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Maximum Survival Capability of an Aircraft in a Severe Windshear[1]

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Abstract. This paper is concerned with guidance strategies and piloting techniques which ensure near-optimum performance and maximum survival capability in a severe windshear. The take-off problem is considered with reference to flight in a vertical plane. In addition to the horizontal shear, the presence of a downdraft is assumed.

First, six particular guidance schemes are considered, namely: constant alpha guidance; maximum alpha guidance; constant velocity guidance; constant absolute path inclination guidance; constant rate of climb guidance; and constant pitch guidance. Among these, it is concluded that the best one is the constant pitch guidance.

Next, in an effort to improve over the constant pitch guidance, three additional trajectories are considered: the optimal trajectory, which minimizes the maximum deviation of the absolute path inclination from a reference value, while employing global information on the wind flow field; the gamma guidance trajectory, which is based on the absolute path inclination and which approximates the behavior of the optimal trajectory, while employing local information on the windshear and the downdraft; and the simplified gamma guidance trajectory, which is the limiting case of the gamma guidance trajectory in a severe windshear and which does not require precise information on the windshear and the downdraft.

The essence of the simplified gamma guidance trajectory is that it yields a quick transition to horizontal flight. Comparative numerical experiments show that the survival capability of the simplified gamma

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guidance trajectory is superior to that of the constant pitch trajectory and is close to that of the optimal trajectory.

Next, with reference to the simplified gamma guidance trajectory, the effect of the feedback gain coefficient is studied. It is shown that larger values of the gain coefficient improve the survival capability in a severe windshear; however, excessive values of the gain coefficient are undesirable, because they result in larger altitude oscillations and lower average altitude.

Finally, with reference to the simplified gamma guidance trajectory, the effect of time delays is studied, more specifically, the time delay τ_1 in reacting to windshear onset and the time delay τ_2 in reacting to windshear termination. While time delay τ_2 has little effect on survival capability, time delay τ_1 appears to be critical in the following sense: smaller values of τ_1 correspond to better survival capability in a severe windshear, while larger values of τ_1 are associated with a worsening of the survival capability in a severe windshear.

Key Words. Flight mechanics, take-off, windshear problems, optimal trajectories, guidance strategies, piloting techniques, feedback control, gamma guidance, simplified gamma guidance, quick transition to horizontal flight.

1. Introduction

Over the past several years, considerable attention has been given to the study of a severe meteorological condition known as microburst (Ref. 1). This condition involves a descending column of air, which then spreads horizontally in the neighborhood of the ground. This condition is hazardous, because an aircraft in take-off or landing might encounter a headwind coupled with a downdraft, followed by a tailwind coupled with a downdraft.

The attention to microbursts and associated windshears has been heightened by two recent aircraft accidents involving considerable loss of life: one accident occurred at New Orleans International Airport (PANAM Flight 759 of July 9, 1982) and involved a Boeing B-727 in take-off (Ref. 2); the other accident occurred at Dallas-Fort Worth International Airport (Delta Airlines Flight 191 of August 2, 1985) and involved a Lockheed L-1011 in landing (Refs. 3-4). Therefore, it is imperative (i) to understand the optimal behavior of an aircraft in a windshear, Refs. 5-7; (ii) to develop effective guidance schemes, Refs. 8-13; and (iii) to develop effective piloting strategies, Ref. 14.

Concerning (i), optimal trajectories were investigated in Refs. 5-6 under the assumption that global information on the wind flow field is known in advance. With particular reference to take-off, it was concluded that: (P1) for weak-to-moderate windshears, the optimal trajectories are characterized by a monotonic climb; and (P2) for severe windshears, the optimal trajectories are characterized by an initial climb, followed by nearly horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region.

Concerning (ii), since global information on the wind flow field is not available, one is forced to employ local information on the windshear and the downdraft. Under this assumption, guidance schemes were developed in Refs. 10–13 so as to approximate the behavior of the optimal trajectories and preserve properties (P1)-(P2). Noteworthy among the guidance schemes are the acceleration guidance scheme of Ref. 12 and the gamma guidance scheme of Ref. 13.

Concerning (iii), we observe that the previous guidance schemes require local information on the windshear and the downdraft. While this information will be available in future aircraft, it might not be available on current aircraft. Therefore, developing effective piloting strategies for flight in a severe windshear is an urgent task for flight safety; among the known piloting strategies, one should mention the one based on constant pitch attitude angle, which is both simple and effective (Ref. 14).

In this paper, we present a simplified gamma guidance scheme, based on the properties of the optimal trajectory and associated near-optimum gamma guidance scheme. This simplified gamma guidance scheme yields a quick transition to horizontal flight and, therefore, a trajectory which is close to an optimal trajectory in a severe windshear; in addition, it is easy to implement as a practical piloting technique.

The survival capability of the simplified gamma guidance scheme is studied and is compared with the survival capability of other guidance schemes. The effect of some operational parameters on flight performance and survival capability is also investigated. These parameters include: the gain coefficient K; the time delay τ_1 in reacting to windshear onset; and the time delay τ_2 in reacting to windshear termination.

2. Notations

Throughout the paper, the following notations are employed:

C_D = drag coefficient;
C_L = lift coefficient;
D = drag force, lb;
g = acceleration of gravity, ft sec^{-2};
h = altitude, ft;
K = gain coefficient, dimensionless or dimensional, depending on the feedback control law;

L = lift force, lb;
m = mass, lb $\text{ft}^{-1}\,\text{sec}^2$;
S = reference surface, ft^2;
T = thrust force, lb;
V = relative velocity, ft sec^{-1};
V_e = absolute velocity, ft sec^{-1};
$W = mg$ = weight, lb;
W_h = h-component of wind velocity, ft sec^{-1};
W_x = x-component of wind velocity, ft sec^{-1};
x = horizontal distance, ft.

Greek Symbols

α = relative angle of attack, rad;
α_e = absolute angle of attack, rad;
β = engine power setting;
γ = relative path inclination, rad;
γ_e = absolute path inclination, rad;
δ = thrust inclination, rad;
θ = pitch attitude angle, rad;
λ = wind intensity parameter;
ρ = air density, lb $\text{ft}^{-4}\,\text{sec}^2$.

3. System Description

In this paper, we make use of the relative wind-axes system in connection with the following assumptions: (a) the aircraft is a particle of constant mass; (b) flight takes place in a vertical plane; (c) Newton's law is valid in an Earth-fixed system; (d) the wind flow field is steady; and (e) maximum power setting is employed.

With the above premises, the equations of motion include the kinematical equations

$$\dot{x} = V\cos\gamma + W_x, \tag{1a}$$

$$\dot{h} = V\sin\gamma + W_h, \tag{1b}$$

and the dynamical equations

$$\dot{V} = (T/m)\cos(\alpha+\delta) - D/m - g\sin\gamma - (\dot{W}_x\cos\gamma + \dot{W}_h\sin\gamma), \tag{2a}$$

$$\dot{\gamma} = (T/mV)\sin(\alpha+\delta) + L/mV - (g/V)\cos\gamma + (1/V)(\dot{W}_x\sin\gamma - \dot{W}_h\cos\gamma). \tag{2b}$$

Because of assumption (d), the total derivatives of the wind velocity components and the corresponding partial derivatives satisfy the relations

$$\dot{W}_x = (\partial W_x/\partial x)(V \cos \gamma + W_x) + (\partial W_x/\partial h)(V \sin \gamma + W_h), \tag{3a}$$

$$\dot{W}_h = (\partial W_h/\partial x)(V \cos \gamma + W_x) + (\partial W_h/\partial h)(V \sin \gamma + W_h). \tag{3b}$$

These equations must be supplemented by the functional relations

$$T = T(h, V, \beta), \tag{4a}$$

$$D = D(h, V, \alpha), \qquad L = L(h, V, \alpha), \tag{4b}$$

$$W_x = W_x(x, h), \qquad W_h = W_h(x, h), \tag{4c}$$

and by the analytical relations

$$V_{ex} = V \cos \gamma + W_x, \qquad V_{eh} = V \sin \gamma + W_h, \tag{5a}$$

$$V_e = \sqrt{(V_{ex}^2 + V_{eh}^2)}, \qquad \gamma_e = \arctan(V_{eh}/V_{ex}), \tag{5b}$$

$$\theta = \alpha + \gamma, \qquad \alpha_e = \alpha + \gamma - \gamma_e. \tag{5c}$$

For a given value of the thrust inclination δ, the differential system (1)-(4) involves four state variables [the horizontal distance $x(t)$, the altitude $h(t)$, the velocity $V(t)$, and the relative path inclination $\gamma(t)$] and two control variables [the angle of attack $\alpha(t)$ and the power setting $\beta(t)$]. However, the number of control variables reduces to one (the angle of attack α), if the power setting β is specified in advance [assumption (e)]. The quantities defined by the analytical relations (5) can be computed *a posteriori*, once the values of x, h, V, γ, α, β are known.

Inequality Constraints. The angle of attack α and its time derivative $\dot{\alpha}$ are subject to the inequalities

$$\alpha \leq \alpha_*, \tag{6a}$$

$$-\dot{\alpha}_* \leq \dot{\alpha} \leq \dot{\alpha}_*, \tag{6b}$$

where α_* is a prescribed upper bound and $\dot{\alpha}_*$ is a prescribed, positive constant.

For the guidance schemes discussed in Sections 4, 6, 7, Ineqs. (6) are enforced directly. On the other hand, for the optimal trajectories discussed in Section 5, Ineqs. (6) are enforced indirectly via the following transformation technique:

$$\alpha = \alpha_* - u^2, \tag{7a}$$

$$\dot{u} = -(\dot{\alpha}_*/2u) \sin w, \qquad |u| \geq \epsilon, \tag{7b}$$

$$\dot{u} = -(\dot{\alpha}_*/2u) \sin^2(\pi u/2\epsilon) \sin w, \qquad |u| \leq \epsilon. \tag{7c}$$

Here, $u(t)$, $w(t)$ are auxiliary variables and ϵ is a small, positive constant, which is introduced to prevent the occurrence of singularities. Incidentally, the right-hand sides of Eqs. (7b)–(7c) are continuous and have continuous first derivatives at $|u| = \epsilon$. Clearly, when using Eqs. (7) in conjunction with Eqs. (1)–(3), one must regard $\alpha(t), u(t)$ as state variables and $w(t)$ as control variable.

Approximations for the Forces. The thrust, the drag, and the lift can be approximated with the following functions:

$$T = A_0 + A_1 V + A_2 V^2; \tag{8}$$

$$D = (1/2) C_D \rho S V^2, \tag{9a}$$

$$C_D = B_0 + B_1 \alpha + B_2 \alpha^2, \qquad \alpha \le \alpha_*; \tag{9b}$$

$$L = (1/2) C_L \rho S V^2, \tag{10a}$$

$$C_L = C_0 + C_1 \alpha, \qquad \alpha \le \alpha_{**}, \tag{10b}$$

$$C_L = C_0 + C_1 \alpha + C_2 (\alpha - \alpha_{**})^2, \qquad \alpha_{**} \le \alpha \le \alpha_*. \tag{10c}$$

The coefficients A_i, B_i, C_i, $i = 0, 1, 2$, appearing in Eqs. (8)–(10) can be determined with a least-square fit of manufacturer-supplied data. Numerical experiments show that the resulting precision in the thrust function $T(V)$, drag coefficient function $C_D(\alpha)$, and lift coefficient function $C_L(\alpha)$ is of order 1% or better in the range of velocities and angles of attack having interest in a windshear encounter. These functions are plotted in Fig. 1 with reference to the Boeing B-727 aircraft powered by three JT8D-17 turbofan engines.

Approximations for the Windshear. Over the past several years, considerable attention has been given to the study of a severe meteorological condition known as microburst. This condition involves a descending column of air, which then spreads horizontally in the neighborhood of the ground. This condition is hazardous, because an aircraft in take-off or landing might encounter a headwind coupled with a downdraft, followed by a tailwind coupled with a downdraft (Refs. 15–17).

The representation of the flow field characteristic of a downburst is usually done in one of two ways: (i) by solving numerically the basic fluid mechanics equations and associated boundary conditions; (ii) by employing simple analytical approximations, suggested by the analysis of aircraft accidents. For optimization and guidance studies, the disadvantage of (i) lies in excessive CPU time and excessive memory requirements; on the other hand, the disadvantage of (ii) lies in the possible dissatisfaction of the basic fluid mechanics equations and limited accuracy.

In the light of (i) and (ii), an alternative point of view is taken in this paper; the flow field characteristic of a downburst is represented (iii) by solving numerically the basic fluid mechanics equations and associated boundary conditions and then developing analytical approximations to the numerical solutions. This point of view leads to the following windshear model, valid for $h \leq 1000$ ft (see Fig. 2):

$$W_x = \lambda A(x), \tag{11a}$$

$$W_h = \lambda (h/h_*) B(x); \tag{11b}$$

here, the parameter λ characterizes the intensity of the windshear/downdraft combination; the function $A(x)$ represents the distribution of the horizontal wind versus the horizontal distance; the function $B(x)$ represents the distribution of the vertical wind versus the horizontal distance; and h_* is a reference altitude, $h_* = 1000$ ft.

Concerning the horizontal wind (11a), the function $A(x)$ represents a linear transition from a uniform headwind of -50 ft sec^{-1} to a uniform tailwind of $+50 \text{ ft sec}^{-1}$; hence, the wind velocity difference is $\Delta W_x = 100 \text{ ft sec}^{-1}$ if $\lambda = 1$. The transition takes place over a distance $\Delta x = 4600$ ft, starting at $x = 0$ ft and ending at $x = 4600$ ft; hence, the average wind gradient over the horizontal distance interval $300 \leq x \leq 4300$ ft is $\Delta W_x/\Delta x \cong 0.025 \text{ sec}^{-1}$ if $\lambda = 1$.

Concerning the vertical wind (11b), the function $B(x)$ has a bell-shaped form; in particular, the downdraft vanishes at $x = 0$ ft and $x = 4600$ ft and achieves maximum negative value at $x = 2300$ ft; this maximum negative value is -50 ft sec^{-1} if $h = 1000$ ft and $\lambda = 1$; hence, $\Delta W_h = 50 \text{ ft sec}^{-1}$ if $h = 1000$ ft and $\lambda = 1$.

To sum up, the windshear model (11) has the following properties: (a) it represents the transition from a headwind to a tailwind, with nearly constant shear in the core of the downburst; (b) the downdraft achieves maximum negative value at the center of the downburst; (c) the downdraft vanishes at $h = 0$; and (d) the functions W_x, W_h nearly satisfy the continuity equation and the irrotationality condition in the core of the downburst. Points (a) through (d) are illustrated in Table 1, which compares the simplified wind model W_x, W_h represented by Eqs. (11) with the theoretical model W_{xt}, W_{ht}. Note that the theoretical solutions W_{xt}, W_{ht} were obtained by solving numerically the partial differential equations of an axially-symmetric downburst and associated boundary conditions.

Aircraft Data. The numerical examples of the subsequent sections refer to a Boeing B-727 aircraft powered by three JT8D-17 turbofan engines. It is assumed that: the aircraft has become airborne from a runway located at sea-level altitude; the ambient temperature is 100 deg Fahrenheit; the gear is up; the flap setting is $\delta_F = 15$ deg; the engines are operating at maximum power setting; and the take-off weight is $W = 180{,}000$ lb.

Complete data for this aircraft are omitted here, for the sake of brevity; they can be found in Ref. 5. It is of interest to note that the maximum lift-to-drag ratio of this configuration is $(L/D)_{max} = 10.52$ and that the average thrust-to-weight ratio over the velocity interval of interest is $(T/W)_{av} = 0.22$.

The inequality constraints (6) are enforced with

$$\alpha_* = 16 \text{ deg}, \qquad \dot{\alpha}_* = 3 \text{ deg sec}^{-1}. \tag{12}$$

Initial State. The following initial conditions are assumed:

$$x(0) = 0 \text{ ft}, \qquad h(0) = 50 \text{ ft}, \tag{13a}$$

$$V(0) = 276.8 \text{ ft sec}^{-1}, \qquad \gamma(0) = 6.989 \text{ deg}, \tag{13b}$$

$$\alpha(0) = 10.36 \text{ deg}. \tag{13c}$$

We note that the values (13) correspond to quasi-steady flight; also, the initial velocity $V(0)$ is FAA certification velocity V_2 augmented by 10 knots; in turn, the velocity $V_2 + 10$ (in knots) corresponds approximately to the steepest climb condition in quasi-steady flight.

Final Time. The final time is set at the value

$$\tau = 40 \text{ sec}. \tag{14}$$

This is about twice the duration of the windshear encounter ($\Delta t = 18$ sec).

Final State. For the guidance schemes discussed in Sections 4, 6, 7, all of the state variables are free at the final point. On the other hand, for the optimal trajectories discussed in Section 5, gamma recovery is imposed; that is, it is required that

$$\gamma(\tau) = \gamma(0) = 6.989 \text{ deg}. \tag{15}$$

4. Particular Guidance Schemes

Prior to analyzing optimal trajectories and prior to introducing new guidance schemes, we discuss in this section six well-known guidance schemes, which do not require precise windshear information.

(GS1) Constant Alpha Guidance. Here,

$$\alpha = \alpha_0, \tag{16a}$$

$$\alpha_0 = 10.36 \text{ deg} = 0.1808 \text{ rad}. \tag{16b}$$

(GS2) Maximum Alpha Guidance. Here,

$$\alpha = \alpha_*, \tag{17a}$$

$$\alpha_* = 16.00 \text{ deg} = 0.2792 \text{ rad}. \tag{17b}$$

The symbol α_* denotes the maximum permissible value of the angle of attack, the so-called stick-shaker angle of attack.

(GS3) Constant Velocity Guidance. Here,

$$V = V_0, \tag{18a}$$

$$V_0 = 276.8 \text{ ft sec}^{-1}. \tag{18b}$$

This guidance scheme is implemented through the feedback control law

$$\alpha - \alpha_0 = -K(V_0 - V), \tag{18c}$$

$$K = 0.01 \text{ rad ft}^{-1} \text{ sec}. \tag{18d}$$

(GS4) Constant Absolute Path Inclination Guidance. Here,

$$\gamma_e = \gamma_{e0}, \tag{19a}$$

$$\gamma_{e0} = 8.52 \text{ deg} = 0.1487 \text{ rad}, \qquad \text{for } \lambda = 1. \tag{19b}$$

This guidance scheme is implemented through the feedback control law

$$\alpha - \alpha_0 = -K(\gamma_e - \gamma_{e0}), \tag{19c}$$

$$K = 10. \tag{19d}$$

(GS5) Constant Rate of Climb Guidance. Here,

$$\dot{h} = \dot{h}_0, \tag{20a}$$

$$\dot{h}_0 = 33.68 \text{ ft sec}^{-1}. \tag{20b}$$

This guidance scheme is implemented through the feedback control law

$$\alpha - \alpha_0 = -K(\dot{h} - \dot{h}_0), \tag{20c}$$

$$K = 0.01 \text{ rad ft}^{-1} \text{ sec}. \tag{20d}$$

(GS6) Constant Pitch Attitude Angle Guidance. Here,

$$\theta = \theta_0, \tag{21a}$$

$$\theta_0 = 17.35 \text{ deg} = 0.3028 \text{ rad}. \tag{21b}$$

This guidance scheme is implemented through the feedback control law

$$\alpha - \alpha_0 = -K(\theta - \theta_0), \tag{21c}$$

$$K = 10. \tag{21d}$$

Case $\lambda = 1$. For each of the above guidance schemes, the differential system (1)–(6) was integrated subject to the initial conditions (13). The windshear model (11) was considered, with $\lambda = 1$. Therefore,

$$\Delta W_x = 100 \text{ ft sec}^{-1}, \quad \Delta W_h = 0 \text{ ft sec}^{-1}, \quad \text{at } h = 0 \text{ ft}, \tag{22a}$$

$$\Delta W_x = 100 \text{ ft sec}^{-1}, \quad \Delta W_h = 25 \text{ ft sec}^{-1}, \quad \text{at } h = 500 \text{ ft}, \tag{22b}$$

$$\Delta W_x = 100 \text{ ft sec}^{-1}, \quad \Delta W_h = 50 \text{ ft sec}^{-1}, \quad \text{at } h = 1000 \text{ ft}. \tag{22c}$$

The numerical results are shown in Fig. 3, which refers to guidance schemes (GS1)–(GS3), and in Fig. 4, which refers to guidance schemes (GS4)–(GS6). Each figure includes three parts: the flight altitude h, the velocity V, and the angle of attack α. The following comments are pertinent.

(i) For $\lambda = 1$, $\Delta W_x = 100 \text{ ft sec}^{-1}$, the trajectories of guidance schemes (GS1)–(GS5) hit the ground. On the other hand, the trajectory of guidance scheme (GS6), $\theta = \theta_0$, barely avoids the ground.

(ii) For guidance scheme (GS1), $\alpha = \alpha_0$, the angle of attack is too low on account of the decreased velocity due to the windshear action. Therefore, a severe altitude drop ensues.

(iii) For guidance scheme (GS2), $\alpha = \alpha_*$, the altitude increases too much at the onset of the windshear. This leads to excessive velocity loss, which is followed by considerable altitude drop later on.

(iv) For guidance scheme (GS3), $V = V_0$, there is a severe altitude drop. This is due to the fact that the feedback control scheme tries to maintain a constant velocity in spite of the windshear action. As in guidance scheme (GS1), the angle of attack is too low.

(v) For guidance scheme (GS4), $\gamma_e = \gamma_{e0}$, the main negative characteristic is the same as in guidance scheme (GS2). An analogous remark holds for guidance scheme (GS5), $\dot{h} = \dot{h}_0$.

(vi) Guidance scheme (GS6), $\theta = \theta_0$, is the best in the group of guidance schemes considered in this section. Note that the altitude does not increase too much at the onset of the windshear; hence, excessive velocity loss is avoided. Also, note that the angle of attack is increased gradually as the velocity decreases due to the windshear action. These are the main reasons for the superiority of guidance scheme (GS6) over guidance schemes (GS1)–(GS5).

For the case $\lambda = 1$, a summary of the numerical results is shown in Table 2A, which contains the following information: the initial altitude h_0; the maximum altitude h_{max} (first relative maximum); the minimum altitude h_{min}; the wind intensity parameter λ; and the wind velocity difference ΔW_x.

Case $\lambda = \lambda_c$. For each of the above guidance schemes, the differential system (1)–(6) was integrated subject to the initial conditions (13). The windshear model (11) was considered for different values of the parameter λ. Therefore,

$$\Delta W_x = 100\, l, \qquad \text{ft sec}^{-1}, \tag{23a}$$

$$\Delta W_h = 50\, \lambda (h/h_*), \quad \text{ft sec}^{-1}. \tag{23b}$$

By changing the value of λ, a critical value λ_c can be found, such that the corresponding windshear/downdraft combination yields a trajectory with $h_{\min} = 0$ for a given guidance scheme.

The numerical results are shown in Table 2B, which contains the following information: the initial altitude h_0; the maximum altitude $h_{\max}$ (first relative maximum); the minimum altitude $h_{\min}$; the critical value of the wind intensity parameter λ_c; and the critical value of the wind velocity difference ΔW_{xc}. Table 2B is consistent with the results of Figs. 3–4 and Table 2A; it shows the superior survival capability of guidance scheme (GS6) over guidance schemes (GS1)–(GS5). Table 2B also shows why it is important to develop guidance schemes which are better than guidance scheme (GS6). While the critical value of the wind velocity difference of guidance scheme (GS6) is $\Delta W_{xc} = 101.8$ ft sec^{-1}, it is known that some recent aircraft accidents have involved higher wind velocity differences, of the order of 120 to 130 ft sec^{-1}. This explains the need for studying optimal trajectories (Section 5). It also explains the need for developing advanced guidance schemes (Section 6) and simplified guidance schemes (Section 7), arising from the optimal trajectories. By studying optimal trajectories, one can arrive at an ideal benchmark against which the goodness of any guidance scheme can be measured.

5. Optimal Flight Trajectories

We refer to take-off trajectories and we assume that: global information on the wind flow field is available, that is, the functions (4c) are known in advance; the power setting $\beta(t)$ is given; and the angle of attack $\alpha(t)$ is subject to Ineqs. (6). Hence, upon converting the inequalities into equalities, we refer to the differential system described by Eqs. (1)–(4) and (7). In this system, the state variables are $x(t)$, $h(t)$, $V(t)$, $\gamma(t)$, $\alpha(t)$, $u(t)$ and the control variable is $w(t)$. We formulate the following optimization problem.

Problem (P). Subject to the previous constraints, minimize the peak value of the modulus of the difference between the absolute path inclination and a reference value, assumed constant. In this problem, the performance index is given by

$$I = \max_t |\gamma_e - \gamma_{eR}|, \qquad 0 \leq t \leq \tau, \tag{24a}$$

where

$$\gamma_e = \arctan[(V \sin \gamma + W_h)/(V \cos \gamma + W_x)], \tag{24b}$$

$$\gamma_{eR} = \gamma_{e0}. \tag{24c}$$

This is a minimax problem or Chebyshev problem of optimal control. It can be reformulated as a Bolza problem of optimal control (Refs. 18-19), in which one minimizes the integral performance index

$$J = \int_0^\tau (\gamma_e - \gamma_{eR})^q \, dt, \tag{24d}$$

for large values of the positive, even exponent q.

Boundary Conditions. Concerning the initial conditions, it is assumed that the values of x, h, V, γ, α are specified at $t = 0$, that is,

$$x(0) = x_0, \qquad h(0) = h_0, \qquad V(0) = V_0, \tag{25a}$$

$$\gamma(0) = \gamma_0, \qquad \alpha(0) = \alpha_0. \tag{25b}$$

Upon combining (7a) and (25b), we see that the specification of the initial value of α implies the specification of the initial value of u, that is,

$$u(0) = u_0 = \sqrt{(\alpha_* - \alpha_0)}. \tag{25c}$$

Concerning the final conditions, it is assumed that the value of γ is specified at $t = \tau$, that is,

$$\gamma(\tau) = \gamma_0. \tag{26}$$

The remaining state variables are free at the final point. The final time τ is chosen to be large enough to correspond to a no-windshear condition.

Clearly, use of (26) means that, at the final point, one intends to achieve gamma recovery, namely, restore the initial value of the relative path inclination.

Sequential Gradient-Restoration Algorithm. Problem (P), governed by Eqs. (1)-(4), (7), (24)-(26), is a Bolza problem of optimal control. It can be solved using the family of sequential gradient-restoration algorithms for optimal control problems (SGRA, Refs. 20–23), in either the primal formulation (PSGRA, Refs. 20–21) or the dual formulation (DSGRA, Refs. 22–23).

Regardless of whether the primal formulation is used or the dual formulation is used, sequential gradient-restoration algorithms involve a sequence of two-phase cycles, each cycle including a gradient phase and a restoration phase. In the gradient phase, the value of the augmented functional is decreased, while avoiding excessive constraint violation. In the restoration phase, the value of the constraint error is decreased, while avoiding excessive change in the value of the functional. In a complete gradient-restoration cycle, the value of the functional is decreased, while the constraints are satisfied to a preselected degree of accuracy. Thus, a succession of suboptimal solutions is generated, each new solution being an improvement over the previous one from the point of view of the value of the functional being minimized.

The convergence conditions are represented by the relations

$$P \leq \epsilon_1, \qquad Q \leq \epsilon_2. \tag{27}$$

Here, P is the norm squared of the error in the constraints; Q is the norm squared of the error in the optimality conditions; and ϵ_1, ϵ_2 are preselected, small, positive numbers.

In this work, the sequential gradient-restoration algorithm is employed in conjunction with the dual formulation. The algorithmic details can be found in Refs. 22–23; they are omitted here, for the sake of brevity. For the numerical results on the optimal trajectory, see Section 8.

6. Gamma Guidance

In Section 5, we introduced optimal trajectories, based on the availability of global information on the wind flow field. In practice, an optimal trajectory is difficult to implement for two reasons: global information on the wind flow field might not be available; even if it were available, there might not be enough computing capability onboard to process it adequately. Therefore, in developing a guidance scheme, one is forced to employ local information on the windshear and the downdraft. Under this assumption, several guidance schemes were derived in Refs. 12–13 so as to approximate the behavior of the optimal trajectories and preserve properties (P1) and (P2) of Section 1. Noteworthy among the guidance schemes are the acceleration guidance scheme of Ref. 12 and the gamma guidance scheme of Ref. 13.

In this section, we refer to the absolute gamma guidance scheme of Ref. 13, which is described by the feedback control law

$$\alpha - \tilde{\alpha}(V) = -K[\gamma_e - \tilde{\gamma}_e(\dot{W}_x/g, W_h/V)], \tag{28a}$$

$$\alpha \leq \alpha_*, \qquad \tilde{\gamma}_{e1} \leq \tilde{\gamma}_e \leq \tilde{\gamma}_{e2}. \tag{28b}$$

Here, K is the gain coefficient, $\tilde{\alpha}(V)$ is the nominal angle of attack; $\tilde{\gamma}_e(\dot{W}_x/g, W_h/V)$ is the nominal absolute path inclination; and $\tilde{\gamma}_{e1}, \tilde{\gamma}_{e2}$ are specified lower and upper bounds for the absolute path inclination, for instance,

$$\tilde{\gamma}_{e1} = 0, \qquad \tilde{\gamma}_{e2} = \gamma_{e0}. \tag{28c}$$

The nominal absolute path inclination $\tilde{\gamma}_e(\dot{W}_x/g, W_h/V)$ is discussed in Refs. 12-13. An important property of $\tilde{\gamma}_e$ is that it decreases monotonically as $\dot{W}_x/g$ and W_h/V increase. Therefore, for severe shear-downdraft combinations, one has

$$\tilde{\gamma}_e = \tilde{\gamma}_{e1} = 0. \tag{29}$$

The nominal angle of attack $\tilde{\alpha}(V)$ is also discussed in Refs. 12-13. In the range of velocities of interest for windshear studies, $\tilde{\alpha}$ decreases monotonically as V increases. For the Boeing B-727 aircraft powered by three JT8D-17 turbofan engines, the functions $\tilde{\gamma}_e(\dot{W}_x/g, W_h/V)$ and $\tilde{\alpha}(V)$, computed in accordance with Refs. 12-13, are shown in Table 3.

7. Simplified Gamma Guidance

We recall that the essence of the absolute gamma guidance scheme (28) is that the nominal absolute path inclination $\tilde{\gamma}_e$ decreases as the intensity of the windshear/downdraft combination increases, tending to zero for severe windshear/downdraft combinations. Therefore, in the light of (29), the feedback control law (28) simplifies to

$$\alpha - \tilde{\alpha}(V) = -K\gamma_e, \qquad \alpha \leq \alpha_*, \tag{30}$$

with the following implication: in a severe windshear, the feedback control law is independent of the intensity of the shear and the downdraft.

Because it might be difficult to measure γ_e, we convert the simplified absolute gamma guidance scheme (30) into a rate-of-climb format, more suitable for in-flight measurements. Upon combining Eqs. (1), (5), and using the assumptions

$$\tan \gamma_e \cong \gamma_e, \qquad \cos \gamma \cong 1, \qquad |W_x/V| \ll 1, \tag{31}$$

we see that the absolute path inclination can be rewritten as

$$\gamma_e = \dot{h}/V. \tag{32}$$

As a consequence, the feedback control law (30) becomes

$$\alpha - \tilde{\alpha}(V) = -K(\dot{h}/V), \qquad \alpha \leq \alpha_*. \tag{33}$$

In a windshear encounter, the relative change of V is small by comparison with the relative change of $\dot{h}$. Therefore, upon replacing V with its average value $\bar{V}$ and upon redefining the gain coefficient as follows:

$$\bar{K} = K/\bar{V}, \tag{34}$$

Eq. (33) becomes

$$\alpha - \tilde{\alpha}(V) = -\bar{K}\dot{h}, \qquad \alpha \leq \alpha_*. \tag{35}$$

Then, upon dropping the bar, Eq. (35) is formally rewritten as

$$\alpha - \tilde{\alpha}(V) = -K\dot{h}, \qquad \alpha \leq \alpha_*. \tag{36}$$

The feedback control law represented by Eq. (36) yields a quick transition to horizontal flight if the gain coefficient K is chosen properly. This law is particularly suitable for flight in a severe windshear. Indeed, the essence of safe flight in a severe windshear is to try to avoid the altitude drop, while simultaneously containing the velocity loss.

Comment. The feedback control law (36) is to be employed in the windshear portion of the trajectory. In the aftershear portion, Eq. (36) should be followed by

$$\alpha - \tilde{\alpha}(V) = -K(\dot{h} - \dot{h}_R), \qquad \alpha \leq \alpha_*, \tag{37}$$

where $\dot{h}_R$ denotes a constant reference value, for instance, $\dot{h}_R = \dot{h}(0)$.

8. Comparison of Trajectories

In this section, we compare the numerical results obtained for three trajectories: (GS6) the constant pitch trajectory of Section 4; (GS7) the simplified gamma guidance trajectory of Section 7; and (GS8) the optimal trajectory of Section 5. More specifically:

(GS6) the constant pitch trajectory is computed using the feedback control scheme (21);

(GS7) the simplified gamma guidance trajectory is computed using the feedback control laws (36)-(37), with $K = 0.004$ rad ft^{-1} sec and $\dot{h}_R = \dot{h}_0 = 33.68$ ft sec^{-1};

(GS8) the optimal trajectory is computed by minimizing the performance index (24d), for $q = 6$.

The comparison of trajectories (GS6)-(GS8) is done using the data of Section 3 for the aircraft (Boeing B-727), the boundary conditions, and the windshear model. We recall that the windshear model is represented by Eqs. (11), where λ is a parameter which characterizes the intensity of the windshear/downdraft combination. Two values of λ are employed:

(a) the fixed value $\lambda = 1$ corresponds to a relatively severe windshear; the associated values of $\Delta W_x, \Delta W_h$ are given by Eqs. (22);

(b) the critical value $\lambda = \lambda_c$ corresponds to a windshear whose intensity is such that $h_{\min} = 0$ for a given guidance scheme; the associated values of $\Delta W_x, \Delta W_h$ are given by Eqs. (23).

Case $\lambda = 1$. The numerical results are given in Fig. 5, which includes six parts: the wind velocity components W_x, W_h; the flight altitude h; the absolute path inclination γ_e; the angle of attack α; the velocity V; and the pitch attitude angle θ. The following points must be noted.

Altitude. The function $h(t)$ of the simplified gamma guidance trajectory (SGGT) is almost identical with the function $h(t)$ of the optimal trajectory (OT); that is, the SGGT is characterized by an initial climb, followed by nearly-horizontal flight, followed by renewed climbing after the aircraft has passed through the shear region. For the $\theta = \theta_0$ guidance trajectory, the function $h(t)$ is oscillatory, in the following sense: the aircraft climbs initially, then loses altitude, then barely avoids the ground, and finally climbs again.

Absolute Path Inclination. The function $\gamma_e(t)$ of the SGGT is almost identical with the function $\gamma_e(t)$ of the OT; that is, the absolute path inclination of the SGGT decreases initially until $\gamma_e \cong 0$; then, this value is kept for a relatively long time interval; after passing through the shear region, the value of γ_e is gradually increased. For the $\theta = \theta_0$ guidance trajectory, the function $\gamma_e(t)$ first decreases slowly, and then achieves negative values associated with the altitude drop.

Angle of Attack. The function $\alpha(t)$ of the SGGT is close to the function $\alpha(t)$ of the OT; that is, the angle of attack of the SGGT exhibits an initial decrease, followed by a gradual, sustained increase until the angle of attack boundary is reached. The quick transition to horizontal flight, which characterizes the SGGT, is mainly due to the initial decrease in the angle of attack. For the $\theta = \theta_0$ guidance trajectory, the function $\alpha(t)$ is nearly constant at the beginning of the shear, increasing afterward.

Velocity. The function $V(t)$ of the SGGT is close to the function $V(t)$ of the OT. In particular, the maximum drop in relative velocity for the SGGT occurs at about the time when the shear ends.

For the case $\lambda = 1$, a summary of the numerical results is shown in Table 4A, which contains the following information: the initial altitude h_0; the maximum altitude $h_{\max}$ (first relative maximum); the minimum altitude $h_{\min}$; the wind intensity parameter λ; and the wind velocity difference ΔW_x.

Case $\lambda = \lambda_c$. By increasing the value of λ, more intense windshear/downdraft combinations are generated until a critical value λ_c is found such that $h_{min} = 0$ for a given guidance scheme. The numerical results are shown in Table 4B, which contains the following information: the initial altitude h_0; the maximum altitude h_{max} (first relative maximum); the minimum altitude h_{min}; the critical value of the wind intensity parameter λ_c; and the critical value of the wind velocity difference ΔW_{xc}. From Table 4B, the following conclusions can be derived:

(i) the survival capability of the simplified gamma guidance trajectory is close to the survival capability of the optimal trajectory;

(ii) the survival capability of the simplified gamma guidance trajectory is superior to that of the constant pitch trajectory $\theta = \theta_0$.

9. Effect of the Gain Coefficient

In this section, we study the effect of the feedback gain coefficient K on the behavior of the simplified gamma guidance scheme in a severe windshear. Generally speaking, larger values of K correspond to a quicker transition to horizontal flight.

With reference to Eq. (36), we consider five equally spaced values of the gain coefficient, ranging from $K = 0.004$ to $K = 0.020$ rad ft^{-1} sec. For each value of K, we integrate the differential system (1)-(6) subject to the initial conditions (13). Once more, we consider the windshear model (11) and employ two values of the parameter λ which characterizes the intensity of the windshear/downdraft combination: the fixed value $\lambda = 1$, corresponding to a relatively severe windshear, such that $\Delta W_x = 100$ ft sec^{-1}; and the critical value $\lambda = \lambda_c$, corresponding to a windshear whose intensity is sufficient to cause a crash.

The numerical results, shown in Table 5, indicate that larger values of K improve the survival capability of the simplified gamma guidance scheme in a severe windshear; however, excessive values of K are undesirable, because they result in larger altitude oscillations and lower average altitude.

10. Effect of a Time Delay in Reacting to Windshear Onset

In this section, we study the effect of a time delay τ_1 in the pilot reaction to windshear onset on the behavior of the simplified gamma guidance scheme. With reference to Eq. (36), we assume that the gain coefficient is set at the level $K = 0.004$ rad ft^{-1} sec. Also, we assume that, in the time interval $0 \le t \le \tau_1$, a different type of guidance (prewindshear guidance) is employed. Specifically, we consider three cases: prewindshear guidance $\theta = \theta_0$; prewindshear guidance $\alpha = \alpha_0$; and prewindshear guidance $V = V_0$.

We consider seven equally spaced values of τ_1, ranging from $\tau_1 = 0$ to $\tau_1 = 6$ sec, followed by seven equally spaced values of τ_1, ranging from $\tau_1 = 6$ to $\tau_1 = 18$ sec. For each value of τ_1, we integrate the differential system (1)-(6) subject to the initial conditions (13). Once more, we consider the windshear model (11) and employ two values of the parameter λ which characterizes the intensity of the windshear/downdraft combination: the fixed value $\lambda = 1$, corresponding to a relatively severe windshear, such that $\Delta W_x = 100$ ft sec^{-1}; and the critical value $\lambda = \lambda_c$, corresponding to a windshear whose intensity is sufficient to cause a crash.

The numerical results, shown in Tables 6-8, indicate that the time delay τ_1 has a critical effect on the survival capability of the simplified gamma guidance scheme, in the following sense: smaller values of τ_1 correspond to better survival capability in a severe windshear, while larger value of τ_1 are associated with a worsening of the survival capability in a severe windshear.

The effect of the time delay τ_1 depends to some degree on the type of prewindshear guidance employed. It is particularly critical for prewindshear guidance $V = V_0$. At any rate, regardless of the type of prewindshear guidance, the simplified gamma guidance scheme is superior to the constant pitch guidance scheme if $\tau_1 \leq 4$ sec. The following comments are pertinent.

(i) If prewindshear guidance $\theta = \theta_0$ is employed, early transition to horizontal flight improves the survival capability. However, if the transition is delayed to values of $\tau_1 \geq 4$ sec, no harmful effect arises: the simplified gamma guidance and the $\theta = \theta_0$ guidance yield almost identical results (see Table 6B).

(ii) If prewindshear guidance $\alpha = \alpha_0$ is employed, early transition to horizontal flight is important to the safety of flight. The survival capability worsens as the time delay τ_1 increases (see Table 7B).

(iii) If prewindshear guidance $V = V_0$ is employed, early transition to horizontal flight is extremely important; indeed, it is vital to the safety of flight. This is explained below.

If $\tau_1 = 0$, both the $V = V_0$ guidance and the simplified gamma guidance involve an initial decrease in the angle of attack after the windshear onset; later on, the $V = V_0$ guidance involves a continued decrease of the angle of attack, while the simplified gamma guidance involves an increase in the angle of attack. If $\tau_1 \neq 0$, the dynamical effects depend on the value of τ_1, in the following sense: if $\tau_1 \leq 4$ sec, the survival capability is affected only in a minor way by the transition delay; however, if $\tau_1 \geq 4$ sec, the survival capability is affected critically by the transition delay (see Table 8B).

(iv) Finally, we note that the value of the gain coefficient $K = 0.004$ rad ft^{-1} sec employed in the numerical experiments of this section is conservative. If larger values of K are employed, then the effect of the time delay τ_1 becomes less critical.

11. Effect of a Time Delay in Reacting to Windshear Termination

In this section, we study the effect of a time delay τ_2 in the pilot reaction to windshear termination on the behavior of the simplified gamma guidance scheme. With reference to Eqs. (36)-(37), we assume that $\tau_1 = 0$; that Eq. (36), with $K = 0.004$ rad ft^{-1} sec, is employed in the time interval $0 \leq t \leq t_* + \tau_2$; and that Eq. (37) is employed in the time interval $t_* + \tau_2 \leq t \leq \tau$. Here, t_* denotes the time instant at which the windshear actually terminates.

We consider four equally spaced values of τ_2, ranging from $\tau_2 = 0$ to $\tau_2 = 8$ sec. For each value of τ_2, we integrate the differential system (1)-(6) subject to the initial conditions (13). Once more, we consider the windshear model (11) and employ two values of the parameter λ which characterizes the intensity of the windshear/downdraft combination: the fixed value $\lambda = 1$, corresponding to a relatively severe windshear, such that $\Delta W_x = 100$ ft sec^{-1}; and the critical value $\lambda = \lambda_c$, corresponding to a windshear whose intensity is sufficient to cause a crash.

The numerical results, shown in Table 9, indicate that the time delay τ_2 has little effect on the survival capability of the simplified gamma guidance scheme.

12. Piloting Implications

In Section 7, we introduced the simplified gamma guidance scheme and we stressed the fact that it yields a quick transition to horizontal flight. In Sections 8-11, we showed that this technique has an excellent survival capability in a severe windshear, nearly as good as that of an optimal trajectory.

The simplified gamma guidance scheme is simple in concept as well as in flight implementation: upon sensing that he is in a shear, the pilot performs a quick transition from climbing flight to horizontal flight, and then keeps the plane in nearly-horizontal flight until the shear region is past.

With reference to the quick transition to horizontal flight, the following comments are pertinent:

(i) it is assumed that the pilot has performed the take-off using maximum power setting; it is also assumed that there are no obstacles ahead;

(ii) the fact that the aircraft has entered into the shear region can be detected by sudden change in the velocity or the rate of climb, depending on the type of prewindshear guidance employed, more specifically: a sudden velocity loss if prewindshear guidance $\theta = \text{const}$ or $\alpha = \text{const}$ is employed; or a sudden rate-of-climb loss if prewindshear guidance $V = \text{const}$ is employed;

(iii) the fact that the aircraft has exited from the shear region can be detected by velocity increase occurring without altitude drop;

(iv) the adjective "quick" in the phrase "quick transition to horizontal flight" means that such transition should be done with the smallest possible time delay after the onset of the windshear; it should also be done by bringing $\dot{h}$ to zero rapidly;

(v) after starting the transition, the pilot has a wide choice of transition speeds, corresponding to different values of the gain coefficient K in Eq. (36); obviously, smaller values of K are easier to implement; larger values of K improve the survival capability in a severe windshear; when choosing a large value of K, attention should be paid to the avoidance of altitude overshoots or oscillations.

13. Conclusions

This paper is concerned with guidance strategies and piloting techniques which ensure near-optimum performance and maximum survival capability in a severe windshear. The take-off problem is considered with reference to flight in a vertical plane. In addition to the horizontal shear, the presence of a downdraft is assumed. The major conclusions are as follows.

(i) Among six popular guidance schemes, the $\theta = \theta_0$ guidance is the best and is easy to implement. In a severe windshear, the $\theta = \theta_0$ guidance has better survival capability than the $\alpha = \alpha_0$ guidance, $\alpha = \alpha_*$ guidance, $V = V_0$ guidance, $\gamma_e = \gamma_{e0}$ guidance, and $\dot{h} = \dot{h}_0$ guidance.

(ii) Based on the idea of approximating the properties of the optimal trajectories, a gamma guidance scheme and a simplified gamma guidance scheme are developed; the latter is the limiting case of the former in a severe windshear.

(iii) In a severe windshear, the simplified gamma guidance scheme yields a quick transition to horizontal flight. Its survival capability is superior to that of the $\theta = \theta_0$ guidance and is close to that of the optimal trajectory.

(iv) Concerning the feedback gain coefficient K, larger values of K improve the survival capability of the simplified gamma guidance scheme in a severe windshear; however, excessive values of K are undesirable, because they result in larger altitude oscillations and lower average altitude.

(v) If the pilot reacts to windshear termination with a time delay τ_2, such time delay has little effect on the survival capability of the simplified gamma guidance scheme.

(vi) If the pilot reacts to windshear onset with a time delay τ_1, such time delay has a critical effect on the survival capability of the simplified gamma guidance scheme, in the following sense: smaller values of τ_1 correspond to better survival capability in a severe windshear, while larger values of τ_1 are associated with a worsening of the survival capability in a severe windshear. The effect of the time delay τ_1 depends to some degree on the type of prewindshear guidance employed. Regardless of the type of prewindshear guidance, the simplified gamma guidance scheme is superior to the constant pitch guidance scheme if $\tau_1 \leq 4$ sec.

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Table 1A. Horizontal wind: Comparison of simplified wind model and theoretical wind model.

	$x = 1300$	$x = 1800$	$x = 2300$	$x = 2800$	$x = 3300$	
W_x	−25.0	−12.5	0.0	12.5	25.0	$h = 0$
W_{xt}	−23.3	−12.4	0.0	12.4	23.3	$h = 0$
W_x	−25.0	−12.5	0.0	12.5	25.0	$h = 200$
W_{xt}	−23.6	−12.6	0.0	12.6	23.6	$h = 200$
W_x	−25.0	−12.5	0.0	12.5	25.0	$h = 400$
W_{xt}	−24.5	−13.0	0.0	13.0	24.5	$h = 400$

Values of x, h are in ft; values of W_x, W_{xt} are in ft sec^{-1}.

Table 1B. Vertical wind: Comparison of simplified wind model and theoretical wind model.

	$x = 1300$	$x = 1800$	$x = 2300$	$x = 2800$	$x = 3300$	
W_h	0.0	0.0	0.0	0.0	0.0	$h = 0$
W_{ht}	0.0	0.0	0.0	0.0	0.0	$h = 0$
W_h	−8.1	−9.9	−10.0	−9.9	−8.1	$h = 200$
W_{ht}	−8.6	−9.8	−10.2	−9.8	−8.6	$h = 200$
W_h	−16.1	−19.9	−20.0	−19.9	−16.1	$h = 400$
W_{ht}	−17.4	−19.8	−20.6	−19.8	−17.4	$h = 400$

Values of x, h are in ft; values of W_h, W_{ht} are in ft sec^{-1}.

Table 2A. Performance of particular guidance schemes, Case $\lambda = 1$.

Guidance scheme	Remark	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ	ΔW_x (ft sec^{-1})
(GS1)	$\alpha = \alpha_0$	50.0	270.6	−593.5	1.000	100.0
(GS2)	$\alpha = \alpha_*$	50.0	657.2	−577.6	1.000	100.0
(GS3)	$V = V_0$	50.0	158.6	−234.5	1.000	100.0
(GS4)	$\gamma_e = \gamma_{e0}$	50.0	441.5	−187.8	1.000	100.0
(GS5)	$\dot{h} = \dot{h}_0$	50.0	414.0	−143.9	1.000	100.0
(GS6)	$\theta = \theta_0$	50.0	278.8	21.7	1.000	100.0

Table 2B. Survival capability of particular guidance schemes, Case $\lambda = \lambda_c$.

Guidance scheme	Remark	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ_c	ΔW_{xc} (ft sec^{-1})
(GS1)	$\alpha = \alpha_0$	50.0	313.7	0.0	0.585	58.5
(GS2)	$\alpha = \alpha_*$	50.0	765.4	0.0	0.577	57.7
(GS3)	$V = V_0$	50.0	171.1	0.0	0.666	66.6
(GS4)	$\gamma_e = \gamma_{e0}$	50.0	465.8	0.0	0.874	87.4
(GS5)	$\dot{h} = \dot{h}_0$	50.0	432.3	0.0	0.901	90.1
(GS6)	$\theta = \theta_0$	50.0	277.0	0.0	1.018	101.8

Table 3A. Gamma guidance scheme, nominal absolute path inclination $\tilde{\gamma}_e = \tilde{\gamma}_e(\dot{W}_x/g, W_h/V)$.

	$\dot{W}_x/g = 0.00$	$\dot{W}_x/g = 0.05$	$\dot{W}_x/g = 0.10$	$\dot{W}_x/g = 0.15$	$\dot{W}_x/g = 0.20$
$W_h/V = 0.00$	6.99	5.59	4.19	2.80	1.40
$W_h/V = -0.05$	6.99	5.02	3.05	1.08	0.00
$W_h/V = -0.10$	6.99	4.45	1.90	0.00	0.00
$W_h/V = -0.15$	6.99	3.87	0.76	0.00	0.00
$W_h/V = -0.20$	6.99	3.30	0.00	0.00	0.00

Values of $\tilde{\gamma}_e$ are in deg.

Table 3B. Gamma guidance scheme, nominal angle of attack $\tilde{\alpha} = \tilde{\alpha}(V)$.

V (ft sec^{-1})	$\tilde{\alpha}$ (deg)	V (ft sec^{-1})	$\tilde{\alpha}$ (deg)
200	16.00	250	13.06
210	16.00	260	11.96
220	16.00	270	11.03
230	16.00	280	10.19
232.63	16.00	290	9.43
240	14.51	300	8.74

Table 4A. Performance of particular guidance schemes, Case $\lambda = 1$.

Guidance scheme	Remark	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ	ΔW_x (ft sec^{-1})
(GS6)	$\theta = \theta_0$	50.0	278.8	21.7	1.000	100.0
(GS7)	SGGT	50.0	140.6	128.0	1.000	100.0
(GS8)	OT	50.0	124.5	115.8	1.000	100.0

Table 4B. Survival capability of particular guidance schemes, Case $\lambda = \lambda_c$.

Guidance scheme	Remark	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ_c	ΔW_{xc} (ft sec^{-1})
(GS6)	$\theta = \theta_0$	50.0	277.0	0.0	1.018	101.8
(GS7)	SGGT	50.0	140.8	0.0	1.135	113.5
(GS8)	OT	50.0	118.5	0.0	1.195	119.5

Table 5A. Effect of the gain coefficient K on the performance of the simplified gamma guidance scheme, Case $\lambda = 1$.

Guidance scheme	Remark	K (rad ft^{-1} sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ	ΔW_x (ft sec^{-1})
(GS7)	SGGT	0.004	50	140.6	128.0	1.000	100.0
(GS7)	SGGT	0.008	50	117.0	106.1	1.000	100.0
(GS7)	SGGT	0.012	50	115.5	80.1	1.000	100.0
(GS7)	SGGT	0.016	50	115.2	62.2	1.000	100.0
(GS7)	SGGT	0.020	50	115.1	46.5	1.000	100.0

Table 5B. Effect of the gain coefficient K on the survival capability of the simplified gamma guidance scheme, Case $\lambda = \lambda_c$.

Guidance scheme	Remark	K (rad ft^{-1} sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ_c	ΔW_{xc} (ft sec^{-1})
(GS7)	SGGT	0.004	50	140.8	0.0	1.135	113.5
(GS7)	SGGT	0.008	50	116.9	0.0	1.162	116.2
(GS7)	SGGT	0.012	50	115.5	0.0	1.179	117.9
(GS7)	SSGT	0.016	50	115.1	0.0	1.189	118.9
(GS7)	SGGT	0.020	50	115.1	0.0	1.195	119.5

Table 6A. Effect of the time delay τ_1 on the performance of the simplified gamma guidance scheme, Case $\lambda = 1$, prewindshear guidance $\theta = \theta_0$, gain coefficient $K = 0{\cdot}004$ rad ft^{-1} sec.

Guidance scheme	Remark	τ_1 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ	ΔW_x (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.6	128.0	1.000	100.0
(GS7)	SGGT	1	50.0	173.2	121.0	1.000	100.0
(GS7)	SGGT	2	50.0	205.0	96.4	1.000	100.0
(GS7)	SGGT	3	50.0	230.4	71.2	1.000	100.0
(GS7)	SGGT	4	50.0	252.6	47.9	1.000	100.0
(GS7)	SGGT	5	50.0	267.9	32.0	1.000	100.0
(GS7)	SGGT	6	50.0	279.2	20.7	1.000	100.0
(GS7)	SGGT	8	50.0	288.1	12.4	1.000	100.0
(GS7)	SGGT	10	50.0	280.7	17.9	1.000	100.0
(GS7)	SGGT	12	50.0	278.8	22.1	1.000	100.0
(GS7)	SGGT	14	50.0	278.8	21.7	1.000	100.0
(GS7)	SGGT	16	50.0	278.8	21.7	1.000	100.0
(GS7)	SGGT	18	50.0	278.8	21.7	1.000	100.0
(GS6)	$\theta = \theta_0$	0	50.0	278.8	21.7	1.000	100.0

Table 6B. Effect of the time delay τ_1 on the survival capability of the simplified gamma guidance scheme, Case $\lambda = \lambda_c$, prewindshear guidance $\theta = \theta_0$, gain coefficient $K = 0.004$ rad ft^{-1} sec.

Guidance scheme	Remark	τ_1 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ_c	ΔW_{xc} (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.8	0.0	1.135	113.5
(GS7)	SGGT	1	50.0	173.3	0.0	1.104	110.4
(GS7)	SGGT	2	50.0	205.0	0.0	1.075	107.5
(GS7)	SGGT	3	50.0	230.2	0.0	1.053	105.3
(GS7)	SGGT	4	50.0	252.1	0.0	1.035	103.5
(GS7)	SGGT	5	50.0	267.3	0.0	1.023	102.3
(GS7)	SGGT	6	50.0	278.6	0.0	1.015	101.5
(GS7)	SGGT	8	50.0	287.3	0.0	1.009	100.9
(GS7)	SGGT	10	50.0	278.8	0.0	1.014	101.4
(GS7)	SGGT	12	50.0	277.0	0.0	1.018	101.8
(GS7)	SGGT	14	50.0	277.0	0.0	1.018	101.8
(GS7)	SGGT	16	50.0	277.0	0.0	1.018	101.8
(GS7)	SGGT	18	50.0	277.0	0.0	1.018	101.8
(GS6)	$\theta = \theta_0$	0	50.0	277.0	0.0	1.018	101.8

Table 7A. Effect of the time delay τ_1 on the performance of the simplified gamma guidance scheme, Case $\lambda = 1$, prewindshear guidance $\alpha = \alpha_0$, gain coefficient $K = 0.004$ rad ft^{-1} sec.

Guidance scheme	Remark	τ_1 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ	ΔW_x (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.6	128.0	1.000	100.0
(GS7)	SGGT	1	50.0	173.3	120.9	1.000	100.0
(GS7)	SGGT	2	50.0	205.8	95.6	1.000	100.0
(GS7)	SGGT	3	50.0	231.9	69.5	1.000	100.0
(GS7)	SGGT	4	50.0	255.1	44.9	1.000	100.0
(GS7)	SGGT	5	50.0	271.3	28.0	1.000	100.0
(GS7)	SGGT	6	50.0	282.8	16.4	1.000	100.0
(GS7)	SGGT	8	50.0	283.5	14.9	1.000	100.0
(GS7)	SGGT	10	50.0	270.9	29.2	1.000	100.0
(GS7)	SGGT	12	50.0	270.9	−22.3	1.000	100.0
(GS7)	SGGT	14	50.0	270.9	−111.4	1.000	100.0
(GS7)	SGGT	16	50.0	270.9	−211.7	1.000	100.0
(GS7)	SGGT	18	50.0	270.9	−311.9	1.000	100.0
(GS1)	$\alpha = \alpha_0$	0	50.0	270.6	−593.5	1.000	100.0

Table 7B. Effect of the time delay τ_1 on the survival capability of the simplified gamma guidance scheme, Case $\lambda = \lambda_c$, prewindshear guidance $\alpha = \alpha_0$, gain coefficient $K = 0.004$ rad ft^{-1} sec.

Guidance scheme	Remark	τ_1 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ_c	ΔW_{xc} (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.8	0.0	1.135	113.5
(GS7)	SGGT	1	50.0	173.4	0.0	1.104	110.4
(GS7)	SGGT	2	50.0	205.7	0.0	1.074	107.4
(GS7)	SGGT	3	50.0	231.6	0.0	1.052	105.2
(GS7)	SGGT	4	50.0	254.7	0.0	1.033	103.3
(GS7)	SGGT	5	50.0	270.8	0.0	1.020	102.0
(GS7)	SGGT	6	50.0	282.3	0.0	1.012	101.2
(GS7)	SGGT	8	50.0	282.2	0.0	1.011	101.1
(GS7)	SGGT	10	50.0	269.5	0.0	1.026	102.6
(GS7)	SGGT	12	50.0	272.3	0.0	0.978	97.8
(GS7)	SGGT	14	50.0	279.0	0.0	0.884	88.4
(GS7)	SGGT	16	50.0	287.8	0.0	0.785	78.5
(GS7)	SGGT	18	50.0	296.7	0.0	0.705	70.5
(GS1)	$\alpha = \alpha_0$	0	50.0	313.7	0.0	0.585	58.5

Table 8A. Effect of the time delay τ_1 on the performance of the simplified gamma guidance scheme, Case $\lambda = 1$, prewindshear guidance $V = V_0$, gain coefficient $K = 0.004$ rad ft^{-1} sec.

Guidance scheme	Remark	τ_1 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ	ΔW_x (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.6	128.0	1.000	100.0
(GS7)	SGGT	1	50.0	169.7	122.8	1.000	100.0
(GS7)	SGGT	2	50.0	181.8	115.9	1.000	100.0
(GS7)	SGGT	3	50.0	183.5	114.8	1.000	100.0
(GS7)	SGGT	4	50.0	158.7	114.3	1.000	100.0
(GS7)	SGGT	5	50.0	158.6	7.0	1.000	100.0
(GS7)	SGGT	6	50.0	158.6	−96.8	1.000	100.0
(GS7)	SGGT	8	50.0	158.6	−169.2	1.000	100.0
(GS7)	SGGT	10	50.0	158.6	−186.1	1.000	100.0
(GS7)	SGGT	12	50.0	158.6	−199.9	1.000	100.0
(GS7)	SGGT	14	50.0	158.6	−208.4	1.000	100.0
(GS7)	SGGT	16	50.0	158.6	−228.9	1.000	100.0
(GS7)	SGGT	18	50.0	158.6	−234.0	1.000	100.0
(GS3)	$V = V_0$	0	50.0	158.6	−234.5	1.000	100.0

Table 8B. Effect of the time delay τ_1 on the survival capability of the simplified gamma guidance scheme, Case $\lambda = \lambda_c$, prewindshear guidance $V = V_0$, gain coefficient $K = 0.004$ rad ft^{-1} sec.

Guidance scheme	Remark	τ_1 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ_c	ΔW_{xc} (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.8	0.0	1.135	113.5
(GS7)	SGGT	1	50.0	169.7	0.0	1.107	110.7
(GS7)	SGGT	2	50.0	181.2	0.0	1.097	109.7
(GS7)	SGGT	3	50.0	182.1	0.0	1.096	109.6
(GS7)	SGGT	4	50.0	155.4	0.0	1.167	116.7
(GS7)	SGGT	5	50.0	158.3	0.0	1.014	101.4
(GS7)	SGGT	6	50.0	162.8	0.0	0.847	84.7
(GS7)	SGGT	8	50.0	167.7	0.0	0.727	72.7
(GS7)	SGGT	10	50.0	170.3	0.0	0.679	67.9
(GS7)	SGGT	12	50.0	171.6	0.0	0.659	65.9
(GS7)	SGGT	14	50.0	171.3	0.0	0.662	66.2
(GS7)	SGGT	16	50.0	171.1	0.0	0.666	66.6
(GS7)	SGGT	18	50.0	171.1	0.0	0.666	66.6
(GS3)	$V = V_0$	0	50.0	171.1	0.0	0.666	66.6

Table 9A. Effect of the time delay τ_2 on the performance of the simplified gamma guidance scheme, Case $\lambda = 1$.

Guidance scheme	Remark	τ_2 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ	ΔW_x (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.6	128.0	1.000	100.0
(GS7)	SGGT	2	50.0	140.6	127.8	1.000	100.0
(GS7)	SGGT	4	50.0	140.6	125.4	1.000	100.0
(GS7)	SGGT	8	50.0	140.6	122.8	1.000	100.0

Table 9B. Effect of the time delay τ_2 on the survival capability of the simplified gamma guidance scheme, Case $\lambda = \lambda_c$.

Guidance scheme	Remark	τ_2 (sec)	h_0 (ft)	h_{max} (ft)	h_{min} (ft)	λ_c	ΔW_{xc} (ft sec^{-1})
(GS7)	SGGT	0	50.0	140.8	0.0	1.135	113.5
(GS7)	SGGT	2	50.0	140.8	0.0	1.135	113.5
(GS7)	SGGT	4	50.0	140.8	0.0	1.135	113.5
(GS7)	SGGT	8	50.0	140.8	0.0	1.128	112.8

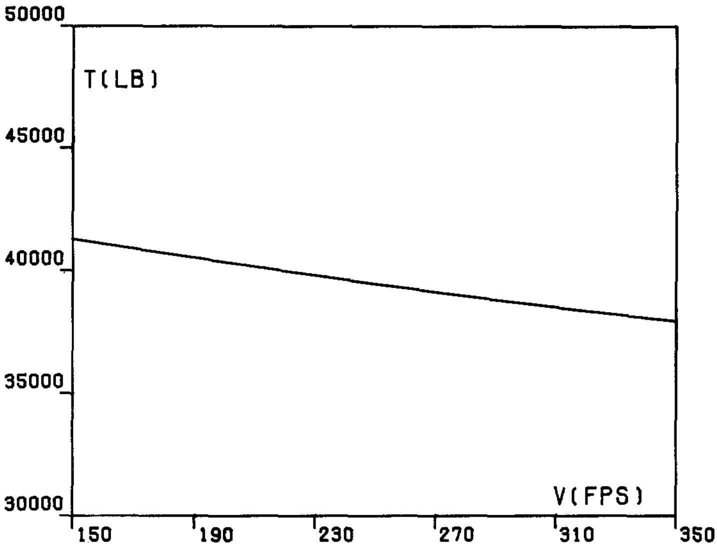

Fig. 1A. Thrust T versus velocity V for the Boeing B-727 aircraft powered by three JT8D-17 turbofan engines (maximum power setting, sea-level altitude, ambient temperature = 100 deg Fahrenheit).

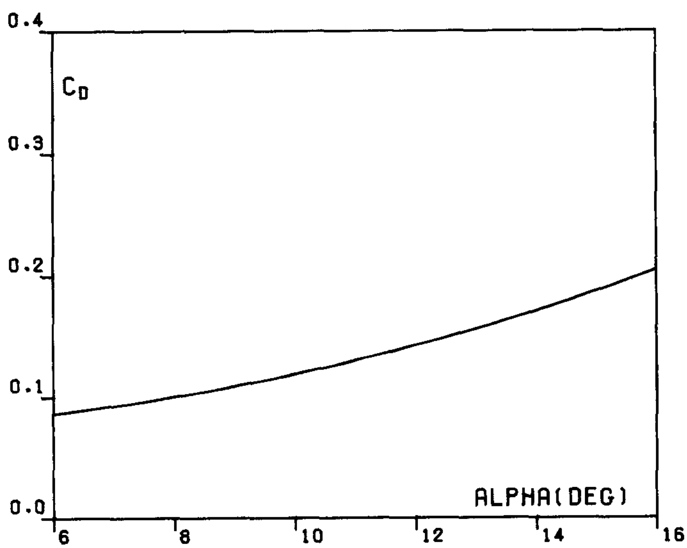

Fig. 1B. Drag coefficient C_D versus angle of attack α for the Boeing B-727 aircraft (gear up, flap setting $\delta_F = 15$ deg).

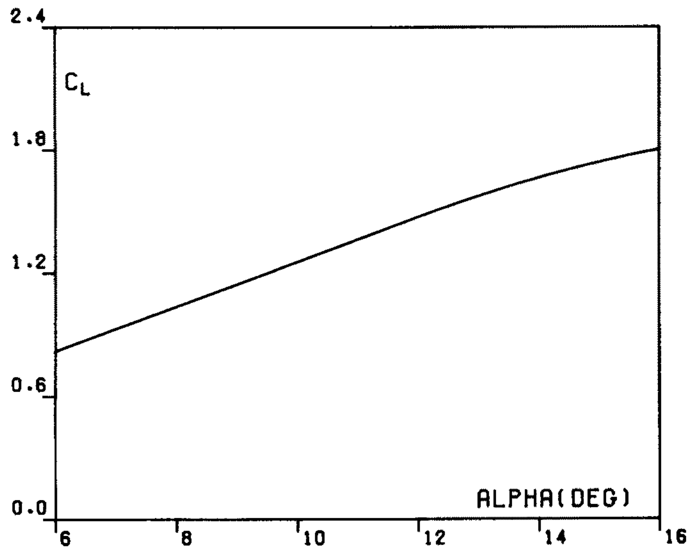

Fig. 1C. Lift coefficient C_L versus angle of attack α for the Boeing B-727 aircraft (gear up, flap setting $\delta_F = 15$ deg).

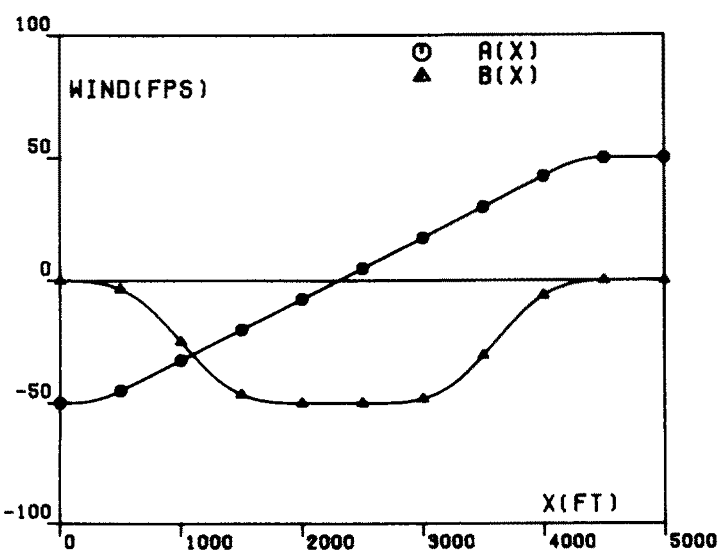

Fig. 2. Horizontal wind function $A(x)$ and vertical wind function $B(x)$.

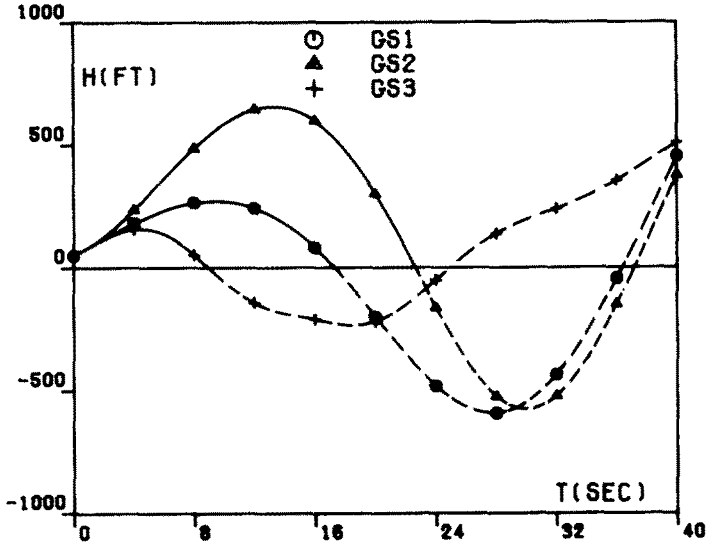

Fig. 3A. Particular guidance schemes: altitude h versus time t.

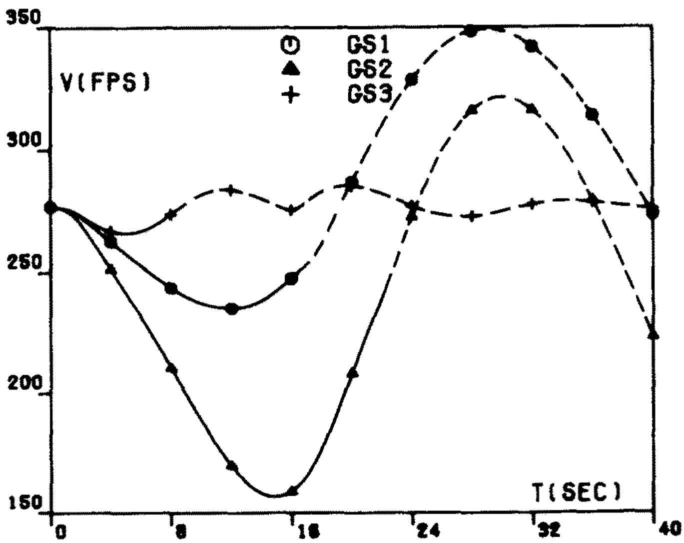

Fig. 3B. Particular guidance schemes: velocity V versus time t.

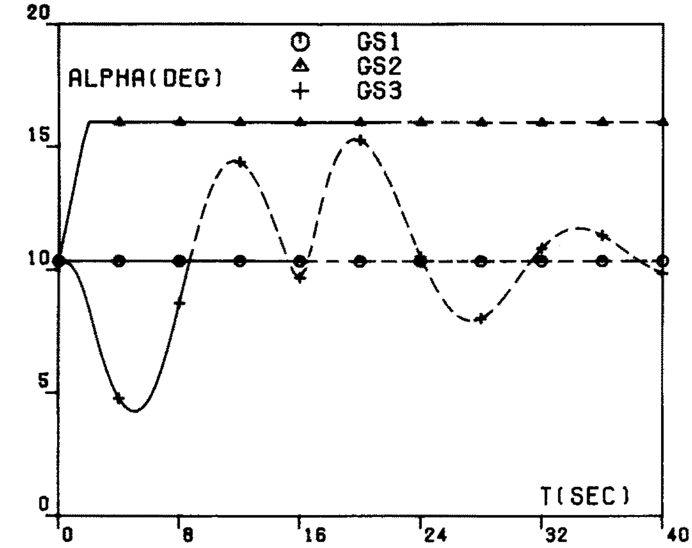

Fig. 3C. Particular guidance schemes: angle of attack α versus time t.

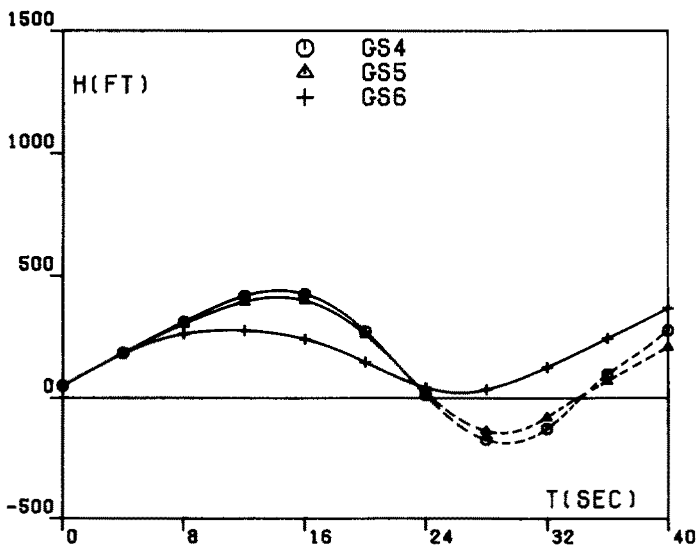

Fig. 4A. Particular guidance schemes: altitude h versus time t.

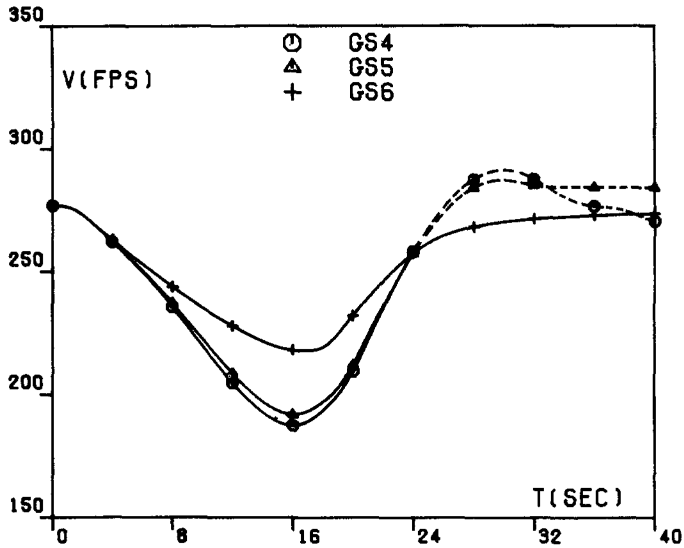

Fig. 4B. Particular guidance schemes: velocity V versus time t.

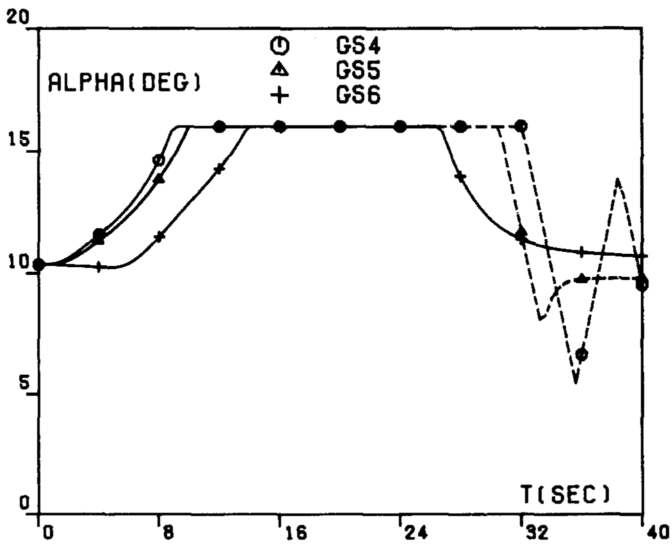

Fig. 4C. Particular guidance schemes: angle of attack α versus time t.

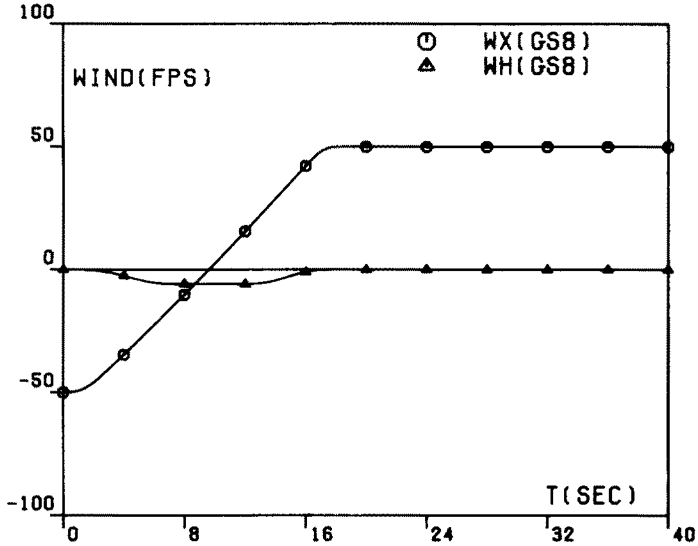

Fig. 5A. Comparison of trajectories: horizontal wind W_x and vertical wind W_h versus time t.

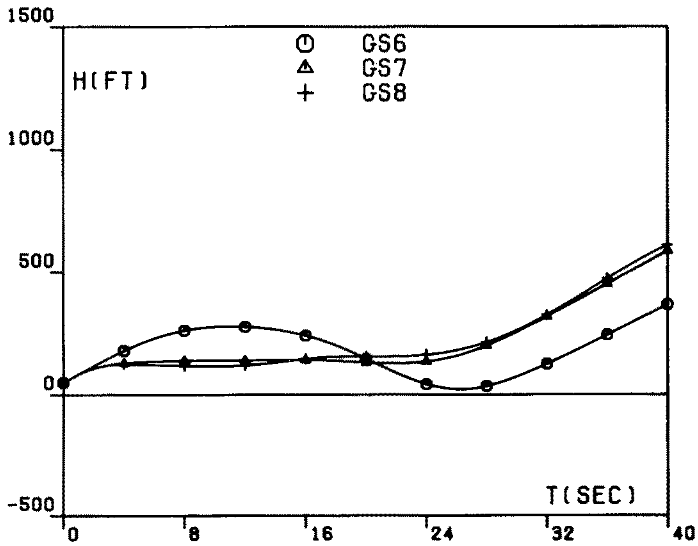

Fig. 5B. Comparison of trajectories: altitude h versus time t.

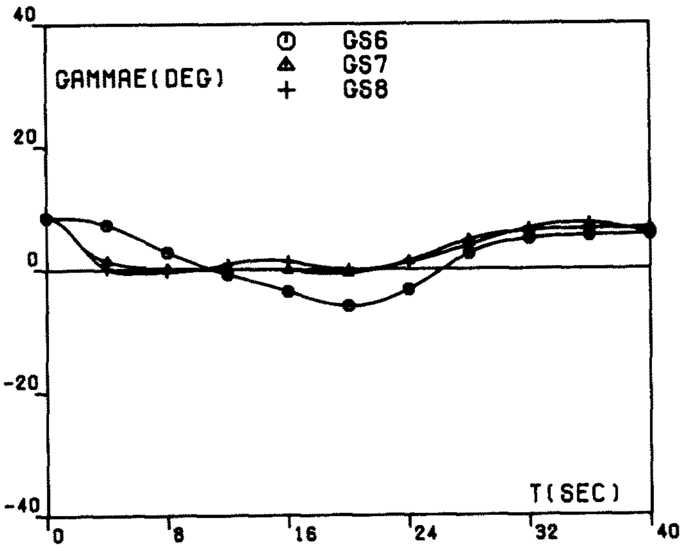

Fig. 5C. Comparison of trajectories: absolute path inclination γ_e versus time t.

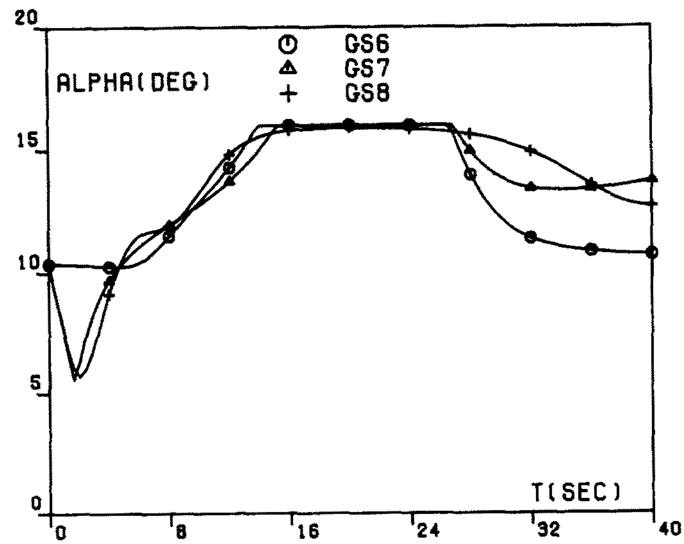

Fig. 5D. Comparison of trajectories: angle of attack α versus time t.

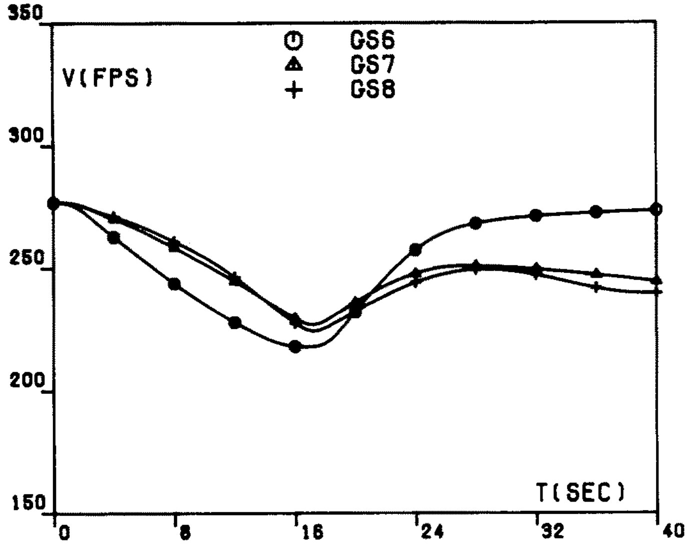

Fig. 5E. Comparison of trajectories: velocity V versus time t.

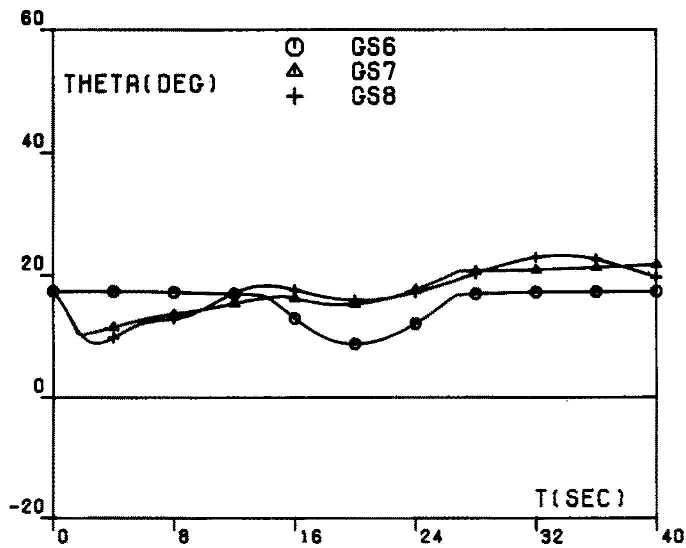

Fig. 5F. Comparison of trajectories: pitch attitude angle θ versus time t.