CHARGE CONJUGATION OF NON-ABELIAN GAUGE FIELDS

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Questions associated with the determination of the charge conjugation in non-Abelian gauge fields are investigated. It is shown that the properties of gauge groups allow the charge conjugation to be simply determined.

Together with other discrete symmetry transformations, a large role in physics is played by the operation of charge conjugation, performing the transition from particle to antiparticle and back. The charge invariance of Lagrangians is discussed in all the classical texts; a detailed exposition is given in [1]. In electrodynamics, the corollary of charge invariance is known as the Farri theorem [2]; the potentials of the electromagnetic field are transformed in charge conjugation in the simple manner

$$A^{\mu c} = -A^{\mu},$$

where $A^{\mu c}$ is the charge-conjugate potential. It is found that generalization of C transformation to the case of theories with non-Abelian gauge fields does not reduce to the simple extension of the law of transforming an Abelian vector field into a non-Abelian field. It is assumed that the scalar φ_i and spinor ψ_i fields transform under charge conjugation in the usual manner

$$\varphi^i \to \varphi^{ic} = \varphi^{i+},\ \psi^i \to \psi^{ic} = C\bar{\psi}^i.$$

Here C is the charge-conjugation matrix. Then the condition of C invariance of the interaction of scalar fields with gauge fields $A^{a\mu}$ and the interaction of spinor fields with $A^{a\mu}$ of vector type is

$$\Gamma_a A^{a\mu} = -\Gamma_a^T A^{a\mu c}. \tag{1}$$

where Γ_a are generators of gauge transformations; Γ_a^T are transposed matrices. The same conditions arise for the CP invariance of axial-vector interactions of spinor and gauge fields.

The following lemma offers the possibility of introducing C conjugation in the non-Abelian case.

LEMMA. The basis of a finite-dimensional, irreducible, complex representation of a semisimple compact Li group may always be chosen so that each of the generators Γ_a will be a symmetric or antisymmetric matrix; the symmetry properties do not depend on the choice of representation.

Using this lemma, it is simple to define the operation of charge conjugation of gauge fields so that the condition in Eq. (1) is satisfied.

The lemma will now be proven. Consider the complex broadening L of a real compact semisimple algebra L_k with basis Γ_a (Γ_a are assumed to be anti-Hermitian matrices); L_k is defined by the involution $\Theta = -\ni$, where $\ni$ is the operation of Hermite conjugation. Then [3], in L there is a Kartan–Beilya basis $\{H_i, E_\alpha\}$ with the properties

$$\Theta(H_i) = -H_i,\ \Theta(E_\alpha) = E_{-\alpha}.$$

Here H_i may be regarded as diagonal matrices, while the algebra L_k is a real linear envelope of the vectors H_i, $U_\alpha = i(E_\alpha + E_{-\alpha})$, $V_\alpha = E_\alpha - E_{-\alpha}$. It may be assumed that the generators Γ_a coincide with H_i, U_α, V_α.

Now consider the representation in the same space of the Li algebra L' produced by the matrices $\{H_i, E^T_{-\alpha}\}$. Transposing from the Kartan–Beilya commutation relations

$$[E_\alpha, E_\beta] = \begin{cases} H_\alpha, & \alpha + \beta = 0 \\ N_{\alpha\beta}E_{\alpha+\beta}, & \alpha+\beta \neq 0,\ \alpha+\beta \in \Delta, \\ 0, & \alpha+\beta \neq 0,\ \alpha+\beta \subseteq \Delta \end{cases}$$

$$[H_i, E_\beta] = \beta(H_i)E_\beta,$$

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it is readily seen that this basis has the same commutation relations as the basis of the matrix $\{H_i, E_\alpha\}$. In addition, the Kartan subalgebras of the algebras L and L' coincide, and hence all the irreducible representations coincide. The latter are thus equivalent, i.e., the following relation holds

$$SE_\alpha S^{-1} = E^{T}_{-\alpha},\ SH_iS^{-1} = H_i \tag{2}$$

for some nonsingular unitary matrix S. It follows from Eq. (2) that $S^T = \pm S$ and that S commutates with H_i. Since H_i may be chosen to be diagonal, $(H_i)_{lj} = \lambda_l^{(i)}\delta_{lj}$, where $\lambda_l^{(i)}$ is the weight of the representation, it follows that

$$S_{lj}(\lambda_l^{(i)} - \lambda_j^{(i)}) = 0.$$

Let $\lambda_1^{(i)}$ be the old weight (which is unique). Then $S_{1i} = S_{i1} = a\delta_{i1}$, i.e., S is a symmetric matrix. Other of its properties are also evident from Eq. (2): $S^2 = 1$, $S^+ = S = S^{-1}$.

The matrix P is now introduced [4]

$$P \equiv \sqrt{S} = \frac{1-i}{2}E + \frac{1+i}{2}S,\ P^2 = S.$$

It is obvious that P^{-1} exists, and that $P^T = P$, $P^+ = P^{-1}$. The next step is to pass from the initial representation with generators $\Gamma_a = \{H_i, U_\alpha, V_\alpha\}$ to an equivalent representation with generators $\Gamma'_a = \{H'_i, U'_\alpha, V'_\alpha\}$

$$H'_i = PH_iP^{-1} = H_i,\ U'_\alpha = PU_\alpha P^{-1},\ V'_\alpha = PV_\alpha P^{-1},\ E'_\alpha = PE_\alpha P^{-1}.$$

It is not difficult to establish that

$$E'^{T}_\alpha = E'_{-\alpha},\ U'^{T}_\alpha = U'_\alpha,\ V'^{T}_\alpha = -V'_\alpha. \tag{3}$$

The generators Γ'_a also form the basis of the representation referred to in the lemma. It will be assumed, in addition, that the generators Γ_a satisfy Eq. (3). The matrix C_a^b is now introduced

$$\Gamma^T_a = C^b_a\Gamma_b,$$

C_a^b is a diagonal matrix with diagonal elements of ±1. The matrix C_a^b evidently has the properties $C^T = C$, $C^2 = 1$. The operation of charge conjugation of gauge fields is defined as follows

$$A^{a\mu c} = -C^a_b A^{b\mu}. \tag{4}$$

Obviously, Eq. (4) satisfies Eq. (1). Using the explicit form of the structural constants in the Kartan–Beilya basis or the relation $f^a_{bc} \sim \mathrm{Sp}([\Gamma_b, \Gamma_c]\Gamma_a)$, it is found that

$$f^a_{b'c'}C^{b'}_b C^{c'}_c = -C^a_{a'}f^{a'}_{bc}.$$

This allows the C invariance of the kinetic term of the gauge fields to be established. The operation of charge conjugation takes on a more usual form in terms of the fields W^μ defined by the identity

$$\Gamma_a A^{a\mu} \equiv H_i W^{i\mu} + E_\alpha W^{\alpha\mu},\ W^{-\alpha\mu} = (W^{\alpha\mu})^+.$$

For W^μ, the operation of charge conjugation takes the form

$$W^{\mu c} = (W^\mu)^+.$$

Finally, for the Faddeev–Popov visitors C^a and $\bar{C}^a$, the operation of charge conjugation is defined as follows

$$C^{ac} = C^a_b C^b,\ \bar{C}^{ac} = C^a_b \bar{C}^b.$$

Then, if the term $-t^a t^a/2$ in the Lagrangian, which fixes the gauge, is chosen so that t^a behaves like $A^{a\mu}$ under charge conjugation (e.g., $t^a = \partial_\mu A^{a\mu}$), the Lagrangian of the visitors will have C invariance.

After this article had gone to press, the authors became aware of a preprint by Smolyakov [5], in which analogous results were obtained.

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A NON-SELF-CONSISTENT ELECTRON-BEAM CONTROLLED DISCHARGE IN METHANE

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The use of methane in a non-self-consistent discharge controlled by an electron beam permits obtaining high discharge currents for relatively low electric fields. The current gain is 10^3 for 500 V/cm fields and a 14 mA/cm² injection current density. For fields greater than 7-8 kV/cm and atmospheric pressure, punch-through of the gas discharge gap occurs. It is shown that a breakpoint in the CVC in the area of low currents is associated with the appearance of spots on the cathode. A domain instability, related to the nonmonotonic dependence of the drift velocity on the reduced field in methane, is detected.

The possibility of realizing a gas-discharge commutating high-pressure instrument with total control whose conductivity is controlled by a fast electron beam is shown in [1, 2]. High discharge current densities j for small voltage drops can be obtained in such an instrument by using a gas with high electron drift velocities for low electric fields. One of the gases satisfying the requirement mentioned is methane [3], in which the drift velocity $v \approx 10^7$ cm/sec is achieved for a very low ratio of the electric field intensity to the charged particle concentration $E/N = 3 \cdot 10^{-17}$ V·cm² (Fig. 1) [4].

Results of investigation of the current–voltage characteristics (CVC) of a discharge in methane are presented in this paper. The electric field distribution is measured in the gap in different discharge combustion modes, and the range of conditions in which complete current control is accomplished is determined.

APPARATUS AND METHOD

The experiments were performed at atmospheric pressure in natural gas with the following composition: methane 89-91; ethane 1.2-2; propane 0.4-0.7; unsaturated hydrocarbons 6, carbon dioxide 0.1, and oxygen (0.5-0.6)%.

The diagram of the experimental apparatus is represented in Fig. 2. The electron beam is formed by a tetrode gun with directly heated cathode [5] and is injected into the space of the discharge chamber through a window of 1.6 cm diameter sealed by a 20-μm-thick titanium foil. The energy of the accelerated electrons is 135 eV, the current density behind the foil is 14 mA/cm². The current pulse was rectangular in shape, of duration 8 ±0.5 μsec with 10 nsec rise and fall times. In order to measure the current distribution over the section, the electrode 1 was sectioned off in the form of concentric rings No. 1-6, respectively, with the diameter 0-25, 25-50, 50-75, 75-100, 100-125, 125-175 mm. The diameter of the electrode 2 was 12 cm and the interelectrode spacing was d = 5 cm.

A negative, or positive, potential was delivered to the electrode 2 from the capacitor c_p, whose capacitance was selected sufficiently high so that the voltage thereon would not drop during the current pulse. The potential distribution along the gap was measured by a probe. The probe 3 was a 100-μm-diameter tungsten wire with a 1.25-mm spacing stretched on a ring and mounted at spacings of 1, 4, 10 mm from electrode 2, which was grounded during the probe measurements, while a positive potential was delivered to electrode 1 (shown by dashes in Fig. 2). The signal from the probe was taken off through a high-resistance divider 18.5 kΩ + 75 Ω, the current sample was here a small fraction of the total discharge current.

A photograph of the discharge was made by using an image brightness amplifier (IBA) on a photographic apparatus with an open shutter. In order to improve the outline, the shape of electrode 2 was selected close to a Rogovskii profile during the survey.

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