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Book Review

Random Field Models in the Earth Sciences
By George Christakos
Academic Press, San Diego, 1992, 474 p., $99.00 (U.S.)

Often statisticians complain that much of geostatistics is a re-naming or re-discovery of well-known results. In turn, some working in the application of statistics and probability to problems in the earth sciences complain that classical methods concentrate too much on techniques that require unreasonable assumptions such as independence or random sampling. While there are few applications of stochastic processes (other than that represented by geostatistics) to mining, there is a long history of the use of stochastic models in hydrology. In that respect, this book is perhaps closer to the spirit of applications in hydrology than those in other earth sciences. As is indicated by the title, the emphasis is on the use of random field models, i.e., random functions or stochastic processes defined on n-dimensional Euclidean space. While most of the results and techniques of geostatistics are found somewhere in this volume, it does not pretend to be such a presentation. Alternatively, it is not particularly intended for the reader familiar with stochastic processes who wishes to learn about geostatistics. There are examples from the earth sciences but in most cases the practical aspects of how to use the results and techniques are missing. This is not a textbook in that there are no exercises. Some chapters can be omitted on a first reading or can be read somewhat independently. There is a sizeable list of references, many of which may be new to the practitioner from the earth sciences.

While the first chapter, *Prolegomena*, discusses philosophical ideas that are too often skipped over, namely the relationship between models that allow for prediction or description and the phenomenon of interest, it is not needed for reading the subsequent chapters. The reader can return to it at their leisure.

Chapter 2, *The Spatial Random Field Model*, largely contains material that would be contained in a treatise on stochastic processes. There is a strong emphasis on processes (= random fields) defined in Euclidean space rather than in 1-space and hence few of the tools of time series are introduced. As with any advanced exposition, and particularly one which addresses applications,

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there is the question of what to assume the reader will know. While the reader will not need to know most of the details of modern measure theory, that is the perspective of the presentation. Kolmogorov's axioms for probability are stated but the reader is assumed to known about measurable sets, measurable functions/mappings, Borel sets, measures without atoms, basic properties of the Lebesgue integral including the Dominated Convergence Theorem, linear normed spaces, completeness (of linear spaces), Hilbert spaces, unitary operators, and the various Schwartz spaces of generalized functions. While in many cases the reader can simply skip over the references to these topics, it will likely be disconcerting. An SRF (spatial random field) is defined three times (definitions 5, 6, and 7) and they do not agree. Although it is not completely adhered to, an important comment buried at the bottom of p. 31 states that unless otherwise noted all SRF's will be assumed to have continuous realizations and to have finite means, variances. This restriction is unnecessarily invoked in Section 4.1 when defining stochastic convergence for a sequence of random variables: of the four definitions given, only one requires second-order properties. There are numerous errors or misstatements in this as well as other chapters. Remark 1 (p. 55) is incorrect in that it confuses second-order stationarity with (strict) stationarity. The description of ergodicity on p. 57 is incorrect in that it ignores the role of the limit. Example 7 on p. 65 is wrong unless the result is considered in the context of generalized functions, i.e., any function that is zero except at one point has a zero integral. The last paragraph on p. 68 is wrong since it fails to distinguish between the matrix-valued covariance and a matrix whose entries are generated by the values of the matrix-valued covariance. This is discussed in Myers (1982, 1984, 1988, 1991a, and 1991b). There is a significant typo in the statement of the "third criterion of permissibility" on p. 73. On p. 96, there seems to be an assumption that the existence of a first derivative ensures the existence of a Taylor Series expansion.

Chapter 3, "Intrinsic Spatial Random Field Models," is largely a review of the development of intrinsic random functions as given in Matheron (1973) and Delfiner (1976). There is a small section at the end on stochastic differential and difference equations. Equation (77) on p. 134 is wrong since it implies that cross-covariances are always nonnegative. The last paragraph on this page repeats the error on p. 68.

Chapter 4, "The Factorable Random Field Model," synthesizes and generalizes the ideas that underlay the development of disjunctive kriging.

Chapter 5, "The Spatiotemporal Random Field Model," introduces random functions defined on a product space, n-dimensional Euclidean space and time. Many of the results in Chapter 2 are extended to spatiotemporal random functions but with no surprises. There is no discussion of the problems identified in Rouhani and Myers (1990).

"Space Transformations of Random Fields," Chapter 6, begins with an idea implicit in the Turning Bands simulation method and generalizes it in several ways. To generate a simulation in n-space, one generates many 1-dimensional simulations. The realization, i.e., the random function, in higher dimensional space is obtained as a linear combination of the uncorrelated 1-dimensional random functions. This link is dual to the relationship between the covariances. These ideas are then applied to problems arising in the solution of stochastic partial differential equations.

The first real discussion of any applications or problems occurs in Chapter 7, "Random Field Modeling of Natural Processes." In addition to presenting the application of geostatistics to specific examples, there is a brief overview of indicator kriging.

Chapter 8, "Simulation of Natural Processes," provides a brief overview of the Turning Bands method, including the use of the spectral density in lieu of the usual covariance, the LU decomposition method (for scalar random functions), and a discussion of 1-dimensional simulation methods.

"Estimation in Space and Time," Chapter 9, essentially presents the standard results on linear estimators, albeit in somewhat different form than is common in the geostatistical literature. The extension to space–time is largely limited to treating time as another Euclidean dimension. There is an interesting section on the relationship between Bayesian/Maximum Likelihood models and those derived from second-order properties.

The last chapter, "Sampling Design," presents the problem of designing a sampling design which is optimal in some sense. There is a discussion of different loss functions (in addition to the common choice of the minimized estimation variance) and different optimization techniques such as simulated annealing.

The reader will want to consider three comparable or related books. *Random Functions and Hydrology* by R. L. Bras and I. Rodriguez-Iturbe (1985) and *Random Functions and Hydrology* (1984) by G. de Marsilly are both somewhat less mathematical with a greater emphasis on applications in hydrology. *Correlation Theory of Stationary and Related Random Functions Basic Results* (1986) by A. M. Yaglom is a standard reference on stochastic processes in n-dimensional Euclidean space.

There are a number of minor annoyances for the reader: (1) a tendency to make up long names (with associated acronyms) that are either unneeded or replace more standard terminology ("pure polynomials" instead of monomials, "geostatistical semivariogram," "spatial polynomials"), (2) notational discrepancies or obfuscations (e.g., $\mathbf{Y}_h(\mathbf{s})$ denotes the first order increment of $\mathbf{X}(\mathbf{s})$ with respect to the vector h whereas $\mathbf{Y}_i(\mathbf{s})$ denotes the partial derivative of $\mathbf{X}(\mathbf{s})$ with respect to the ith component of $\mathbf{s}$, there are numerous instances of where a function is written with one argument and then on the next line with a different number of arguments, there are frequent instances of where vectors are treated as scalars or *vice versa*, e.g., squaring a vector), (3) the numbering system for definitions, examples, and remarks (in each chapter the numbering starts over and is not coordinated with the section numbering system), and (4) the repeated use of stilted, improper phrases such as "amounts to the fact," "it must hold," "so-called," "properties are valid," "on the strength of," "properties below are valid."

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