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Comparative Statics Under Multiple Sources of Risk with Applications to Insurance Demand

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Abstract

In this paper we propose an answer to the following problem of comparative statics in models with multiple sources of risk: How a risk averse agent will change his coinsurance demand when the distribution of the insurable loss is shifted? To answer the question, we first comment on Jack Meyer's results and then we show how an alternate approach leads to more definitive comparative statics.

Key words: Insurance Demand, Multiple Sources of Risk, Comparative Statics, Increases in Risk.

1. Introduction

In his article (this issue), Jack Meyer addresses the difficult problem of comparative statics analysis in models with multiple sources of risk. The discussion is presented in terms of coinsurance demand[1] with two sources of randomness that can be independently distributed or stochastically dependent. The comparative statics concerns the following question: How will a risk averse agent change his coinsurance demand when the distribution of the insurable loss is shifted? In this paper, we first comment on Meyer's results and then show how an alternate approach leads to more definitive comparative statics.

2. Comment

J. Meyer analyzes the effects of four different shifts: first degree stochastically dominant decrease, first degree risk reducing deterministic transformations of insurable losses, Rothschild-Stiglitz mean preserving decrease in risk, and simple risk reducing deterministic transformations of insurable losses around the insurance premium. Four theorems are discussed in detail; two with independent random variables and two with stochastically dependent variables. More generally, he proposes a methodology to handle the specific characteristics associated with the comparative statics analysis of problems with more than one random variable:

the notion of beneficial changes in a random parameter is introduced and a ceteris paribus assumption concerning the other random parameters is proposed. When the random parameters are independently distributed, the ceteris paribus assumption considered is that of Hadar and Seo [1990] who assumed that the distribution function of the other random variables remains fixed when shifts in the distribution of one asset are considered for comparative statics analyses. When the random parameters are not independently distributed, we face the difficulty of defining a shift in risk of one random parameter without altering the riskiness of the other random parameters; i.e. without altering the marginal distribution of the other parameters. In his article, J. Meyer proposes the use of deterministic transformations to solve this problem.

Following Meyer [1992], we consider an insurance model with two random variables $\tilde{y}$ and $(M-\tilde{x})$ where $\tilde{y}$ is the background risk and $(M-\tilde{x})$ is the value of the insurable asset.[2] The problem of the risk averse individual is to maximize

$$EU(\delta,h_1) = \int_0^B\int_0^M U(Z(\delta,x,y))\, h_1(x,y)\, dxdy \tag{1}$$

where

$$\begin{aligned} \tilde{Z} &\equiv M - \tilde{x} + \tilde{y} + \delta(\tilde{x} - P); \\ P &= \text{marginal cost in insurance, } 0 < P < M; \\ \delta &= \text{coinsurance rate;} \\ h_1(x,y) &= \text{joint density of the initial risky situation;} \\ [0,M] \times [0,B] &= \text{support of } (\tilde{x},\tilde{y}). \end{aligned}$$

It is important to emphasize here that $\tilde{Z}$ is a linear payoff of both δ and the random variables $\tilde{x}$ and $\tilde{y}$. (See Dionne, Eeckhoudt and Gollier [1990] for more details.)

The first order condition for an interior solution of the above problem is

$$\frac{\partial EU}{\partial \delta}(\delta_1,h_1) = \int_0^B\int_0^M U'(Z(\delta_1,x,y))\,(x - P)\, h_1(x,y)\, dxdy = 0 \tag{2}$$

Under strict risk aversion, $\frac{\partial^2 EU}{\partial \delta^2} < 0$, and the above first order condition is necessary and sufficient for δ_1 to be the optimal (interior) coverage. We are now ready to present some comparative statics analyses.

Theorem 1 and 3 in Meyer's article extend to the insurance problem Hadar and Seo's results on portfolio selection with two independent risky assets.[3] In theorem 3 it is shown that if relative risk aversion is (weakly) less than one and if decision makers are increasingly relative and decreasing absolute risk averse, then the demand for insurance is reduced when the insurable loss undergoes a Rothschild and Stiglitz decrease in risk. Theorem 1 shows the same result when the insurable loss undergoes a first degree stochastically dominant decrease. In this case the sufficient conditions to obtain the desired result are simply that the individual is risk averse and that relative risk aversion is (weakly) lower than one. In both cases, since the studied shifts are general, restrictions on utility functions are necessary to obtain acceptable results or results that are intuitively appealing. However the same results can be obtained more directly by using the concept of partial relative risk aversion (Menezes and Hanson [1970] and Zeckhauser and Keeler [1970]) instead of combinations of both absolute and relative risk aversion. Suppose that wealth (Z) has two components ($Z_1 + \tilde{Z}_2 \equiv Z$). Then partial relative risk aversion (R_p) is defined by

$$R_p(Z_1,Z_2) = - Z_2 \frac{U''(Z_1+Z_2)}{U'(Z_1+Z_2)} .$$

As shown by Diamond and Stiglitz (1974)

$$\frac{dR_p}{dZ_2} = \frac{dR_r}{dZ_2} - Z_1 \frac{dR_a}{dZ_2}$$

where R_a and R_r are used for absolute and relative risk aversion respectively. When R_r is nondecreasing and R_a is nonincreasing, R_p is nondecreasing. The reverse is not true which reinforces the idea that the direct use of partial relative risk aversion for comparative statics analyses may generate more general results. Applying the above definitions to the insurance problem one can restate the results of Theorem 1 and Theorem 3 as follows:

Theorem 3': *If $\tilde{x}$ and $\tilde{y}$ are independent random parameters and $\tilde{x}$ undergoes a Rothschild and Stiglitz decrease in risk, then decisions makers who are nondecreasingly partial risk averse with R_p $(Z_1, Z_2) \leq 1$ for all Z_1, Z_2, will (weakly) decrease* δ.

Proof: It is sufficient to show that $U'(Z)\,(x-P)$ is convex in x for all values for y. The first derivative is equal to $U'(Z)\,[1 - R_p(Z_1, Z_2)]$ where $Z_1 \equiv M - P + y$ and $Z_2 \equiv (\delta - 1)\,(x - P)$. Differentiating again we obtain $(\delta - 1)\,\{U''(Z)\,[1 - R_p(Z_1, Z_2)] - U'(Z) \frac{\partial R_p}{\partial Z_2} (Z_1, Z_2)\}$ which is (weakly) positive when the two sufficient conditions of the theorem are used and when $(\delta - 1) < 0$.

Theorem 1': *If $\tilde{x}$ and $\tilde{y}$ are independent random parameters and $\tilde{x}$ undergoes a first degree stochastically dominant decrease, then risk averse decision makers with $R_p(Z_1,Z_2) \leq 1$ for all Z_1, Z_2, will not increase* δ.

The proof is straightforward from the first derivative of U′(Z) (x − P) above and the use of the sufficient conditions. The requirement that $R_p \leq 1$ for all Z_1 and Z_2 implies that $R_r \leq 1$ for all Z. Let $Z_1 = 0$ to see the point. This means that Theorem 1′ is simply a restatement of Theorem 1 in Meyer's article.[4] However Theorem 3′ is a generalization of Meyer's Theorem 3 since it allows the possibility of $\frac{dR_a}{dZ_2} \geq 0$. Moreover, $\frac{dR_p}{dZ_2} \geq 0$ for all Z_1 and Z_2, implies that $\frac{dR_r}{dZ} \geq 0$.

The above result about first degree stochastic dominance has to be qualified more carefully. In his article, Meyer assumes explicitly that the insurance premium is not affected when shifts in the distribution of losses are considered. This restriction is natural when we consider mean preserving increases in risk.[5] Indeed, when the insurer is risk neutral, only expected losses matter in setting premiums. The existence of a positive loading is then rationalized by the presence of transaction costs. This restriction is also natural when we are concerned with a specific decision maker's loss distribution. However, it is clear that when expected losses are shifted for all the insured in the portfolio, the insurer's profitability is shifted and premiums have to be adjusted. More investigation of this particular assumption is necessary since in many circumstances exogenous shifts in risk may affect a significant group of individuals in the insurer's portfolio.

The same comment applies to Meyer's Theorem 2 where it is shown that a first degree stochastically dominant decrease in $\tilde{x}$ reduces δ under the same sufficient conditions as in Theorem 1. However here $\tilde{x}$ and $\tilde{y}$ are stochastically dependent and the result was obtained by using a deterministic transformation of the random variable $\tilde{x}$.

We now concentrate the discussion on the effect of a second degree decrease in risk on δ, when random variables are not necessary stochastically independent. This analysis is related to Theorem 4 in Meyer's contribution where three ingredients are used to obtain the desired intuitive comparative statics results: 1) the change in the random parameter is (semi-) deterministic; 2) the transformation of $\tilde{x}$ corresponds to a simple decrease in risk around P (to be defined); however, the mean is not required to be kept fixed; 3) finally, the transformation is expected utility increasing.

We comment on these ingredients before presenting the main results. Deterministic transformations of random variables were used by Sandmo [1970] in his classical article on optimal saving under uncertainty where he applied a linear transformation of the random variable (or a stretching of a random variable around a constant mean) to represent a particular type of risk increase.[6] This transformation approach was recently generalized by Meyer [1989] and Meyer and Ormiston [1989] who showed that a particular class of deterministic transformations can be interpreted as a fourth characterization of a Rothschild-Stiglitz decrease in risk. Let us suppose that the random parameter $\tilde{x}$ is transformed by using the function $t(\tilde{x})$ which is assumed to be non decreasing, continuous and differentiable. Then, the transformation from $\tilde{x}$ to $\tilde{z} = t(\tilde{x})$ represents a Rothschild and Stiglitz decrease in risk if it guarantees that the mean of the random variable is preserved and guarantees that the initial random variable ($\tilde{x}$) is second-order stochastically dominated by the transformed random variable ($\tilde{z}$). Therefore, deterministic transformations that share these characteristics represent a general definition of a decrease in risk and restrictions on utility functions are needed to obtain unambiguous comparative statics results. In fact, in their 1989 article, Meyer and Ormiston proposed restrictions on both the utility functions *and* the deterministic transformations to obtain intuitive comparative statics results. They showed that a risk averse decision maker will invest more in a risky asset if $t(\tilde{x})$ is a simple deterministic decrease in risk and if its utility corresponds to a decreasing absolute risk aversion function (see Alarie, Dionne and Eeckhoudt [1992] for an application to the coinsurance problem). It should be noted that the above definition of a simple deterministic decrease in risk is not restricted to be around a particular parameter.

An alternative method is to use an even more restrictive definition of decrease in risk. For example, the definition of a strong decrease in risk was introduced in the literature by Meyer and Ormiston [1985]. It has proved to be a useful restriction of the Rothschild-Stiglitz [1970] definition of a decrease in risk which obtains intuitive comparative statics results. The same restriction can be applied to deterministic transformations: probability mass of the original distribution can be redistributed to points inside its support. This case will be discussed later. Another type of restriction is to use a simple decrease in risk across a particular point. In his article, J. Meyer uses a simple decrease in risk across P, the marginal cost of insurance.

The third ingredient (increasing expected utility) is particularly important since this was never emphasized in modeling comparative statics results in the presence of a single random variable. Of course, all existing changes in risk exhibiting the intuitively appealing comparative statics results for the case of one random parameter are restricted cases of the Rothschild-Stiglitz definition of decreases in risk. Therefore, they all unambiguously increase expected utility. This point is emphasized in Gollier [1991] who shows that it is possible to find changes in risk which are beneficial, but which increase the demand for insurance, even with a single random parameter. As pointed out in Meyer's paper, in the presence of two risky assets, a decrease in one risk can either increase or decrease the expected utility of a risk averse individual. For example, as shown in the first example of Meyer's article, a mean preserving decrease in risk can eliminate the possibility of diversifying the decision maker's risk when the two risks are negatively correlated.

Theorem 4 uses an assumption on the variation of the expected utility to get the result. We will show that this last restriction is not necessary to obtain comparative statics results in presence of two dependent random variables. We will also show that fully deterministic transformations of the insurable loss are not necessary; semi-deterministic transformations of x will be sufficient.

3. Extension

A necessary and sufficient condition for $\delta_2 < \delta_1$ is that

$$\frac{\partial EU(\delta_1,h_2)}{\partial\delta} = \int_0^B\int_0^M U'(Z(\delta_1,y))\,(x-P)\,h_2(x,y)\,dxdy < 0. \tag{3}$$

In order to introduce our main results with two dependent random variables, let us first consider the simple case where $y = y_0$ with probability one (no background risk) so that $h_i(x,y)$ is degenerate: $h_i(x,y) = f_i(x)\,\beta(y)$, $i=1,2$, with $\beta(y)$ being the Kronecker function.

Let $F_i(x)$, $i=1,2$, denote the cumulative distribution function of x. A useful definition is the following.

Definition 1: *(Dionne-Gollier [1991]): We say that $F_2(x)$ is a simple decrease in risk (sDR) across P of $F_1(x)$ if and only if*

1.1) the mean is preserved
1.2) $F_2(x)$ is larger than $F_1(x)$ whenever x is larger than P and $F_2(x)$ is less than $F_1(x)$ whenever x is less than P.

A simple decrease in risk is illustrated in Figure 1.

Definition 2: *(Dionne-Gollier [1991] and Meyer [1992].*[7] *We say that t(x) represents a simple risk reducing deterministic transformation across P of $\tilde{x}$ if and only if it is a deterministic transformation satisfying the following properties:*

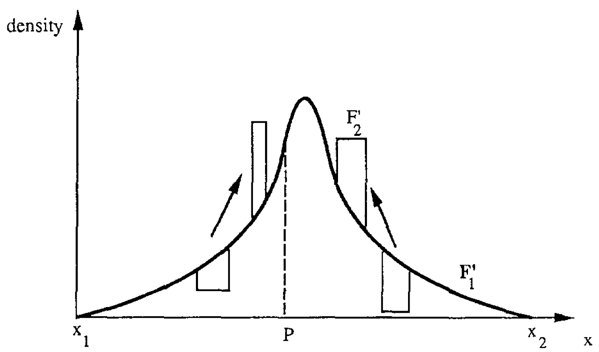

Figure 1. A simple decrease in risk across P.

2.1) $E[t(\tilde{x})] = E(\tilde{x})$

2.2) $t(x)$ *is nondecreasing and* $t(x) \leq x$ *whenever* $x > P$ *and* $t(x) \geq x$ *whenever* $x < P$.

It should be emphasized here that the above definition of a simple decrease in risk across P differs from that in Meyer and Ormiston [1989] which is not restricted to be around a particular parameter.

Proposition 1: *If* $F_2(x)$ *is a simple decrease in risk across P of* $F_1(x)$ *or if* $\tilde{x}$ *undergoes a simple risk reducing deterministic transformation across P, then all risk-averse individuals (weakly) reduce their coinsurance rate;* $\delta_2 \leq \delta_1$.

The proof consists of showing that the above conditions are sufficient to verify that

$$\frac{\partial EU}{\partial \delta}(\delta_1, f_2) = \int_0^M U'(Z(\delta_1, x, y))\,(x-P)\,(f_2(x) - f_1(x))\,dx < 0 \tag{4}$$

for all strictly concave utility functions and all admissible values of δ_1. See Gollier [1991] and Dionne and Gollier [1991] for more details. In fact, Dionne and Gollier [1991] show that simple decreases in risk are equivalent to simple risk reducing deterministic transformations.

We now extend the above results to the case of two dependent random variables where any type of stochastic dependence is permitted between $\tilde{x}$ and $\tilde{y}$. Define conditional density of x given y ($f_i(x|y), i=1,2$) and the marginal density of y by $g_i(y), i=1,2$. Thus $h_i(x,y) = f_i(x|y)\,g_i(y)$. As in Meyer's contribution, we consider a change in the distribution of the insurable loss while assuming that the marginal distribution of y is held fixed $g_1(y) \equiv g_2(y)$. We can also consider deterministic transformations of the insurable loss distribution but need not. However, as pointed out above, when we consider deterministic transformations we do not impose the same deterministic transformation t(x) for all y. Our methodology allows us to consider more general transformations $t(x|y)$, that is, semi-deterministic transformations of x that are conditional on the realizations of y (See Dionne and Gollier [1991] for more details).

We are now ready to present our main result:

Proposition 2: *If, for all possible realisations of the background risk y, the distribution of the insurable losses x conditional on y undergoes a simple decrease in risk across P, then all risk averse individuals will (weakly) decrease their demand for insurance.*

Proof: From the above discussion we have that

$$\frac{\partial EU(\delta_1,h_2)}{\partial \delta} = \int_0^B \left[\int_0^M U'(Z(\delta_1,x,y))\,(x-P)\; d(F_2(x|y)\; -\; F_1(x|y)) \right] g(y)\,dy\;. \qquad (5)$$

We have shown in Proposition 1 that the square bracketed term in (5) is negative if $F_2(x|y)$ is a simple decrease in risk across P. Since this property holds for every y, it implies that $\frac{\partial EU(\delta_1,h_2)}{\partial \delta}$ above is negative. The concavity of EU concludes the proof.

4. Other results

Dionne, Eeckhoudt and Gollier [1991] have shown that, for the case of one random variable and for linear payoffs, if $f_2(x)$ is a relatively weak decrease in risk of $f_1(x)$, then $\delta_2 \leq \delta_1$. A direct implication of that result will be useful for the following discussion. Let us first rephrase the definition of strong increases in risk (Meyer and Ormiston, [1985]):

Definition 3: *We say that f_2 is a strong decrease in risk compared with f_1 if and only if there exist x_1,x_2 in $[0,M]$, $x_1 < x_2$, such that*

3.1) the mean is preserved
3.2) $f_2(x) = 0\ \forall x \notin [x_1,x_2]$, $f_2(x) \geq f_1(x)\ \forall x \in [x_1,x_2]$.

Proposition 3: *If f_2 is a strong decrease in risk compared with f_1 then all risk averse individuals will (weakly) decrease their insurance coverage ($\delta_2 \leq \delta_1$).*

The proof is similar to that of Proposition 1 where it is shown that (4) is negative.

The following definition of a strong deterministic transformation of $\tilde{x}$ can be useful for some applications.

Definition 4: *$t(x)$ is a strong risk reducing deterministic transformation of x if and only if there exist x_1, x_2 ε $[0,M]$, $x_1 < x_2$ such that*

4.1) $t(x)$ and x have the same mean;
4.2) $t(x) \in [x_1,x_2]$ for all x outside $[x_1, x_2]$ and $t(x) = x$ for all $x \in [x_1, x_2]$.

Any strong decrease in risk can be represented by a strong risk reducing deterministic transformation of the insurable loss. Therefore, the result of Proposition

1 also applies to a strong deterministic transformation of $\tilde{x}$. We can also extend the result of Proposition 3.

Proposition 4: *If, for all possible realisations of the background risk y, the distribution of the insurable losses x conditional to y undergoes a strong decrease in risk, then all risk averse insured will (weakly) decrease their demand for insurance.*

The proof is similar to that of Proposition 2 where we obtain that (5) is negative.

A more general result is obtained in Dionne-Gollier [1991] when combinations of different decreases in risk of x are applied. Notice, that as said earlier, any strong decrease in risk conditional on y can be represented by a semi-deterministic risk reducing strong transformation t(x|y) of x. Similarly any simple decrease in risk conditional on y can be represented by a semi-deterministic simple risk reducing transformation t(x|y) of x. This is illustrated in Figures 2 and 3 where different strong risk reducing deterministic transformations and simple risk reducing deterministic transformations are observed at y_1 and y_2 respectively.

5. Conclusion

Let us conclude with the following remarks about portfolio selection with more than one random asset and demand for insurance with a background risk. The two problems are not formally identical. In the portfolio problem with two random assets the payoff Z_p is defined as

$$\begin{aligned}\tilde{Z}_p &\equiv b\tilde{x} + (1-b)\tilde{y} + Z_0 \\ &\equiv b(\tilde{x}-\tilde{y}) + \tilde{y} + Z_0\end{aligned}$$

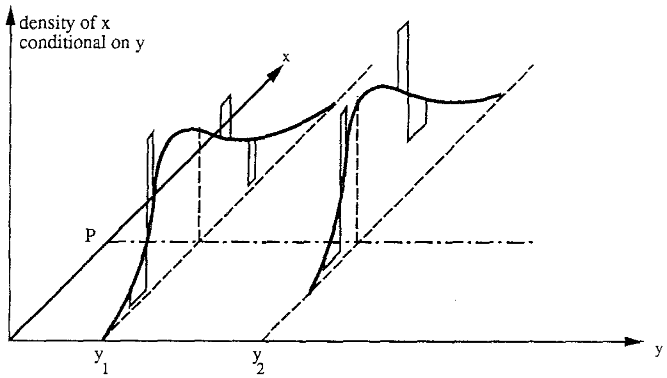

Figure 2. Conditional distributions undergo simple decreases in risk.

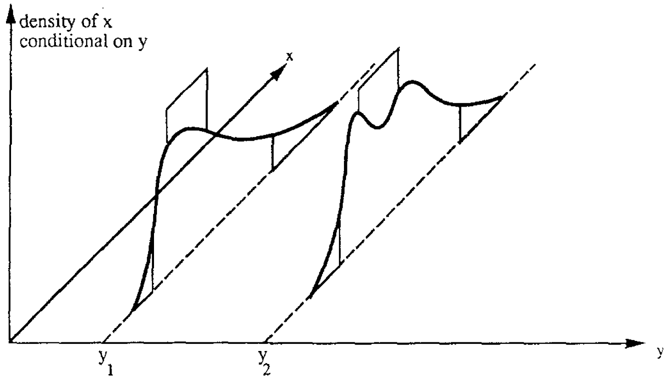

Figure 3. Conditional distributions undergo strong decreases in risk.

where b is the decision variable and $\tilde{x}$ and $\tilde{y}$ are random returns for assets x and y and Z_0 is a constant parameter.

In the coinsurance problem, the payoff Z_a is equal to

$$\begin{aligned}\tilde{Z}_a &\equiv (M-\tilde{x}) + \delta(\tilde{x}-P) + \tilde{y} \\ &\equiv (\delta-1)(\tilde{x}-P) + \tilde{y} + Z_0\end{aligned}$$

where $Z_0 \equiv (M-P)$. When EU is maximized over δ, background risk is not adjusted, although it influences the optimal level of insurance coverage. The same comment applies when a shift in the distribution of x occurs; the adjustment of δ does not affect directly the level of the background risk. In portfolio models, when b is chosen, $(1-b)$ is determined. Moreover, in the analysis of the effects of shifts in the distribution of x on b, $(1-b)$ is also adjusted and there is a substitution of assets in the portfolio. In that respect, the portfolio problem is a different problem than the simple coinsurance problem with an exogenous background risk, a particularity not observed in presence of a single risk since both payoffs are equivalent:

$$\begin{aligned}\tilde{Z}_p &\equiv b(\tilde{x}-r) + Z_0 \\ \tilde{Z}_a &\equiv (\delta-1)(\tilde{x}-P) + Z_0 .\end{aligned}$$

In the presence of more than one random variable, the two problems are equivalent when the background risk is not exogenously determined, as in the model of Mayers and Smith [1983]. An extension of the analysis to this general model is welcome. Another extension is to consider changes in background risk.

Professor J. Meyer has presented a very important contribution to the theory of economic behavior under uncertainty. His paper will rapidly become a pathbreaking contribution to the literature on the comparative statics analysis of changes in risk under multiple sources of risk.

Acknowledgments

This paper was written while C. Gollier was visiting the Department of Economics at the University of Montreal. An earlier version was presented as a comment to Jack Meyer's Geneva Risk Economics Lecture, Mons, Belgium, September 1991. Often, invited speakers to different meetings take the life easy and present either an assessment of a particular point in the literature or a survey of their recent contributions to the field. Not surprisingly, Professor Meyer did not follow that pattern and decided to present a contributing paper. In order to respond adequately to this challenge we have decided to present an extended discussion of his paper. We thank Louis Eeckhoudt for his invitation to be discussants of this lecture and, more importantly, for his continuous teaching about changes in risk. We also thank Josée Lafontaine for her valuable contribution in the preparation of this article and H. Schlesinger and J. Meyer for their comments on a previous version.

Notes

1. For an analysis of the effect of increases in risk on the optimal deductible in presence of a single risk, see Eeckhoudt, Gollier and Schlesinger [1991].
2. On the demand for insurance in presence of uninsurable background risks see also Doherty and Schlesinger [1983, 1985], Eeckhoudt and Kimball [1992], Levy-Garboua and Montmarquette [1991], Ramaswami and Roe [1992] and Scarmure [1991]. Mayers and Smith [1983] studied the simultaneous optimal choices of insurance and assets portfolio; i.e. they developed a general model where the "background" risk is endogenously determined.
3. See also Meyer and Ormiston [1991] for an analysis of optimal portfolio decisions under multiple sources of risk.
4. We thank J. Meyer for pointing out some results in this paragraph. For others applications of the definition of the partial relative risk aversion function, see Dionne [1984], Briys and Eeckhoudt [1985] and Eeckhoudt and Gollier [1991].
5. See however Eeckhoudt, Gollier and Schlesinger [1991].
6. See Eeckhoudt and Hansen [1980] for a squeeze of a random variable around a constant mean which is also a deterministic transformation.
7. For another definition of simple transformations of random variables see Hammond [1974].

References

ALARIE, Y., G. DIONNE and L. EECKHOUDT [1992]: "Increases in Risk and the Demand for Insurance," in *Contributions to Insurance Economics*, G. Dionne (ed.), Kluwer Academic Publishers.

BRIYS, E. and L. EECKHOUDT [1985]: "Relative Risk Aversion in Comparative Statics: Comment," *American Economic Review*, 75, (March 1985), 281–283.
DIAMOND, P. and J. STIGLITZ [1974]: "Increases in Risk and Risk Aversion," *Journal of Economic Theory*, 8, 337–360.
DIONNE, G. [1984]: "Search and Insurance," *International Economic Review*, 25 (June 1984), 357–367.
DIONNE, G., L. EECKHOUDT and C. GOLLIER [1990]: "Increases in Risk and Linear Payoffs," Mimeo, Economics Department and CRT, Université de Montréal.
DIONNE, G. and C. GOLLIER [1991]: "Simple Increases in Risk and Their Comparative Statics for Portfolio Management," Mimeo, Economics Department and CRT, Université de Montréal.
DOHERTY, N. and H. SCHLESINGER [1983]: "Optimal Insurance in Incomplete Markets," *Journal of Political Economy*, 91 (November-December 1983), 1945–1954.
EECKHOUDT, L. and M. KIMBALL [1992]: "Background Risk, Prudence and the Demand for Insurance," in *Contributions to Insurance Economics*, G. Dionne (ed.), Kluwer Academic Publishers.
EECKHOUDT, L., C. GOLLIER and H. SCHLESINGER [1991]: "Increases in Risk and Deductible Insurance," *Journal of Economic Theory*, 55 (December 1991), 435–440.
EECKHOUDT, L. and C. GOLLIER [1991]: *Le risque financier: évaluation, gestion et partage*, (forthcoming).
EECKHOUDT, L. and P. HANSEN [1980]: "Minimum and Maximum Prices, Uncertainty and the Theory of the Competitive Firm," *American Economic Review*, 70 (December 1980), 1064–1068.
HADAR, J. and T. K. SEO [1990]: "The Effects of Shifts in a Return Distribution on Optimal Portfolios," *International Economic Review*, 31, (August 1990), 721–736.
HAMMOND, J. S. [1974]: "Simplifying Choice Between Uncertain Prospects Where Preference is Nonlinear," *Management Science*, 20, 1047–1072.
GOLLIER, C. [1991]: "The Comparative Statics of an Increase in Risk Revisited," Mimeo, HEC, School of Management, Paris.
LÉVY-GARBOUA, L. and C. MONTMARQUETTE [1991]: "The Demand for Insurance Against More Than One Risk, with An Application to Social Insurance," Working paper 9121, Economics Department, Université de Montréal.
MAYERS D. and C. W. SMITH [1983]: "The Interdependence of Individual Portfolio Decisions and the Demand for Insurance," *Journal of Political Economy*, 91 (March-April 1983), 304–311.
MENEZES, C. F. and D. L. HANSON [1970]: "On the Theory of Risk Aversion," *International Economic Review*, 11 (October 1970), 481–487.
MEYER, J. [1989]: "Stochastic Dominance and Transformations of Random Variables," in *Studies in the Economics of Uncertainty*, Fomby, T. B. and T. K. Seo (Eds.), Springer Verlag, 1989.
MEYER, J. [1992]: "Beneficial Changes in Random Variables Under Multiple Sources of Risk and Their Comparative Statics," The Geneva Risk Economics Lecture, Mons, Belgium, September, 1991. *The Geneva Papers on Risk and Insurance Theory*, 17, (June 1992), 7–19.
MEYER, J. and M. B. ORMISTON [1985]: "Strong Increases in Risk and Their Comparative Statics," *International Economic Review*, 26 (June 1985), 425–437.
MEYER, J. and M. B. ORMISTON [1989]: "Deterministic Transformations of Random Variables and the Comparative Statics of Risk," *Journal of Risk and Uncertainty*, 2, (June 1989), 179–188.
MEYER, J. and M. B. ORMISTON [1991]: "The Effects of Return Distribution Changes on Optimal Portfolios: The Case of Stochastically Dependent Returns," Working paper, Department of Economics, Michigan State University.
RAMASWAMI, B. and T. ROE [1992]: "Crop Insurance in Incomplete Markets," in *Contributions to Insurance Economics*, G. Dionne (ed.), Kluwer Academic Publishers.
ROTHSCHILD, M. and J. STIGLITZ [1970]: "Increasing Risk I: A Definition," *Journal of Economic Theory*, 2 (September 1970), 225–243.
SANDMO, A. [1970]: "The Effect of Uncertainty on Saving Decisions," *Review of Economic Studies*, 37 (July 1970), 353–360.

SCARMURE, P. [1991]: "Compulsory Insurance, Uninsurable Risks and the Demand for Insurance," Miméo, Facultés Universitaires Catholiques de Mons, Belgique.
SCHLESINGER, H. and N. DOHERTY [1985]: "Incomplete Markets for Insurance: An Overview," *Journal of Risk and Insurance*, 52 (September 1985), 402–423.
ZECKHAUSER, R. and E. KEELER [1970]: "Another Type of Risk Aversion," *Econometrica*, 38 (September 1970), 661–665.