ERRATA CORRIGE

On a Domination Property for
Vector Maximization with Respect to Cones[1]

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Abstract. We give a corrected version of Theorem 2.1 of Ref. 1.

Key Words. Vector maximization, multiple-objective optimization, efficient points, proper efficiency, nondominated points.

1. Introduction

In Ref. 1, we give two results concerning a domination property for vector maximization with respect to cones. The first of these, Theorem 2.1, is incorrect, as shown by the following counterexample. See also Ref. 2.

Let

$$X = \{x \in R^3 | 0 \leq x_1 \leq 1, \; x_2^2 \leq (1 - x_1)x_3, \; x_3 \geq 0\},$$

$$S = \{(s_1, s_2) \in R^2 | s_1 - s_2 \geq 0, \; s_1 + s_2 \geq 0\},$$

$$\langle c_1, x \rangle = x_1, \qquad \langle c_2, x \rangle = x_2, \qquad \langle c_3, x \rangle = 0.$$

Then

$$\bar{x} = (0.1, 0.1, 1.0) \in X;$$

but, since

$$X_E = X_{PRE} = \{x \in R^3 | x = (1, 0, x_3) \text{ for some } x_3 \geq 0\},$$

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there does not exist an $x^0 \in X_E$ such that

$$\langle c_i, x^0 \rangle \geqq \langle c_i, \bar{x} \rangle, \qquad i = 1, 2, 3.$$

Since X is a closed, convex set, and S is a closed, convex, acute cone, it is clear that Theorem 2.1 is violated.

2. Results

For any $c_j \in R^n$, $j \in J$, let C denote the $p \times n$ matrix whose jth row is given by c_j^T for each $j \in J$. A correct restatement of Theorem 2.1 can then be made as follows.

Theorem 2.1. Let X be closed and convex and assume, for each $j \in J$, that

$$f_j(x) = \langle c_j, x \rangle, \quad \text{for all } x \in X, \tag{1}$$

where $c_j \in R^n$. Assume that S is closed, convex, and acute. If X_{PRE} is nonempty and rank $C = n$, then (1) satisfies the domination property.

Proof. Suppose that $\bar{x} \in X$. If $\bar{x} \in X_E$, then, since

$$C\bar{x} \geqq_S C\bar{x},$$

there obviously exists an efficient point x^0 such that

$$Cx^0 \geqq_S C\bar{x}.$$

So, assume that $\bar{x} \notin X_E$. Let

$$L(\bar{x}) = \{x \in X \,|\, Cx \geqq_S C\bar{x}\},$$

and consider the function

$$\delta^*[a | L(\bar{x})] = \sup \langle a, x \rangle, \qquad \text{subject to } x \in L(\bar{x}),$$

defined on

$$A[L(\bar{x})] = \{a \in R^n \,|\, \sup\{\langle a, x \rangle \,|\, x \in L(\bar{x})\} < +\infty\}.$$

From the proof of Theorem 2.1 in Ref. 1, $L(\bar{x})$ is a nonempty, closed, convex set, $\delta^*[\cdot | L(\bar{x})]$ is a convex function on $A[L(\bar{x})]$, and

$$C^T(\text{int } S^*) \subseteq A[L(\bar{x})], \tag{2}$$

where int S^* is the interior of the dual cone S^* of S and, for any set $D \subseteq R^p$, $C^T D$ denotes $\{C^T d \,|\, d \in D\}$.

Since rank $C = n$, we have

rank $C^T = n$.

Thus, the dimension of the convex set $C^T(\text{int } S^*)$ is n. From (2), this implies that the dimension of $A[L(\bar{x})]$ is also n. Therefore,

$\quad$ int $A[L(\bar{x})] \neq \varnothing$ $\quad$ and $\quad$ ri $A[L(\bar{x})] = $ int $A[L(\bar{x})]$;

see Ref. 3. Hence,

$\quad$ ri $A[L(\bar{x})] \neq \varnothing$.

From (2), since $C^T(\text{int } S^*)$ and $A[L(\bar{x})]$ are nonempty, convex sets of the same dimension, there exists an $s^* \in (\text{int } S^*)$ such that

$\quad C^T s^* \in$ ri $A[L(\bar{x})]$;

see Ref. 3, Corollary 6.3.3. Choose any such s^*. Then, since $\delta^*[\cdot \, | L(\bar{x})]$ is a convex function on $A[L(\bar{x})]$, the set of subgradients of $\delta^*[\cdot \, | L(\bar{x})]$ at $C^T s^*$ is nonempty (Ref. 3, Theorem 23.4). From Corollary 23.5.3 in Ref. 3, this set of subgradients consists of the points x^0 such that

$\quad \langle C^T s^*, x^0 \rangle = \sup \langle C^T s^*, x \rangle, \qquad$ subject to $x \in L(\bar{x})$.

Choose any such point x^0. Then, $x^0 \in X_E$ [Ref. 4, Theorem 3.2(ii)] and

$\quad Cx^0 \geqq_S C\bar{x}$. $\hfill \square$

The second result in Ref. 1 concerning the domination property is Corollary 3.1. This result, while correct, does not follow from the restatement of Theorem 2.1.

The counterexample given at the beginning of this corrigendum shows that, if the assumption that the rank of C equals n is omitted from the restatement of Theorem 2.1, then the domination property may not hold.

References

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