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TECHNICAL NOTE

Importance of Search-Domain Reduction in Random Optimization

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Abstract. The importance of incorporating systematic search-domain reduction into random optimization is illustrated. In the absence of domain reduction, even an enormous number of function evaluations does not ensure convergence sufficiently close to the optimum as was recently reported by Sarma. However, when the search domain is reduced systematically after every iteration as recommended by Luus and Jaakola, convergence is obtained in a relatively small number of function evaluations, even when the initial search region is large and the starting point is far from the optimum.

Key Words. Random optimization, search-domain reduction, numerical convergence.

1. Introduction

Recently, when Sarma (Ref. 1) considered random optimization where the search is uniformly distributed over the admissible region, it was reported that even an enormous number of function evaluations did not yield convergence to the optimum unless the search domain was chosen to be the immediate vicinity of the optimum. To overcome that problem, Luus and Jaakola (Ref. 2) suggested the use of systematic reduction of the search domain after every iteration. The purpose of this note is to illustrate the effect of

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incorporating domain contraction into random optimization for the first problem used by Sarma.

2. Problem

It is required to minimize the transformer design function

$$f(x)=(L_1+L_2)(x_1+x_2+x_3)+(L_3+L_4)(x_1+1.57x_2+x_4), \quad (1a)$$

$$L_1=0.0204x_1x_4, \qquad L_2=0.0607x_1x_4x_5^2, \quad (1b)$$

$$L_3=0.0187x_2x_3, \qquad L_4=0.0437x_2x_3x_6^2, \quad (1c)$$

subject to the constraints

$$x_i \geq 0, \qquad i=1, \ldots, 6, \quad (2a)$$

$$x_1x_2x_3x_4x_5x_6=2.07\times 10^3, \quad (2b)$$

$$1-Ax_5^2-Bx_5^2>0, \quad (2c)$$

$$B=0.00058(x_1+1.57x_2+x_4)(2.07)^2\times 10^6/x_1^2x_2x_3x_4^2>0, \quad (2d)$$

$$A=0.00062x_1x_4(x_1+x_2+x_3)>0, \quad (2e)$$

$$1/(2A)+C>x_5^2>1/(2A)-C, \quad (2f)$$

$$C=[1/(2A)][1-4AB]^{1/2}, \quad (2g)$$

$$AB<0.25, \quad (2h)$$

$$x_4>[(x_1+x_2+x_3)+(x_1+1.57x_2)]/D, \quad (2i)$$

$$D=Kx_1x_2x_3-(x_1+x_2+x_3), \quad (2j)$$

$$K=0.16225, \quad (2k)$$

$$x_3>(x_1+x_2)/(Kx_1x_2-1), \quad (2l)$$

$$x_2>1/Kx_1. \quad (2m)$$

Sarma (Ref. 1), using the search domain

$$5.27\leq x_1\leq 5.81, \quad (3a)$$

$$4.29\leq x_2\leq 4.71, \quad (3b)$$

$$10.14\leq x_3\leq 10.56, \quad (3c)$$

$$11.89\leq x_4\leq 12.31, \quad (3d)$$

$$0.73\leq x_5\leq 1.27, \quad (3e)$$

obtained a minimum of 135.0906 after 18,840 function evaluations and of 135.0770 after 4 million function evaluations. When the search domain was widened to

$$1 \leq x_1 \leq 10, \tag{4a}$$

$$1 \leq x_2 \leq 10, \tag{4b}$$

$$5 \leq x_3 \leq 15, \tag{4c}$$

$$7 \leq x_4 \leq 17, \tag{4d}$$

$$0.73 \leq x_5 \leq 1.27, \tag{4e}$$

Sarma obtained after 9 million function evaluations $f = 135.0991$, which is still 0.02 percent above the optimum of 135.0760.

We applied the random optimization procedure employing different search-domain contraction factors (Ref. 2) using 200 iterations with 50, 100, 200, 500, and 1000 random points at each iteration. The initial search domain as given by the constraints (4) above was used with the initial point at the center of the region, namely (5.5, 5.5, 10, 12, 1). Table 1 shows the minimum value of f obtained as a function of the reduction factor and the number of random points used per iteration. It is clear that, even with 1000 random points per iteration, if the reduction factor is 1 (i.e., no reduction), the optimum cannot be reached in 200 iterations. The reduction factor should

Table 1. Effect of search region reduction after every iteration.

Reduction factor	Number of random points per iteration				
	1000	500	200	100	50
1.00	135.6727	136.6199	136.7937	138.3344	137.6978
0.99	135.1245	136.1206	135.1679	135.1599	135.2087
0.98	135.0786	135.0812	135.0795	135.0793	135.0806
0.97	135.0761	135.0763	135.0763	135.0767	135.0768
0.96	135.0760	135.0762	135.0774	135.0762	135.0838
0.95	135.0766	135.0762	135.0765	135.0762	135.0760
0.94	135.0762	135.0764	135.0771	135.0790	135.0767
0.93	135.0763	135.0763	135.0768	135.0778	135.0782
0.92	135.0760	135.0764	135.0763	135.0889	135.2600
0.91	135.0766	135.0774	135.0782	135.0786	135.0836
0.90	135.0760	135.0765	135.0767	135.0861	135.9201
0.89	135.0762	135.0766	135.0883	135.0773	135.1445
0.88	135.0761	135.0760	135.0819	135.0995	135.0879
0.87	136.0766	135.0767	135.0792	135.0802	135.1207
0.86	135.0762	135.0784	135.0787	135.0821	135.7279
0.85	135.0764	135.0769	135.0784	135.1280	135.0962

be less than 0.98 to yield convergence to within 0.001% of the optimum. With 1000 points per iteration, convergence to within 0.001% of the global optimum was obtained when any value of the reduction factor between 0.97 and 0.85 was used, and less than 30,000 function evaluations were required. With 100 points per iteration, the number of function evaluations was reduced, but the minimum value was not always within 0.001 percent of the optimum. The global minimum obtained was 135.07598, with

$$x_1 = 5.33209, \quad x_2 = 4.65429, \quad x_3 = 10.43693,$$
$$x_4 = 12.08580, \quad x_5 = 0.75246, \quad x_6 = 0.87880.$$

For this problem, it is not necessary to use as many as 1000 points per iteration. As seen in Table 1, even with 50 points per iteration, convergence to within 0.001% of the optimum was achieved when a reduction factor in the vicinity of 0.95 was used. Similar convergence results for more complicated systems have been reported with a reduction factor of 0.95 by Wang and Luus (Ref. 3).

3. Conclusions

Incorporation of search-domain reduction into a random optimization technique yields the global optimum in a reasonable number of function evaluations. However, random optimization techniques that do not employ search-domain contraction may not converge to the optimum when the search domain is large, even if a very large number of function evaluations are used. Although the rate of convergence can be improved by incorporating a line search as was done by Luus and Brenek (Ref. 4), the optimum can be reached without such modification, as has been illustrated in this example.

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