THEORY OF CLUSTER SYSTEMS IN PLASMA-LIKE MEDIA

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The conditions of existence of stable formations of charged particles (clusters) moving in a plasma-like medium are discussed in this paper. The interaction of clusters is considered in the framework of a one-dimensional model. It is shown that the stability of a cluster depends strongly on the number of particles that form part of it. When two clusters moving with respect to each other interact, they break up if the relative velocity is fairly low. Fast-moving clusters hardly interact with each other.

1. The formation of bound states (clusters) of several moving particles of the same sign (ions) was observed in their transit through thin films of plasma-like media (see, for example, [1, 2]). As is well known, when a fast charged particle moves in a matter a so-called wake (the radiation field of the longitudinal perturbations of the medium) develops behind it [3-5]. The mechanism of the formation of such clusters of fast charged particles was discussed in [1] and in more recent publications and is due to the capture of charged particles into the potential well of the wake of the ion. The characteristic dimension of the clusters is of order V_0/Ω_p (V_0 is the particle velocity, Ω_p is the plasma frequency of the electrons of the medium) and exceeds the dimensions of the ionized molecules used in the experiments of [1, 2].

It is of interest to consider the stability of clusters of such a type and to discuss their possible means of formation and features of interaction. For a better understanding of the mechanisms of these phenomena, it is meaningful to consider the most simple one-dimensional model first of all. In a considerable number of interesting cases the one-dimensional description preserves the qualitative physical picture of the phenomena with relative simplicity (see [6, 7]).

2. We consider a one-dimensional* bunch of charged particles moving at a constant velocity V_0 in nonconfined plasma. Let the particle density in the bunch be $n(\xi)$, where $\xi = x - V_0t$; the charge of one particle of the bunch is q; the plasma density n_p considerably exceeds $n(\xi)$. Going over to the Fourier representation

$$n(\kappa) = \int_{-\infty}^{+\infty} d\xi e^{-i\kappa\xi} \cdot n(\xi), \tag{1}$$

we find the Fourier transform of the electric field without difficulty by means of Poisson's equation:

$$E(\kappa, \omega) = -i8\pi q\delta(\varepsilon)\, n(\kappa)/[\kappa\cdot\varepsilon(\omega, \kappa)], \tag{2}$$

where $\varepsilon(\omega, \kappa) = 1 - \Omega_e^2/(\omega + \kappa v_0)^2$ is the plasma permittivity in the (ω, κ) representation in the rest frame of the bunch; $\Omega_e^2 = 4\pi e^2 n_p/m_e$; e, m_e are the charge and mass of the electrons of the plasma; $\delta(x)$ is the delta function.

We implement an inverse transformation, under which

$$\begin{bmatrix} E(\xi) \\ n_t(\xi) \\ n_i(\xi) \\ \Phi(\xi) \end{bmatrix} = \int_{-\infty}^{+\infty} d\kappa \frac{e^{i\kappa\xi}}{\varepsilon(0,\kappa)} n(\kappa) \begin{bmatrix} -2iq\kappa^{-1} \\ (2\pi)^{-1} \\ (2\pi)^{-1}[1-\varepsilon(0, \kappa)] \\ 2q\kappa^{-2} \end{bmatrix}, \tag{3}$$

*A "one-dimensional particle" is a uniformly charged infinite plane with surface charge density q.

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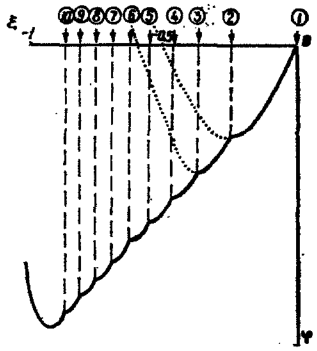

Fig. 1

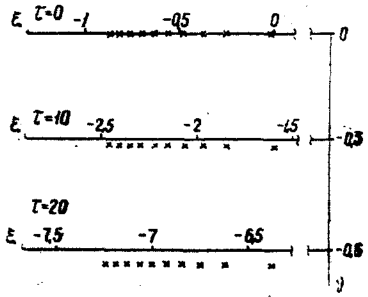

Fig. 2

Fig. 1. Initial distribution of the particles in the cluster (shown by circles) and the shape of the potential well formed by them.

Fig. 2. Phase picture of cluster evolution (N = 10).

where $n_t(\xi)$, $n_i(\xi)$, respectively, are the total charge density and the charge density induced in the plasma, and $\Phi(\xi)$ is the potential of the electric field in the plasma. The expression for $\varepsilon(\omega, \kappa)$ and this entire discussion are valid for $V_0^2 \gg V_{Te}^2 = T_e m_e$, where T_e is the electron temperature of the plasma. Integrating $n_t(\xi)$ with respect to ξ, we find that the total charge in the plasma is zero. Thus, the charge induced in the plasma is equal to the charge of the bunch with the opposite sign. For a single moving particle $n(\xi) = \delta(\xi)$, $n(\kappa) = 1$. Retaining the small imaginary part in $\varepsilon(0, \kappa)$ [8, 9] and using the relation

$$\kappa^{-2}\varepsilon^{-1}(0, \kappa) = (2\kappa_0)^{-1}\left[\frac{P}{\kappa-\kappa_0} - \frac{P}{\kappa+\kappa_0} - i\pi\delta(\kappa-\kappa_0) + i\pi\delta(\kappa+\kappa_0)\right], \tag{4}$$

where P/x is the principal value of x^{-1}, we write down the following expressions for the electric field $E_1(\xi)$, the charge density induced in the plasma, $n_{i1}(\xi)$, and the potential $\Phi_1(\xi)$:

$$\begin{bmatrix} E_1(\xi) \\ n_{i1}(\xi) \\ \Phi_1(\xi) \end{bmatrix} = \theta(-\xi)\begin{bmatrix} -4\pi q\cos(\kappa_0\xi) \\ \kappa_0\sin(\kappa_0\xi) \\ 4\pi\kappa_0^{-1} q\sin(\kappa_0\xi) \end{bmatrix}, \tag{5}$$

where $\kappa_0 = \Omega_p/v_0$; $\theta(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geqslant 0 \end{cases}$ is the theta function.

It is important to note that, allowing for finite absorption in the plasma, the right-hand sides of (5) are multiplied by $\sim\exp(\alpha\xi)$, where $\alpha \sim \nu'/V_0$, ν' is the collision frequency of the electrons in the plasma. In the one-dimensional model there is no electric field ahead of the particle, while, for convenience, the potential is selected equal to zero.

If there are few charged particles moving in the plasma, their behavior can be calculated only by means of numerical methods on a computer. The self-consistent system of equations which takes into account the back reaction of the field on the particles has the form

$$\frac{d\zeta_\beta}{d\tau} = \nu_\beta; \quad \frac{d\nu_\beta}{d\tau} = \varepsilon(\zeta_\beta), \tag{6}$$

$$\varepsilon(\zeta_\beta) = -\frac{4\pi}{N}\sum_{\alpha=1}^{N}\cos\left[2\pi g_\alpha(\zeta_\beta - \zeta_\alpha)\right]\cdot\theta(\zeta_\beta - \zeta_\alpha),$$

where

$$\zeta = \kappa_0\xi/2\pi; \quad \nu = \kappa_0(v - v_0)/(2\pi\gamma); \quad \gamma^2 = q^2\kappa_0/(2\pi m_q),$$
$$g_\alpha = (1 + \rho\nu_\alpha)^{-1}; \quad \rho = 2\pi/(\kappa_0 v_0); \quad \tau = \gamma t;.$$

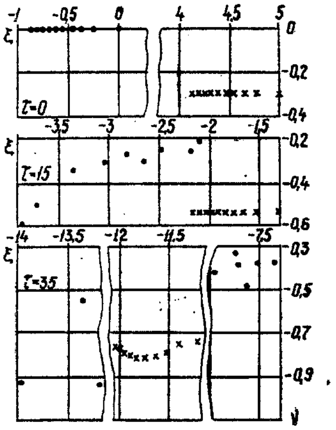

Fig. 3

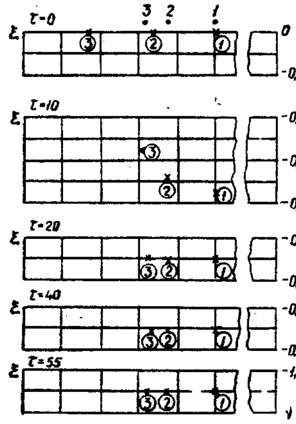

Fig. 4

Fig. 3. Phase picture of the interaction of two clusters with initial $\Delta\nu = -0.3$.

Fig. 4. Evolution of a short bunch of particles into a cluster.

$\dot{N}$ is the total number of particles, $\varepsilon = q\kappa_0 E/(2\pi m q \gamma^2)$.

3. In the general case, for an arbitrary initial distribution of the moving charged particles in the plasma, the original configuration of the bunch will be distorted. However, the particles can be positioned so that the stability of the bunch is improved appreciably. We will show it is possible to construct such a relatively stable bunch – a cluster. Let a charged particle moving in a plasma be the point $\xi = \xi_1 = 0$. We position a particle with the same velocity at the point $\xi = \xi_2 = -\pi/(2\kappa_0)$, where the electric field of the leading particle is zero (or the potential is minimal, which is the same). Then, locating each subsequent particle at the potential minimum of the field produced by the particles positioned ahead (see Fig. 1), we can construct a cluster whose stability is extremely high. The initial position of the particles for such a bunch is determined by the expressions

$$\xi_1 = 0;\ \kappa_0\xi_n = -\sum_{m=2}^{n}\arcsin(1/\sqrt{m-1}), \tag{7}$$

while the initial velocities of all the particles in the considered coordinate system are zero. The forces exerted by the radiation field on each particle of such a cluster are the same and equal to the force of deceleration of a single particle. The amplitude of the electric radiation field behind a cluster in which the number of particles is N exceeds the amplitude of the radiation field of a single particle by a factor $\sqrt{N}$. Thus, the radiation intensity of such a cluster of N particles is minimal and proportional to N. We note that for the cluster particles ρ is usually extremely small and can be disregarded.

Figure 2 shows the evolution of the particles of a cluster formed in accordance with laws (7).

Analysis of the results of the numerical calculations of the problem for the investigation of the dynamics of single bunches of the II kind indicated that when each of the cluster particles is positioned at a point corresponding to the minimum of the field energy of all the preceding particles, the bunch formed is stable for a long time (of the order of tens of time units). It can be seen in the phase picture (Fig. 2) that the cluster particles, strictly observing the initial spatial distribution, are decelerated at the same velocity. The relative spread in velocities for $\tau = 30$ is 0.01%, and this does not exceed the accuracy of the numerical calculations.

If the initial spatial distribution differs from the equilibrium (cluster) distribution but this difference is negligible, the bunch obtained will evolve as a system of bound particles which execute translational–vibrational motion within the limits of the potential well of the adjusted field of all particles. In order to determine the limiting value of the displacement of the particles from the equilibrium distribution, we turn to Fig. 1, which shows the potential well of a cluster and the spatial distribution of the particles in it. It can be seen that for the satisfaction of the condition of cohesion it is necessary that the total energy of each particle (if there is no initial velocity it is determined by the field potential of all the preceding particles) be nonpositive. For a particle under the number 2 the potential well is the negative semiarc of a sine curve; therefore, the maximum displacement for it is 1/4 of the period of the plasma (natural) oscillations. For the next particles the potential well becomes steeper as the number of the particle increases; hence, the maximum displacement is reduced. Hence, it follows that as the number of particles in the bunch increases the probability of the "breakup" of the bunch as a bound system increases and the time of its existence as a beam decreases. Computer calculations for the values N = 3, 10, 50 showed that when there were three particles in the bunch and they were considerably displaced from the position in the cluster the particles retained the bound state during a time ~20-30, while the spread in velocities did not exceed 20%. For N = 10 the significant displacement of half of the particles already led to an appreciable spread in velocities ($\Delta\nu \sim 30\%$) during time $\tau = 10$, while for N = 50 the displacement of the last 20 particles by 0.001 from the equilibrium state led to the complete breakup of the bunch during the same times.

4. We now consider the interaction of two clusters with the relative velocity $\Delta\nu$. If $\Delta\nu$ appreciably exceeds the critical velocity $\Delta\nu_{cr} = \Delta\nu_{cr}(N)$, the influence of the clusters on each other can be disregarded. The value of $\Delta\nu_{cr}(N)$ is determined by the minimum binding energy of the particles in the cluster. The depth of the potential well $\Delta\Phi_m$, confining the (m + 1)-th particle of the cluster between the particles with numbers m + 2 and m, is

$$\Delta\Phi_m \sim \left(\frac{4\pi q}{\varkappa_0}\right)\sqrt{m}\,[1-(1-m^{-1})^{1/2}]. \tag{8}$$

Thus, for fairly large N, we have $\Delta\Phi \sim 1/\sqrt{N}$. Therefore, clusters with a large number of particles break up at lower relative velocities. However, it is unfortunately not possible to give the critical velocity $\Delta\nu_{cr}$ as a function of N in explicit form.

It follows from the results of the numerical analysis of the system of equations (6) for two clusters with $N_1 = N_2 = 10$ and $|\Delta\nu| > \Delta\nu_{cr}$ that the relative position of the particles in the clusters does not vary after interaction. Thus, they retain their stability. But if $|\Delta\nu|$ approaches $\Delta\nu_{cr}(10) \approx 0.3$, first the cluster which is behind at the initial time breaks up, and then (to a somewhat lesser extent) the second cluster breaks up. Figure 3 shows the evolution of a system of two clusters with $N_1 = N_2 = 10$ and $|\Delta\nu| = 0.3$.

5. In conclusion, we discuss the possibility of the formation of a cluster of several charged particles in a system with negative friction. Such a situation is realized if a neutral stream containing ions propagates in a plasma-like medium (we note that collective processes which lead to the development of instabilities predominate when there is a large concentration of charged particles in the stream [10]). In this case, the second equation in system (6) has the form

$$\frac{d\nu_\beta}{d\tau} = \varepsilon(\zeta_\beta) - \chi\nu_\beta,$$

Figure 4 shows the behavior of three arbitrarily positioned charged particles for $\chi = 3$. It is easy to verify that these particles form a cluster for $\tau \geq 40$. Clusters consisting of a larger number of particles can be obtained in exactly the same way, but in this case their formation time increases.

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