A MODEL OF AN ELASTIC-PLASTIC MEDIUM WITH DELAYED YIELD

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Stress-strain relationships for metals at high strain rates have long been studied, but no really reliable and generally accepted theory has emerged. It is sometimes assumed that the dynamic stress-strain diagram is largely insensitive to the rate over a certain range. Another approach is to insert derivatives of the stress and strain with respect to time. One difficulty in establishing the actual relationships is that experiment provides only indirect evidence (direct tests are usually impossible). Any real dynamic experiment tends to produce complicated effects, which can be interpreted only if the basic equations are taken as known. The best that experiment can then do is to confirm or reject some prior assumptions.

Many experimental studies deal with mechanical characteristics such as breaking strength and yield point as functions of strain rate; however, strain rate characterizes a range of conditions rather than defines a parameter. We therefore have to use simple models that allow formulation and solution of definite mechanical problems in relation to the dynamics of elastic-plastic media.

1. Delayed yield is a characteristic feature of mild steel under shock loading. If the stress σ exceeds the static yield point σ_{ys}, the strain is elastic for some time $\tau(\sigma)$, the delay. Cottrell's theory explains this as due to a cloud of solute atoms around a dislocation in carbon steel, and a certain time is needed for a given σ before the dislocation can break out [1–3]. Then the condition for onset of yield can be written as

$$\frac{1}{\tau_0}\int_0^t \varphi(\sigma, T)\,d\tau = 1. \qquad (1.1)$$

While (1.1) is not met, Hooke's law applies; yield starts when (1.1) is met. We put σ = const and $t = \tau$ in (1.1) to get the delay τ:

$$\tau = \frac{\tau_0}{\varphi(\sigma, T)}. \qquad (1.2)$$

Tests on τ as a function of σ allow us to find $\varphi(\sigma, T)$.

Another approach is to stretch or compress the specimen at a constant strain rate $\dot{\varepsilon}$; in the elastic range, the stress also varies at a constant rate, $\dot{\sigma} = E\dot{\varepsilon}$, and Hooke's law applies until σ reaches a value σ_u, the upper yield point, which is dependent on $\dot{\sigma}$. From (1.1) we get

$$\sigma_u = \Phi^{-1}(\dot{\sigma}, \tau_0), \qquad \Phi(\xi) = \int_0^\xi \varphi(\xi)\,d\xi. \qquad (1.3)$$

The basis of (1.1) is summation of delay times, and the equation has been tested repeatedly, e.g., by comparing [4] the τ from the above two testing methods and by a varying σ sinusoidally.

Figure 1 shows the $\sigma(\varepsilon)$ relation for carbon steel at a constant high $\dot{\varepsilon}$; the dashed line represents the static behavior. The dynamic diagram can be constructed quite reliably in the elastic range, and existing methods [5, 6] permit the determination of the dynamic σ_y as a function of $\dot{\sigma}$, which can be kept constant. It is not possible to record the falling part below A, and $\dot{\varepsilon}$ in the plastic range is variable in existing test methods, so the initial part of the plastic-strain curve cannot be considered reliable.

Moreover, the plastic deformation of mild steel in the yield range is very uneven, large plastic deformations being localized in small volumes. Thus it is purely arbitrary to assign the plastic resistance to the mean strain rate. All the same, some conclusions can be drawn on the lower yield point σ_l as a function of $\dot{\varepsilon}$, which is much weaker than that for σ_u. Figure 2 shows Belyaev's results for σ_u/σ_{u0} and σ_l/σ_{l0} as functions of $\dot{\varepsilon}$ for steel 3 at room temperature. While σ_l can actually be considered a function of $\dot{\varepsilon}$ (if we make certain assumptions about the structure of the equations), σ_u is dependent on the history of the loading in the elastic region and can be considered as a function of the rate only for constant-rate testing. Any comparison of σ_u and σ_l as functions of rate can be only qualitative. Warnock and Taylor [7] have given analogous results on the weaker rate dependence of σ_l. This gives us a basis for the following model that describes approximately the dynamic behavior of materials such as mild steel. It is assumed that there is a deformation diagram $\sigma = f(\varepsilon)$ independent of the rate (Fig. 3), for which in first approximation we can use the static diagram (without the yield step). This diagram has an elastic range $\sigma < \sigma_0$. Dynamic loading raises the yield point in accordance with (1.1) to σ_u (point A). If the strain at A is kept constant, after A is reached the stress falls abruptly to the value corresponding to this ε on the static curve $\sigma = f(\varepsilon_A)$. If the load is removed at a point on the elastic part indefinitely close to A on the left, there will be no delay on repeating the loading, and the σ-ε curve follows the static diagram.

The term "static diagram" as used here for $\sigma = f(\varepsilon)$ has only a nominal meaning, because the relationship should contain the rate; however, the dependence is relatively weak, so we can use the appropriate mean rate. For instance, if the rate is rather high, as in wave processes, we use the dynamic diagram, as in the theory of the propagation of elastic-plastic waves. If the problem is not one of waves, we can take the next step and use for $\sigma = f(\varepsilon)$ an ideal-plasticity diagram, as in Fig. 3b; then σ_l = const. If evidence is available on the rate dependence of σ_l, the calculation should be performed as follows. First we calculate for σ_0 equal to the static yield point to find the rate distribution, and then a revised σ_0 is found from the mean rates. There is no reliable evidence on the rate dependence of σ_0, so as σ_0 we have to use the static yield point.

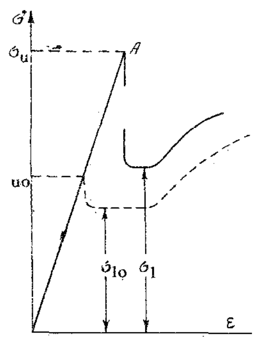

Fig. 1

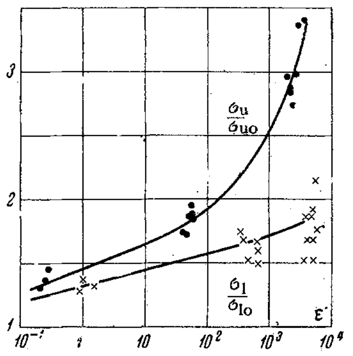

Fig. 2

2. The above scheme needs refinements; however, the necessary experimental evidence for these refinements is not available; nevertheless, they must be specified.

1) When a dislocation has broken away from its solute atoms, the latter begin to migrate to the new position of the dislocation and eventually produce the blocking atmosphere again. This effect must be taken into account for low σ and large delays [2], and it will be particularly important for delayed yield under oscillatory loads.

2) Until recently there has been no evidence on yield delay under conditions of compound stress; the experimental studies relate to compression (the majority) or tension. The published evidence does not allow us to conclude that the delay characteristics in tension and compression are the same, since no systematic evidence has been published on these two types of test for the same material. The dislocation theory of delay indicates that the delay for a single crystal and the plastic deformation are determined by the tangential stress in the corresponding slip system, so we naturally assume that (1.1) still applies for a state of compound stress, with σ replaced by the maximum tangential stress τ_{max} or by the stress intensity σ_i. This would imply identical delay characteristics in tension and compression.

3) It remains unclear how (1.1) is to be applied to sign-varying loads. Cottrell's scheme indicates that partial release of dislocations from their atmospheres in one direction does not facilitate motion in the other direction, so we assume that (1.1) applies only for stresses of one sign, and the delay for an alternating load is reckoned from the instant when the stress reverses, with φ dependent on $|\sigma|$, i.e., $\varphi = \varphi(|\sigma|T)$.

Various forms for $\varphi(\sigma, T)$ in (1.1) have been proposed from theoretical considerations, e.g.,

$$\varphi = (|\sigma| / \sigma_*)^n, \qquad (2.1)$$

$$\varphi = \begin{cases} a(|\sigma| / \sigma_0 - 1)^n & (\sigma > \sigma_0) \\ 0 & (\sigma < \sigma_0) \end{cases}, \qquad (2.2)$$

$$\varphi = K \exp(|\sigma| / \sigma_{**}). \qquad (2.3)$$

Yokobori [8] and Campbell [9] derived (2.1) from Cottrell's theory. The τ_0 in (1.1) allows us to choose

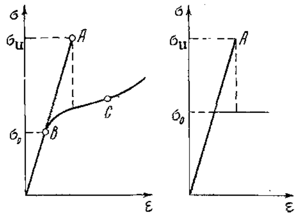

Fig. 3

σ_* arbitrarily, e.g., $\sigma_* = \sigma_0$. It is clear that (2.1) is not suitable for σ near σ_0, since it gives a finite delay at that point; however, this does not prevent practical use of the equation. Also, (2.3) [9] is not applicable for σ small, and only (2.2) is free from these difficulties. The coefficients in (2.1)–(2.3) are functions of T. A theoretical relationship has been derived for n in (2.1), n = const/T, but this is correct only in a certain temperature range [10]. We will consider various formulas for $\varphi(\sigma, T)$ as well as empirical relationships that approximate the experimental results over certain ranges. Here it is not possible to prefer (2.1) to (2.3); the considerable spread in the observed points makes the two equivalent. The position is entirely analogous with that in the theory of creep, where different approximations to the creep law give roughly the same results, and convenience is decisive in choosing between formulas. For this reason we will use (2.1) in what follows.

Numerous careful measurements are needed in order to determine τ_0 and n (or σ_* for a given τ_0); these quantities are very much dependent on the grain size, chemical composition, etc. See [10] on this subject.

3. Consider the propagation of elastic-plastic waves in a medium with delay. We assume that the end of an infinite rod is given a velocity v; then a forward elastic wave propagates along the rod with a stress $\sigma = Ev/c$, in which c is the speed of propagation of longitudinal waves. This stress corresponds to a delay τ defined by (1.2). When the leading edge has moved a distance $c\tau$, the stress at the end falls to σ_0 and, if there is no work-hardening, the critical velocity is zero, and breakaway occurs at the end. If $\sigma = f(\varepsilon)$ corresponds to a work-hardening material, the propagation pattern is as follows (Fig. 4). A forward elastic wave propagates in region A, which carries the stress σ_u over a length $c\tau$, behind which follows an elastic wave of reduced stress, whose distance from the leading edge is constant at $c\tau$. In region B there arises a centered bunch of elastic-plastic waves, and in region C there propagates a wave with a constant velocity,

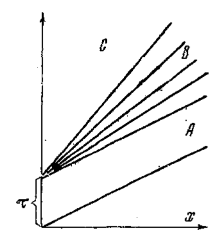

Fig. 4

the speed of the particles being constant at v, while the strain ε' and stress σ' are also constant and are defined in the usual way via the $\sigma = f(\varepsilon)$ diagram as functions of v (point C in the diagram). The stress and strain on the characteristics for the centered bunch correspond to part BC of the $\sigma = f(\varepsilon)$ diagram, and behind the wave of reduced stress (the boundary between regions A and B) the stress is equal to the static yield point σ_0. It is readily seen that any assumption of inelastic strain behind this second front means that the kinematic and dynamic conditions are not met. The propagation of elastic-plastic waves in the presence of delay is the same as in the absence of delay, except for the time shift τ. In a rod of finite length, the head wave is reflected, and this carries a stress σ_u, so it can have a marked effect on the propagation of elastic-plastic waves.

It is not essential to assume that the $\sigma = f(\varepsilon)$ diagram is independent of the rate in discussing wave problems. For instance, we can suppose that the behavior of the material is described by some definite equation after σ_u has been reached, this equation containing $\dot{\sigma}$ and $\dot{\varepsilon}$ [11]. Then the unloading following the delay wave will cause the plastic waves to propagate in accordance with the Sokolovskii-Malvern scheme.

4. Consider the effects of a uniform pressure suddenly produced within a spherical cavity in an unbounded body, whose material has an ideal elastic-plastic nature with delay. If the material is assumed to be incompressible, elastic waves will not propagate in it, and there is only a wave separating the plastic region from the elastic one. We put $\xi = r/a$, in which r is current radius and a is the radius of the spherical cavity. The equation of motion is

$$\frac{\partial \sigma_r}{\partial \xi} - \frac{2(\sigma_\theta - \sigma_r)}{\xi} = \rho \frac{\partial^2 u}{\partial t^2}. \qquad (4.1)$$

The equation of incompressibility $(\partial u/\partial \xi + 2u/\xi) = 0$ implies that

$$u = a\frac{w}{\xi^2}, \qquad \varepsilon_\theta = \frac{w}{\xi^3},$$

$$\varepsilon_r = -\frac{2w}{\xi^3}, \qquad w = w(t). \qquad (4.2)$$

Here the dimensionless time t is referred to the characteristic time $t_* = a/c$, where c is the shear-wave speed.

Hooke's law gives for the elastic region that

$$\sigma_\theta - \sigma_r = 2\mu(\varepsilon_\theta - \varepsilon_r) = 6\mu w / \xi^3.$$

We substitute this expression and the expression for u into (4.1) and integrate with $\sigma_r(\infty) = 0$ to get

$$\sigma_r = -\mu\left(\frac{4w}{\xi^3} + \frac{\ddot{w}}{\xi}\right). \qquad (4.3)$$

The entire medium is in the elastic state for t < < t_1 and the equation of motion is found from (4.3) with $\xi = 1$, $\sigma_r = -q(t)$ as

$$\ddot{w} + 4w = q / \mu. \qquad (4.4)$$

We denote by $w_1(t)$ the integral of (4.4) that satisfies zero initial conditions and get $w = w_1(t)$ $(t \le t_1)$.

The plasticity condition $\sigma_\theta - \sigma_r = \sigma_0$ is obeyed in the plastic region. We insert this into the equation of motion and integrate subject to the boundary condition $\sigma_r(1) = -q(t)$ to get

$$\sigma_r = 2\sigma_0 \ln \xi - \mu\ddot{w}(1 - \xi) / \xi - q. \qquad (4.5)$$

Let x be the radius of the interface between the elastic and plastic regions. Since the material is incompressible, the velocity is continuous at the boundary, so the radial stress σ_r is also continuous, and (4.3) and (4.5) give

$$\ddot{w} + 4\frac{w}{x^3} + \frac{2\sigma_0}{\mu}\ln x = \frac{q}{\mu}. \qquad (4.6)$$

The unknown functions w(t) and x(t) appear in (4.6). A second relation between them is derived from (1.1) which becomes an equality at the interface. If $\xi = x$, $\sigma_0 - \sigma_r = 6\mu w/x$ in the elastic region, so (1.1) becomes

$$\int_0^t \left(\frac{6\mu w}{\sigma_0 x^3}\right)^n dt = \frac{\tau_0 c}{a} = \tau_*. \qquad (4.7)$$

We now introduce the symbols

$$y = \frac{6\mu w}{\sigma_0}, \qquad x^3 = z, \qquad p = \frac{3q}{\sigma_0}.$$

The system of equations is finally written as

$$\ddot{y} + \frac{4y}{z} + 4\ln z = 2p, \quad z^n = 1 + \frac{1}{\tau_*}\int_{t_1}^{t} y^n\, dt. \qquad (4.8)$$

We must put z = 1 in the first equation in the elastic state and for $t < t_1$; then the solution subject to the initial condition $y(0) = \dot{y}(0) = 0$ defines the function y_1. The condition for onset of yield is

$$1 = \frac{1}{\tau_*}\int_0^{t_1} y_1^n\, dt. \qquad (4.9)$$

Functions y(t) and z(t) for $t > t_1$ are defined by solving (4.8); the second equation in this system is derived from (4.7) with use of (4.9). The initial conditions are as follows: for $t = t_1$, $z = 1$, $y = y_1(t_1)$, $\dot{y} = \dot{y}_1(t_1)$. Of course, only numerical methods can actually yield a solution.

5. Consider now the pure bending of a prismatic rod (width b, thickness 2h) of ideally elastic-plastic

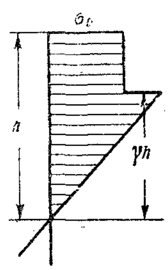

Fig. 5

material showing delay. The stress distribution over the cross-section is linear in the elastic state; if the stress at the edge exceeds σ_0, this stress after a time t_1 falls to σ_0, and the boundary for stress reduction moves towards the neutral axis. Figure 5 shows the stress distribution. We put $\varepsilon = \varkappa y/h$ to find the following relation between the bending moment, the curvature, and the parameter γ that defines the position of the elastic-plastic boundary:

$$M = bh^2\sigma_0\left[\frac{2}{3}\frac{E}{\sigma_0}\gamma^3\varkappa + 1 - \gamma^2\right]. \qquad (5.1)$$

Also, $\sigma = E\varkappa\gamma$ at the boundary of the elastic region, and substitution into (1.1) with the power law for delay gives

$$\int_0^t \left(\frac{E\varkappa\gamma}{\sigma_0}\right)^n dt = \tau_0.$$

This condition refers to a specific point in the cross-section, so γ must be taken as constant there. We rewrite this as

$$\gamma^n \int_0^t \varkappa^n\, dt = \tau_1, \qquad \tau_1 = \left(\frac{\sigma_0}{E}\right)^n \tau_0 \qquad (5.2)$$

We introduce the symbols

$$bh^2\sigma_0 = M_T, \qquad M / M_T = m, \qquad 2E / 3\sigma_0 = p.$$

Then (5.1) is put as

$$m = p\gamma^3\varkappa + 1 - \gamma^2. \qquad (5.3)$$

a) Constant-rate strain. We put $\varkappa = \alpha t$ and get from (5.2) that

$$\gamma^n\alpha^n \frac{t^{n+1}}{n+1} = \tau_1 \qquad (5.4)$$

We put $\gamma = 1$ in (5.4) and get the time t_1 corresponding to onset of yield at the edge; the curvature is

$$\varkappa_1 = [(n+1)\alpha\tau_1]^{1/(n+1)}.$$

We eliminate the time from (5.4) to get

$$\gamma = \left(\frac{\varkappa_1}{\varkappa}\right)^{(n+1)/n} \qquad (5.5)$$

We substitute this γ into (5.3) to relate the bending moment to the curvature:

$$m = m_1\left(\frac{\varkappa_1}{\varkappa}\right)^{2+3/n} + 1 - \left(\frac{\varkappa_1}{\varkappa}\right)^{2+2/n}. \qquad (5.6)$$

Here $m_1 = p\varkappa_1$ is the maximum bending moment at $t = t_1$. Since n is large, we can replace this formula by the approximation

$$m = (m_1 - 1)\left(\frac{\varkappa_1}{\varkappa}\right)^2 + 1.$$

Here n appears only in the expression for m_1, which is proportional to $\alpha^{1/(n+1)}$

b) Bending by a constant moment. We put m = const to get from (5.3) that

$$\varkappa = \frac{m - 1 + \gamma^2}{p\gamma^3}. \qquad (5.7)$$

Here γ decreases from 1 to γ_1, and the interface can move only towards the neutral axis (otherwise stress reduction would occur), and hence the motion stops when $d\varkappa/d\gamma = 0$. We get from (5.7) that

$$\frac{d\varkappa}{d\gamma} = \frac{1}{p\gamma^4}(3 - 3m - \gamma^2).$$

This derivative is negative for $\gamma = 1$ if $m > 2/3$, and motion begins only if this condition is met; in fact, the largest stress is σ_0 if $m = 2/3$. We specify that $d\varkappa/d\gamma =$ $= 0$ when $\gamma = \gamma_1$ to get

$$\gamma_1 = \sqrt{3(1 - m)}. \qquad (5.8)$$

We differentiate (5.2) and substitute for $\varkappa$ from (5.7) to get the following differential equation for $\gamma(t)$:

$$\left(\frac{m - 1 + \gamma^2}{p\gamma^3}\right)^n = -n\tau_1\gamma'\gamma^{-(n+1)}.$$

Then we have the time of motion for the plasticity boundary as

$$t = n\left(\frac{3}{2}\right)^n \tau_0 \int_{\gamma_1}^{\gamma} \frac{\gamma^{2n-1}\, d\gamma}{(m - 1 + \gamma^2)^n}. \qquad (5.9)$$

Relation (5.8) sets the lower limit to γ_1 if $2/3 < m \le$ ≤ 1: if $m > 1$, the applied moment is greater than the static moment, and we must put $\gamma_1 = 0$ in (5.9). Then the time for which the rod retains its carrying capacity is derived as follows. The time t_1 to the onset of yield is found from (5.2), in which we put $\varkappa =$ const $=$ $= m/p$; we get $t_1 = (3/2)^n\tau_0 m^{-n}$. The loss of carrying capacity occurs at $t_1 + t_2$, with t_2 defined by (5.9). It is readily seen that

$$\frac{1}{2} < \frac{t_2}{t_1} < \frac{1}{2}\left(\frac{m}{m-1}\right)^n, \qquad (5.10)$$

and that t_2/t_1 approaches 0.5 as m increases.

6. This last example shows that the time for retention of carrying capacity is not dependent on the elastic modulus. We pass to the limit $E \to \infty$ to get the scheme for a rigid plastic body with delayed yield, whose strain to zero up to the instant of loss of carrying capacity. However, if a rigid body is considered as an elastic one with E very large, we naturally assume that the stress distribution in the rigid state will be as for an elastic body, since this is independent of the modulus for a given load. The equations of motion retain only the inertial terms that correspond to motion of the body as a whole. There is no interest in the stress distribution in the parts of the body assumed to be rigid in the usual rigid-plastic formulation, since the disposition of the plastic zones is determined by the scope for subsequent motion. The history of the stress from the start of loading can play a major part when there is delay.

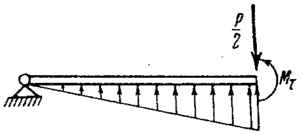

Fig. 6

These arguments may be illustrated via a rigid plastic beam with yield delay. The condition for onset of yield in bending is put as

$$\int_0^t m^n\,dt = \tau_2. \qquad (6.1)$$

From the above, this condition is the one for yield at the edge, and the carrying capacity will be exhausted only when the section is an ideal double T. All the same, we retain (6.1) as an approximate condition for any cross-section, with $m = M/M_u$ as before, in which M_u is the limiting moment, and $\tau_2 = \tau_0(\sigma_0 W/M_u)^n$, where W is the resistance moment of the cross-section.

Consider a beam of length $2l$ on two supports and loaded at the middle by a force P that varies linearly with time. We put $Pl/2M_u = p = p't$ and write (6.1) as

$$\int_0^p m^n\,dp = p't_2. \qquad (6.2)$$

The largest bending moment m = p occurs at the middle while the beam remains rigid. A plastic hinge is formed at this point for $p = p_1$, and (6.2) gives

$$p_1^{n+1} = (n+1)p't_2. \qquad (6.3)$$

Each half of the beam will rotate around its support after the plastic hinge has formed. Figure 6 shows the forces acting, including the linearly distributed inertial force. We put $\xi = x/l$ to get the bending moment as

$$m = -1/2[(p-3)\,\xi - (p-1)\xi^3]. \qquad (6.4)$$

By $p_0(\xi)$ we denote the loading parameter for which the bending moment in the section with coordinate ξ becomes zero. If $p > p_0$, the moment in this cross-section is negative, and in examining the scope for yield delay we need take account of only that part of the history beginning with the instant when $p = p_0$. Condition (6.2) gives as follows after substitution of (6.4) and use of (6.3):

$$\int_{p_0}^{p}\left[p\frac{\xi-\xi^3}{2} - \frac{3\xi-\xi^3}{2}\right]^n dp = \frac{p_1^{n+1}}{n+1}.$$

Then

$$p = \frac{3-\xi^2}{1-\xi^2} + p_1\left[\frac{\xi(1-\xi^2)}{2}\right]^{-n/(n+1)}. \qquad (6.5)$$

We can put $n/(n+1) \approx 1$ for n large, so (6.5) is replaced by the simpler equation

$$p = \frac{3-\xi^2}{1-\xi^2} + \frac{2p_1}{\xi(1-\xi^2)}. \qquad (6.6)$$

Now we can find the point at which yield first occur. We put $dp/d\xi = 0$ to get

$$2\xi^3 - p_1(1-3\xi^2) = 0. \qquad (6.7)$$

If there is no delay, $p_1 = 1$, $\xi = 1/2$, and (6.6) gives $p = 9$. If $p_1 > 1$, the real root of (6.7) lies in the range

$$1/2 < \xi < 1/\sqrt{3}.$$

This example shows that allowance for delay can alter the scheme for the disposition of plastic hinges when a rigid-plastic model is used.

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