth ambiguity model (Einhorn and Hogarth, [1985, 1986]). He examined situations where the roles adopted by the two sides to a transaction (e.g., buyers and sellers, plaintiffs and defendants) imply different frames (cf. Tversky and Kahneman, [1981]) where one party naturally encodes a situation in terms of a probabilistic loss and the other as a probabilistic gain. For certain ranges of probabilities, the Einhorn-Hogarth model implies that whereas the decision of one party to the transaction will be sensitive to ambiguity, the decision of the other will not. This, in turn, can imply strategic advantages and disadvantages in how the parties negotiate the transaction. It would be important to investigate whether the experimental validation of these predictions obtained by Hogarth [1989] would be replicated under market conditions such as buying and selling options in futures markets.

Another class of situations concerns asymmetries in information between two parties to a transaction. Consider, for example, differences between ambiguous buyers (e.g., consumers or industrial firms) and nonambiguous insurers. According to the Einhorn-Hogarth ambiguity model, buyers should often be prepared to

pay much more for coverage than insurers will necessarily want to charge. We would therefore expect to find active insurance markets for such risks. Some that come to mind are warranties for consumer durables, insurance against risks that are relatively rare such as airplane accidents (Eisner and Strotz, [1961]) and certain forms of illness, or acts not impacted by problems of moral hazard. To the student of economic markets, therefore, an interesting issue centers on the extent to which competition forces insurers to reduce prices so that any excess rents due to ambiguity are eliminated.

For other non-standard risks where both buyers and sellers are ambiguous, the equilibrium premium may be greater than the expected value. As an example, consider the case of a promotional campaign in Belgium by one of Europe's largest television manufacturers.[16] They offered to pay for any of their sets purchased during a six month period between the announcement of the 24 teams competing in the final rounds of soccer's 1986 World Cup and the start of the competition, provided Belgium was the overall winner. Although Belgium was a long-shot at odds of 25 to 1,[17] the television manufacturer was willing to pay a premium of approximately $30,000 to insure against a $300,000 loss should Belgium win the competition. In other words, the television manufacturer was willing to pay a premium that was more than 2 and 1/2 times greater than the expected loss that could have been estimated from the bookmakers' odds.[18]

Ambiguity may also contribute to the failure of insurance markets since basic conditions of insurability are not met.[19] For example, in a survey of the insurance industry for *The Economist,* McCullough [1987] stated,

> Some of the areas which insurers refused to cover in 1985–86 were: pollution risks, liquor liability (that is, cases related to drunken driving), day-care centres, medical malpractice, asbestos removal from schools, commercial fishing boats, municipal liability, commercial trucking and high-limit coverage (above $50m) for industrial concerns. The insurers argued that they were pulling out because liberal court decisions made the potential losses incalculably large,or because the risk itself was no longer actuarially predictable.

Note that McCullough mentions two sources of ambiguity. One is the estimation of risk that has been discussed in this paper; the other is ambiguity concerning the amount of the potential loss covered by insurance (see also Priest [1987]). The failure of environmental liability insurance illustrates an interaction between these two dimensions. Indeed, in a study similar to that reported here, we have started to investigate how ambiguity concerning both probabilities and losses affects pricing decisions of underwriters (Kunreuther, Meszaros, Hogarth and Spranca [1991]).

In addition to emphasizing the need to obtain more data on the details of the judgmental processes used in setting prices under ambiguity, our work suggests the need for a more general model of choice that could encompass the behavior observed in this study. Over and above the ability to explain the effects of ambi-

guity, we note at least two requirements of such a model. First, unless one is willing to live with the notion that people have different utility functions in ambiguous and nonambiguous circumstances (cf. Smith [1969]), the model should permit nonadditive probabilities (for further discussion on this point, see Hogarth and Kunreuther [1989]). Second, models should be able to capture effects due to the size of potential payoffs (or losses) which could result either from correlation between risks or, when risks are independent, the size of potential losses per se. (For a descriptive model of choice under risk and uncertainty meeting these requirements, see Hogarth and Einhorn [1990].)

Our findings also raise a broader set of questions as to the decision processes of individuals in organizational settings where their inputs are only part of the final choice process. The interesting study by MacCrimmon and Wehrung [1986] on managerial risk-taking indicates that managers want to collect additional information and delay decisions that involve uncertain risks. As further pointed out by March and Shapira [1987], managers take actions with the goal of controlling risks thereby suggesting that they want to avoid ambiguity if at all possible.

These findings suggest the need to gain a better understanding of the types of interactions between key individuals in firms (e.g., actuaries and underwriters) who have the responsibility for pricing and offering coverage against specific risks. What types of constraint impact on their decision processes? What types of underwriting procedures are utilized by firms? When institutions like Lloyds suffer unusual losses, how much is due to incomplete analyses of risks or just bad luck? How are their actions judged by their superiors? Are there other attributes such as potential regret which influence their final choices? The importance of justification, uncertainty avoidance and other behavior which may not fit directly into the standard utility paradigm suggests a broad agenda for studying the impact of ambiguity and uncertainty by individuals in organizational settings.

6. Conclusion

The presence of ambiguity with respect to probability raises the broad question as to whether economic agents who follow heuristic rules of thumb can survive and prosper in a competitive environment. Recently, this and similar questions have aroused considerable interest in connection with stock market behavior where the existence of several anomalies seem to suggest that the market is not efficient.[20] The lack of availability of insurance in the past few years for a number of different risks raises a set of related questions regarding the performance of insurance markets. Ambiguous probabilities associated with potential losses may be an important determinant of this behavior. If insurers are uncertain as to the likelihood of specific events occurring and there is only limited opportunity to learn from experience due to the low probability nature of the risks, then they may be reluctant to insure except at a price considerably above expected value. By incorporating ambiguity more explicitly into models of choice we may be able

to gain insight into the conditions under which thick and thin markets are likely to emerge and the resulting equilibrium values.

Acknowledgments

Notes

1. For an excellent summary of the problems of cost and availability of liability insurance, see Chapter 3 of the Committee for Economic Development [1989].
2. In a stimulating article on recent developments in choice under uncertainty, Machina [1987] pointed out that until fifteen years ago expected utility theory was considered one of the success stories of economic analysis. Today, the theory is in a state of flux with economists and psychologists providing experimental evidence that it does not describe behavior (see, e.g., Schoemaker, [1982]; Hogarth and Reder, [1987]) and attempting to develop new theoretical models (for an overview, see Weber and Camerer, [1987]; also Camerer, [1989], Sarin, [1991]).
3. This latter issue has been clearly recognized in the financial press. See, for example, the article by Large [1984] in the *Wall Street Journal* (June 21, 1984). Similarly, the French magazine *L'Expansion* (June 6–12, 1986) reported great variations in the price of satellite insurance between 1983 and 1986 with premiums varying between 18% and 30% of the value of satellites.
4. Kagel and Levin [1986] specify the appropriate modification for avoiding the winner's curse in a common value auction situation where bidders are risk neutral.
5. Because we have assumed a risk-averse utility function, it makes no difference to our analysis whether we refer to the firm's or the actuary's utility function.
6. The essential intuition underlying the reasoning given in the Appendix is that the variance of a weighted average of probabilities is greater than the variance of the average itself. The same intuition, more generally applied to mean-preserving spreads, appears also to hold, but this has not been formally proved. This condition is necessary and sufficient to ensure that the noted result $r_2 > r_1$ holds for all risk-averse utility functions (see Rothschild and Stiglitz, [1970]).
7. See Mayerson [1964] for a discussion of credibility theory and its application by casualty actuaries as well as Mayerson, Jones and Bowers [1968] and Lange, et al. [1969].
8. Complementary approaches are to use coinsurance and deductibles so that the insured absorbs part of any given loss.
9. A 42% response rate is relatively high for a mail survey. It does, however, leave open the

question of how selectivity bias might affect results. On the other hand, we take comfort in the fact that we have no reason to suppose that nonrespondents would have had any motivation to provide qualitatively different responses. In addition, it is not clear in what manner they might be different.

10. Copies of the questionnaires used in this paper can be obtained by writing to the authors.

11. That these kinds of failures do occur in the computer industry is attested to by a recent experience of Apple Computer Inc. Hard disk drives for the MacIntosh II produced by one subcontractor were found to have an unusually high failure rate thereby indicating a common-mode failure. Personal communication, Apple Computer Inc., June and November, 1989.

12. We thank David Hildenbrand for suggesting the use of this measure. A similar measure is loss divided by price, i.e., L/price. This indicates the extent of coverage per dollar of premium. Although all our results are presented in terms of the pL/price ratio, we also analyzed our results using the raw data, i.e., quoted prices. Whereas the substantive conclusions from both analyses are the same, we feel more confident in reporting statistical tests based on the pL/price variable.

13. This was for "the defective product scenario" described in Hogarth and Kunreuther [1985, 1989].

14. We would like to express our appreciation to Kenneth Frohlich, Robert DeLiberatto, Rich Ernst, Brian Moore (all of Reliance Corporation) and Jean Lemaire (Department of Insurance, Wharton School) for their participations in this process.

15. The only other study that we are aware of that has examined the effects of ambiguity on decisions taken by professionals operating within their own area of expertise is by Alison Ashton [1985]. She found that ambiguity affected the hypothetical decisions made by audit partners and audit managers in seven large accounting firms.

16. Personal discussion with Jean Lemaire.

17. William Hill & Sons, Bookmakers, London, U.K., personal communication, May, 1988.

18. Interestingly enough, Belgium reached the semi-finals before losing.

19. For a detailed discussion of the conditions of insurability, see Berliner [1982].

20. See DeLong, et al. [1991] for a summary of these studies. These authors also show with a simple model that irrational traders with erroneous stochastic beliefs can both affect prices and earn higher expected returns.

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Appendix

Proof that the expected utility model implies that, for certain utility functions, premiums will be larger under ambiguity

Consider the situation where a firm with a nonlinear, risk-averse utility function is insuring m statistically independent risks each of which can result in a loss of L with probability p. Assume that in one case p is known unambiguously whereas in another an ambiguous estimate of p results from combining k estimates p_i (i = 1,...,k) provided by k experts using weights w_i. Further, impose the restriction that the ambiguous estimate for a single risk p. = $\sum_{i=1}^{k} w_i p_i$ where $\sum_{i=1}^{k} w_i = 1$ and $w_i \geq 0$ (i = 1,....,k), and that the experts provide an unbiased estimate of p (i.e., p. = p). We wish to show that the variance of the ambiguous risk is larger than the variance of the nonambiguous risk.

Denote the random variable representing the total loss from all m risks in the nonambiguous case by X(p). This is distributed according to a binomial distribution B(m,p) with mean of Lmp and variance of $L^2mp(1-p)$. In the ambiguous case, the analog to X(p) is the mixture of the k loss distributions $X(p_i)$, i = 1,..., k, each of which is a binomial random variable $B(m,p_i)$ (multiplied by a constant L). Denote this random mixture by $\otimes_i X(p_i)$. The random variable $\otimes_i X(p_i)$ results from the two-stage process of first choosing $X(p_i)$ with probability w_i, i = 1,.......,k, and then observing the outcome of $X(p_i)$.

We first note that the means of the distributions of X(p) and $\otimes_i X(p_i)$ are the same since the absolute moments of mixture random variables are simply the corresponding mixture of absolute moments of their component random variables.

To show that Var $[\otimes_i X(p_i)] \geq$ Var [X(p)], note that

$$\text{Var}[\otimes_i X(p_i)] = E\{[\otimes_i X(p_i)]^2\} - [E\{\otimes_i X(p_i)\}]^2$$

$$= \sum_{i=1}^{k} w_i \, E\{[X(p_i)]^2\} - [\sum_{i=1}^{k} w_i E\{X(p_i)\}]^2$$

$$= \sum_{i=1}^{k} w_i L^2[m(m-1)p_i^2 + mp_i] - [\sum_{i=1}^{k} w_i Lmp_i]^2 \qquad (A.1)$$

Recalling that $Var[X(p)] = L^2mp(1-p)$, we can write,

$$
\begin{aligned}
&Var[\otimes_i X(p_i)] - Var[X(p)] \\
&= \sum_{i=1}^{k} w_i L^2 m(m-1)p_i^2 + L^2 mp - [Lmp]^2 - L^2 mp(1-p) \\
&= [L^2 m(m-1)]\{(\sum_{i=1}^{k} w_i p_i^2) - p^2\}
\end{aligned}
$$

$$(A.2)$$

Noting that p^2 is a strictly convex function, it must be the case that $(\sum_{i=1}^{k} w_i p_i^2) > p^2$ except in the (trivial) cases where either all experts' opinions are identical (i.e., $p = p_i$ for all $i=1,\ldots,k$) or there is a single risk so that $m = 1$. Because the expression in equation (A.2) is always nonnegative, the desired claim is established. Thus, firms with risk-averse utility functions that depend only on the mean and variance will charge higher premiums in the presence of ambiguity.