SLIT RADIATOR AT THE EDGE OF AN IMPEDANCE WEDGE IN A UNIAXIAL PLASMA

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A rigorous analytical solution is derived for the problem of the excitation of electromagnetic waves by a magnetic flux source located at the top of a wedge with impedance boundaries. The wedge is placed in a uniaxial anisotropic plasma that has been magnetized transverse to the edge of the wedge direction.

Investigation of the processes of propagation and generation of waves in a confined plasma is of interest for its heating, diagnostics, and other aims. The fields excited in a magnetically active plasma have a complex wave structure. Studying diffraction phenomena is significantly simplified in a plasma that is located in a very strong external magnetic field (in a uniaxial plasm). Its properties are characterized by the diagonal tensor of the relative dielectric constant, one of the components of which equals the dielectric constant of an isotropic plasma while the other two are equal to unity. Several problems are examined in the literature on the diffraction of waves in a uniaxial medium for a half-plane [1-3] and rectangular wedge [4]. Between them, the application of the method of changing the scale [5] makes it possible to propagate two-dimensional diffraction in a uniaxial medium at a wedge of arbitrary aperture with impedance boundaries (faces).

Let the boundaries of the wedge and the bisectrix of the wedge angle subtend with the anisotropy axis the angles φ_1, φ_2, and β, respectively (Fig. 1). The faces are characterized by a dimensionless surface impedance $Z_{1,2}$ referenced to the impedance of a vacuum. The electromagnetic field is created by a narrow slit at the edge of the wedge which has equal-strength magnetic field lines. $M = J^m\delta(x)\delta(y)\exp(-i\omega t)z_0$ located there. The exponential time factor has been dropped in what follows. The medium is characterized by the relative dielectric constant tensor

$$\varepsilon = \begin{pmatrix} \varepsilon & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1}$$

where $\varepsilon = 1 - 1/\Omega^2$, Ω is the ratio of the field frequency ω to the plasma frequency of the electrons. Maxwell's equations lead to a differential equation in terms of the unit magnetic component of the field H_z

$$\left(\frac{\partial^2}{\partial x^2} + \frac{1}{\varepsilon}\frac{\partial^2}{\partial y^2} + \kappa_0^2\right) H_z = -i\omega\varepsilon_0 J^m \delta(x)\,\delta(y), \; \kappa_0 = \omega\sqrt{\varepsilon_0\mu_0}, \tag{2}$$

where ε_0, μ_0 are the dielectric constant and magnetic permeability of a vacuum. The impedance boundary condition $E_r = ZH_z$ is given at the faces of the wedge, where E_r is the component of the electric field that is tangent to the face. By introducing the coordinate $\hat{y} = \sqrt{\varepsilon}y$, the differential operator in Eq. (2) becomes a Laplace operator in the coordinates $(x, \hat{y})$, and Eq. (2) becomes a wave equation for a vacuum. The transition to polar coordinates r, φ and the introduction of the modified coordinates and the impedance

$$\hat{r} = rN(\varphi), \; \hat{\varphi} = \operatorname{arctg}(\sqrt{\varepsilon}\,\mathrm{tg}\,\varphi), \; \hat{Z} = \sqrt{\varepsilon}\,Z/N(\varphi) \tag{3}$$

transforms the original boundary problem for a uniaxial medium into an equivalent problem for an isotropic medium. In Eqs. (3), $N(\varphi) = \sqrt{\cos^2\varphi + \varepsilon\sin^2\varphi}$ is the so-called ray index of

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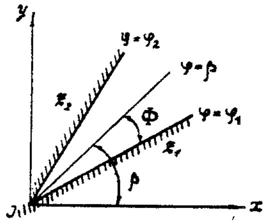

Fig. 1

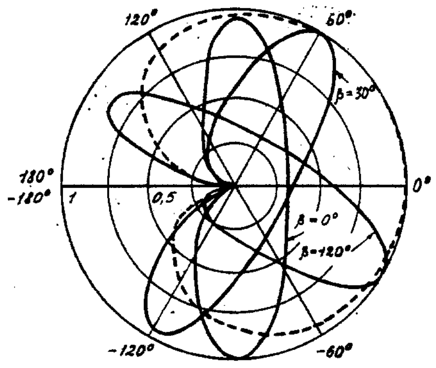

Fig. 2

refraction and the value of the arctangent is chosen such that $\hat{\varphi} \to \varphi$ for $\varepsilon \to 1$. The exact solution of this equivalent problem is known [6]. It has the form of an integral along the contour C located near the $\operatorname{Im}\alpha$ axis and passing through the point $\alpha = 0$:

$$H_z(\hat{r},\ \hat{\varphi}) = A(\hat{\Phi})\int_C f(\alpha + \hat{\varphi} - \hat{\beta})\exp(i\kappa_0\hat{r}\cos\alpha)\,d\alpha, \tag{4}$$

$$A(\hat{\Phi}) = J^m\omega\varepsilon_0\sqrt{\varepsilon}/(4\hat{\Phi}),$$

$$f(\alpha) = \frac{\Psi_{\hat{\Phi}}\left(\alpha - \hat{\Phi} + \frac{\pi}{2} + \hat{\theta}_1\right)\Psi_{\hat{\Phi}}\left(\alpha + \hat{\Phi} - \frac{\pi}{2} - \hat{\theta}_2\right)}{\Psi_{\hat{\Phi}}\left(\alpha - \hat{\Phi} + \frac{\pi}{2} - \hat{\theta}_1\right)\Psi_{\hat{\Phi}}\left(\alpha + \hat{\Phi} - \frac{\pi}{2} + \hat{\theta}_2\right)} \times$$

$$\times \frac{\cos\frac{\pi}{4\hat{\Phi}}(\alpha + \hat{\Phi})\cos\frac{\pi}{4\hat{\Phi}}(\alpha - \hat{\Phi})}{\cos\frac{\pi}{4\hat{\Phi}}(\alpha - \hat{\Phi} + \hat{\theta}_1)\cdot\cos\frac{\pi}{4\hat{\Phi}}(\alpha + \hat{\Phi} - \hat{\theta}_2)}. \tag{5}$$

In Eq. (5), $\Psi_{\hat{\Phi}}(\alpha)$ denotes the special Malyuzinets functions [7, 8], in the composition of the arguments of which enter the half-planes of the wedge $\hat{\Phi} = 0{,}5(\hat{\varphi}_2 - \hat{\varphi}_1)$ and the complex Brewster's angles $\hat{\theta}_{1,2} = \arcsin\hat{Z}_{1,2}$. At great distances from the edge, the integral in Eq. (4) can be approximately calculated by the method of steepest descents, for which the contour C is deformed into the line $\operatorname{Re}\alpha - \operatorname{grad}(\operatorname{Im}\alpha) = 0$. The contribution from the saddle point $\alpha = 0$ yields a cylindrical wave $H_\Sigma(r,\ \varphi)$. If in the process of deforming the contour there takes place an intersection of the poles in the function under the integral in Eq. (5), then their contributions yield surface waves $H_{1,2}^{(s)}(r,\ \varphi)$, which propagate along the faces of the wedge from the edge to the periphery. Thus, in the far field, the slit-radiator field in a uniaxial medium will consist of three terms:

$$H_z(r,\ \varphi) = H_\Sigma + H_1^{(s)}\eta[\operatorname{Im}(\hat{\varphi}_1 - \hat{\varphi} - \hat{\theta}_1)] + H_2^{(s)}\eta[-\operatorname{Im}(\hat{\varphi}_2 - \hat{\varphi} + \hat{\theta}_2)], \tag{6}$$

where

$$H_\Sigma(r,\ \varphi) = A(\hat{\Phi})\cdot\frac{\Psi_{\hat{\Phi}}\left(\hat{\varphi} - \hat{\varphi}_1 - \frac{\pi}{2} - \hat{\theta}_2\right)\Psi_{\hat{\Phi}}\left(\hat{\varphi} - \hat{\varphi}_2 + \frac{\pi}{2} + \hat{\theta}_1\right)}{\Psi_{\hat{\Phi}}\left(\hat{\varphi} - \hat{\varphi}_1 - \frac{\pi}{2} + \hat{\theta}_2\right)\Psi_{\hat{\Phi}}\left(\hat{\varphi} - \hat{\varphi}_2 + \frac{\pi}{2} - \hat{\theta}_1\right)} \times$$

$$\times \frac{\cos\frac{\pi}{4\hat{\Phi}}(\hat{\varphi} - \hat{\varphi}_1)\cos\frac{\pi}{4\hat{\Phi}}(\hat{\varphi} - \hat{\varphi}_2)}{\cos\frac{\pi}{4\hat{\Phi}}(\hat{\varphi} - \hat{\varphi}_1 - \hat{\theta}_2)\cdot\cos\frac{\pi}{4\hat{\Phi}}(\hat{\varphi} - \hat{\varphi}_2 - \hat{\theta}_1)}\left(\frac{2\pi}{\kappa_0\hat{r}}\right)^{1/2} \times$$

$$\times \exp\left[i\left(\kappa_0\hat{r} - \frac{\pi}{4}\right)\right], \tag{7}$$

$$H_{1,2}^{(s)}(r,\varphi)=8i\hat{\Phi}A(\hat{\Phi})\frac{\Psi_{\hat{\Phi}}\left(2\hat{\Phi}-\frac{\pi}{2}\right)\Psi_{\hat{\Phi}}\left(\frac{\pi}{2}+\hat{\theta}_1+\hat{\theta}_2\right)}{\Psi_{\hat{\Phi}}\left(2\hat{\Phi}-\frac{\pi}{2}+\hat{\theta}_{1,2}\right)\Psi_{\hat{\Phi}}\left(\frac{\pi}{2}\pm\hat{\theta}_1\mp\hat{\theta}_2\right)}\times$$

$$\times\frac{\sin\frac{\pi\hat{\theta}_{1,2}}{4\hat{\Phi}}\cdot\cos\frac{\pi\hat{\theta}_{1,2}}{4\hat{\Phi}}}{\cos\frac{\pi}{4\hat{\Phi}}(\hat{\theta}_1+\hat{\theta}_2)}\exp\left[i\kappa_0\hat{r}\cos(\hat{\varphi}-\hat{\varphi}_{1,2}\pm\hat{\theta}_{1,2})\right]. \tag{8}$$

The presence of the Heaviside functions $\eta(\alpha)$ in Eq. (6) indicate the necessity to satisfy the existence condition of the surface waves. it is understood that all parameters in Eqs. (6)-(8) denoted with a (^) sign are transformed according to Eq. (3).

Analyzing the field in the far zone must be done differently for the cases $\varepsilon \gtrless 0$. For $\varepsilon > 0$ the ray index of refraction $N(\varphi)$ is always real and the fields primarily retain the structure that they had in an isotropic medium. A directional diagram of the radiation field $D(\varphi)$ is determined by $|H\Sigma(r,\varphi)|^2$ and it has the simple form:

$$D(\varphi)=\frac{1}{N(\varphi)}\cdot\frac{\cos^2\frac{\pi(\hat{\varphi}-\hat{\beta})}{\hat{\varphi}_2-\hat{\varphi}_1}}{\left|\cos\frac{\pi}{4\hat{\Phi}}(\hat{\varphi}-\hat{\varphi}_1-\hat{\theta}_2)\cos\frac{\pi}{4\hat{\Phi}}(\hat{\varphi}-\hat{\varphi}_2-\hat{\theta}_1)\right|}. \tag{9}$$

The surface waves, as in the isotropic case, are possible only for inductive impedance of the faces ($\operatorname{Im} Z_{1,2} < 0$). If $\operatorname{Re} Z_{1,2} = 0$, these waves propagate along the faces without attenuation with a retardation coefficient of $N(\varphi_{1,2})\cos\hat{\theta}_{1,2}=\sqrt{\cos^2\varphi_{1,2}+\varepsilon(\sin^2\varphi_{1,2}+Q_{1,2}^2)}$, where $Q_{1,2} = iZ_{1,2} > 0$. The retardation coefficient in a uniaxial plasma is less then the retardation coefficient in free space, but greater than the retardation coefficient in an isotropic plasma for the same frequency independent of the slope angle of the impedance surface to the anisotropy axis. The retardation of the surface wave is a maximum when the impedance face is located along the anisotropy axis. For purely reactive impedance faces ($\theta = i\varkappa$), the directional diagram of Eq. (9) does not depend on the sign of the impedance:

$$D(\varphi)=\frac{1}{N(\varphi)}\times$$

$$\times\frac{\cos^2\frac{\pi(\hat{\varphi}-\hat{\beta})}{\hat{\varphi}_2-\hat{\varphi}_1}}{\left[\operatorname{ch}\frac{\pi\hat{\varkappa}_2}{\hat{\varphi}_2-\hat{\varphi}_1}+\cos\frac{\pi(\hat{\varphi}-\hat{\varphi}_1)}{\hat{\varphi}_2-\hat{\varphi}_1}\right]\left[\operatorname{ch}\frac{\pi\hat{\varkappa}_1}{\hat{\varphi}_2-\hat{\varphi}_1}+\cos\frac{\pi(\hat{\varphi}-\hat{\varphi}_2)}{\hat{\varphi}_2-\hat{\varphi}_1}\right]}. \tag{10}$$

In Eqs. (9) and (10), $\hat{\beta}=0{,}5(\hat{\varphi}_1+\hat{\varphi}_2)$ is the bisectrix of the modified wedge. Figure 2 shows the diagrams calculated according to Eq. (10) for a half-plane. In comparison with an isotropic medium (dotted curve), the maximum in the radiation is formed not in a direction close to the bisectrix of the wedge, but near directions perpendicular to the anisotropy axis. The diagram is plotted for $Q_1 = 0.5$, $Q_2 = 1.0$, and $\varepsilon = 0.1$; the angles are referenced from the continuation of the half-plane.

For $\varepsilon < 0$ (resonance frequency of the region), the field structure in the far field sharply changes [5]. The radiation field turns out to be concentrated in the "exposed" region of space, $|\varphi|<\alpha$, $|\varphi-\pi|<\alpha$. The boundary of the shadow is determined by the angle $\alpha = \arctan(|\varepsilon|^{-1/2})$, the far field condition $\kappa_0 rN(\varphi) \gg 1$ is not fulfilled on that boundary, and all components of the field become infinite. In the "dark" region all the fields are exponentially attenuated. The noted discontinuities in the field are characteristic for wave equations of the hyperbolic type and they are removed if losses are introduced into the plasma. In the "exposed" region, the field in the far zone is entirely described by Eq. (6)-(8) if one imposes analytic continuation of the solution into the region where $0 \le \arg\varepsilon \le \pi$. It can be shown that for $\varepsilon < 0$ the modified angles should be expressed according to the equation

$$\hat{\varphi} = 0{,}5i \ln \left| \frac{\sin (\alpha + \varphi)}{\sin (\alpha - \varphi)} \right| + (m - 0{,}5l)\, \pi, \tag{11}$$

where the integer m is determined from the condition $|m\pi - \varphi| < 0.5\,\pi$, and the number $\ell = 0$ in the "exposed" region and $\ell = -\operatorname{sgn}(\sin 2\varphi)$ in the "dark" region. The structure of the surface waves for $\varepsilon < 0$ also varies. They can propagate along the faces without attenuation only in the "exposed" region and only for capacitive impedance of the faces ($Q_j < 0$). The impedance must satisfy the condition

$$|Q_j| < \sqrt{\sin (\alpha + \varphi_j) \sin (\alpha - \varphi_j)} / \cos \alpha. \tag{12}$$

If Eq. (12) is satisfied, then the modified Brewster's angles turn out to be real: $\hat{\theta}_j = \arcsin \left[-Q \cos\alpha / \sqrt{\sin (\alpha + \varphi_j) \sin (\alpha - \varphi_j)}\right]$. From Eq. (7) it follows that in the resonance region the arguments of the Ψ -functions are altered, which substantially limits the possibilities for obtaining a diagram in a form containing only elementary functions. It turns out to be possible to obtain such a representation for angles of the wedge aperture that are multiples of $0.5\,\pi$ if the impedances of the faces differ in sign and their moduli are related by

$$Q_2 = Q_1 \sqrt{\frac{\sin (\alpha + \varphi_2) \cdot \sin (\alpha - \varphi_2)}{\sin (\alpha + \varphi_1) \cdot \sin (\alpha - \varphi_1)}}. \tag{13}$$

Let, for example, conditions (12) and (13) be fulfilled and a rectangular wedge of $\Phi = 0.75\,\pi$ be oriented such that its bisectrix is located in the "exposed" region $|\varphi| < \alpha$. Then the modified angles $\hat{\varphi} = i\hat{\varphi}'$, $\hat{\beta} = i\hat{\beta}'$ are imaginary, the modified Brewster's angles $\hat{\theta}_2 = -\hat{\theta}_1 = \hat{\theta}$ are real, and the directional diagram of the radiation field has the form

$$D(\varphi) = \frac{1}{N(\varphi)} \cdot \frac{\operatorname{ch}^2 \frac{2}{3} (\hat{\varphi}' - \hat{\beta}')}{\left[\operatorname{ch} \frac{2}{3} (\hat{\varphi}' - \hat{\beta}') + \sin \frac{2}{3} \hat{\theta}\right]^2}, \quad |\varphi| < \alpha.$$

The established variation in the directionality of the radiation of a slit source in a confined uniaxial medium as a function of the orientation of the anisotropy axis is preserved in a plasma with a finite magnetic field, which must be allowed for in practical investigations.

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