$$L_1(x) = -\frac{i}{2}\,\partial_{z^\gamma}\tilde{\zeta}(z,\ x)|_{z=0}.$$

3. Notice that the regularization scheme we discuss coincides with the method of [2] for the particular case when the set of fields Φ^A consists of only one field Φ, and corresponds to considering only one differential operator $D(x, y) = D(\partial_x, x)\delta(x - y)$ in our scheme. In the general case, the problem of computing the generalized ζ-function $\tilde{\zeta}(z, x, x')$ of Eq. (18) can be reduced to solving the eigenvalue problem for a system of integro-differential equations (7), which is much more complicated from the mathematical point of view than a similar problem for the system of differential equations (10). It may be possible to simplify the solution of Eqs. (7) and (10) when $D_{\alpha\beta} = 0$ (or $D_{ab} = 0$), or when D_{ab} and $D_{\alpha\beta}$ are partially or completely diagonal.

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EXCITATION OF ATOMIC NUCLEI AND ATOMS BY RELATIVISTIC CHARGE PARTICLES BOUND IN A ONE-DIMENSIONAL POTENTIAL

A. N. Almaliev, I. S. Batkin, and I. V. Kopytin

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The process of exciting atoms and atomic nuclei by relativistic electrons and positrons bound in a one-dimensional potential is investigated theoretically. It is shown that a pole corresponding to the emergence of a virtual photon on a bulk surface occurs in the matrix interaction element under definite kinematic relationships. It is obtained that the probability of the excitation process depends on the lifetime of the level being excited, the virtual photon, and the charged particle in a definite energetic state. An estimate of the magnitude of the excitation section of low-lying nuclear states yields a value exceeding by several orders the section obtained for charged particles in the absence of a binding potential.

1. In a number of physical problems the particle motion turns out to be confined to one direction, for instance, during charged particle motion in a one-dimensional permanent magnetic field, between charged plates, along a charged plane or filament, in artificial or natural channels, etc. The properties and characteristics of the radiation of charged particles bound in a one-dimensional potential were studied theoretically and experimentally with sufficient completeness by many authors [1-3] and in [3] it was proposed to use it to excite atomic nuclei. It is here assumed, however, that the radiation is first formed completely, while the activation process itself penetrates because of photoabsorption. It can be considered that we deal here with a two-stage excitation mechanism for the atomic nucleus. Such a mechanism is not unique, and excitation of a quantum system (atom or atomic nucleus) directly by charged particles bound in a one-dimensional particle can be considered. We call this a single-stage mechanism. In this paper we perform a quantum study of a single-stage mechanism. Our purpose is to obtain the excitation probability for quantum systems (atoms or atomic nuclei) by relativistic charged particles (electrons or positrons) with transversely con-

Lenin Komsomol Pedagogical Institute, Tomsk.

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strained motion. Let us note that it is necessary to deal with a mechanism analogous to ours even in the classical formulation when problems about radiation properties in an absorbing medium are examined. The macroscopic solution of such problems is presented in the book [4] without taking account of quantum effects.

2. The matrix element of electromagnetic charged particle interaction with an insulated quantum system can be represented in the following form in first-order perturbation theory [5]:

$$<F, f|H_{\text{int}}|I, i> = -\frac{4\pi}{c^2}\int\frac{d^3\kappa}{(2\pi)^3}\left\{\frac{<F|j^{(t)}(r)e^{i\kappa r}|I>}{\kappa^2-\kappa_0^2}\times\right.$$
$$\left.\times<f|Je^{-i\kappa r'}|i> - c^2\frac{<F|\rho(r)e^{i\kappa r}|I>}{\kappa^2}<f|\rho^{(e)}e^{-i\kappa r'}|i>\right\}, \quad (1)$$

c is the speed of light; $\hbar c\kappa_0 = E_F - E_I = E_i - E_f$; E_I, E_F (E_i, E_f) are the initial and final state energies of the quantum system (particles); $J = iec\gamma$; $\rho^{(e)} = e\gamma_4$ (γ, γ_4 are Dirac matrices and e is the particle charge); $\rho(r)$ and $j(4)$ are the charge density and current of the quantum system under consideration, respectively (see, e.g., [5]). The superscript t denotes the transverse component of the total current $j^{(t)} = \hat{\kappa}\times(j\times\hat{\kappa})$ and $\hat{\kappa} = \kappa/\kappa$.

We shall perform the examination in a laboratory coordinate system. We assume that the charged particle moves in a one-dimensional potential V(x) while the motion along the y and z coordinates is free. Then the wave functions of the initial and final particle states have the form

$$|i> = L_0^{-1}e^{-i(\kappa_{yi}y+\kappa_{zi}z)}|n_i>;\ |f> = L_0^{-1}e^{-i(\kappa_{yf}y+\kappa_{zf}z)}|n_f>, \quad (2)$$

where $|n>$ is the solution of the Dirac equation with potential V(x); n is the quantum number governing the state of one-dimensional motion; L_0 is the normalizing length; and $\kappa_{i,f} = p_{i,f}/\hbar$, where p is the particle momentum. Integration with respect to κ_y and κ_z in (1) with (2) taken into account is elementary in execution and we obtain the following expression

$$<F, f|H_{\text{int}}|I, i> = \frac{2}{L_0^2}\left\{-\int_{-\infty}^{\infty}d\kappa_x\frac{<F|j^{(t)}(r)e^{i(\kappa_x x+K^{\parallel}r^{\parallel})}|I>A(\kappa_x)}{\kappa_x^2+(K^{\parallel})^2-\kappa_0^2}+\right.$$
$$\left.+\int_{-\infty}^{+\infty}d\kappa_x\frac{<F|\rho(r)e^{i(\kappa_x x+K^{\parallel}r^{\parallel})}|I>B(\kappa_x)}{\kappa_x^2+(K^{\parallel})^2}\right\} \equiv I^{(t)}+I^{(l)}. \quad (3)$$

We here introduced the following notation

$$A(\kappa_x) = <n_f|e^{-i\kappa_x x'}a|n_i>;\ B(\kappa_x) = e<n_f|e^{-i\kappa_x x'}|n_i>;$$
$$a_x = \frac{\hbar e}{E_i}\frac{\partial}{\partial x'},\ a^{\parallel} = i\frac{e}{E_i}p_i^{\parallel},\ K^{\parallel} = \kappa_i^{\parallel} - \kappa_f^{\parallel}. \quad (4)$$

For an arbitrary vector n the projection on the (y, z) plane is denoted by $n^{\parallel}$. It was assumed that $\hbar c\kappa_0 \ll E_i$ in obtaining (3). We will consider motion of a particle with rest mass m along the x axis direction to be nonrelativistic. In this approximation the particle energy has the form $E \approx E^{\parallel} + \varepsilon$, where $E^{\parallel} = [c^2(p^{\parallel})^2 + m^2c^4]^{1/2}$ while the energy of the transverse motion ε is determined from the equation [6]

$$\left[-\frac{\hbar^2c^2}{2E^{\parallel}}\frac{d^2}{dx^2}+V(x)\right]|n> = \varepsilon_n|n>. \quad (5)$$

We limit ourselves henceforth to consideration of symmetric potentials V(x) for which V(x) = V(−x). Under definite kinematic relationships the denominators of the integrands in (3) can vanish. Using the energy conservation law, conditions for the existence of pole singularities in the integrals $I^{(t)}$ and $I^{(l)}$ can be obtained in the ultrarelativistic limit:

$$\hbar c\kappa_0 \leqslant 2(\varepsilon_{n_i}-\varepsilon_{n_f})(E_i/mc^2)^2 \text{ for the integral } I^{(t)}, \quad (6)$$
$$\hbar c\kappa_0 \approx \varepsilon_{n_i}-\varepsilon_{n_f} \text{ for the integral } I^{(l)}. \quad (7)$$

Furthermore, we consider the case of exciting quantum systems for which $E_F - E_I = \hbar c\kappa_0 \gg \varepsilon_{ni} - \varepsilon_{nf}$. Under this condition, existence of a pole is possible here only in the integral $I^{(t)}$, which corresponds to the emergence of a virtual photon on the bulk surface. We also assume henceforth that the quantum system being excited is outside the limits of the localization domain for the wave function of the transverse motion of the bombarding particle ($|x| = d$, where d is the characteristic width of the localization domain). After integrating with respect to κ_x in (3), this permits separating the variables x and x'. Considering the condition (6) satisfied, using the standard rule for bypassing the poles in the photon Green's function and assuming $x > 0$ for definiteness, we obtain

$$I^{(t)} = -\frac{2\pi i}{L_0^2 \varkappa} < F | j^{(t)}(r) e^{iKr} | I > A(\varkappa), \tag{8}$$

$$I^{(l)} = \frac{2\pi}{L_0^2 K^{\parallel}} < F | \rho(r) e^{iQr} | I > B(iK^{\perp}), \tag{9}$$

where

$$K \equiv (\varkappa, K^{\parallel}), \quad Q \equiv (iK^{\parallel}, K^{\perp}), \quad \varkappa = \varkappa_0 + i\delta, \quad \delta = \Gamma \kappa_0^{(0)} / \hbar c \varkappa_0,$$
$$\kappa_0^{(0)} = \operatorname{Re} \kappa_0, \quad \Gamma / \hbar c = \operatorname{Im} \kappa_0, \quad \varkappa_0 = [(\kappa_0^{(0)})^2 - (K^{\perp})^2]^{1/2}.$$

Introduction of the width $\Gamma = \Gamma_N + \Gamma_\omega + \Gamma_e$ is specified by the finite lifetime T_N of the quantum system in the excited state ($\Gamma_N = \hbar / T_N$), the finite path length of the virtual photon $l_\omega (\Gamma_\omega = \hbar c / l_\omega)$ and the finite lifetime T_e of the bombarding particle in a definite quantum state of transverse motion ($\Gamma_e = \hbar / T_e$).

As a rule, the wave functions $|I>$ and $|F>$ are determined in a coordinate system coupled to the center of gravity of the quantum system. We introduce the coordinate of the center of gravity of the system $R = \sum_i m_i r_i / \sum_i m_i$ (r_i are the position coordinates of individual particles of the system, and m_i is the particle mass), and the relative coordinates $\xi_i = r_i - R$.

We will consider that the motion of the quantum system as a whole does not influence the characteristics of its internal excitations. Then the wave functions of the quantum system are factorized, for instance $|I> = |I: \xi> \cdot |\alpha; R>$ (ξ is the whole set of variables ξ_i). The function $|\alpha; R>$ characterizes the system motion as a whole, and α are quantum numbers characterizing the motion of the center of gravity. It is understood later that all the computations are executed in the variables (ξ and R).

3. Using the known expansions of plane and transverse plane waves in multifields and the standard technique of angular moment algebra (see, e.g., [5]), after summing over the magnetic quantum numbers we obtain for the differential probability of quantum system excitation per unit time:

$$dW = \frac{2\pi}{\hbar} | < F, f | H_{\text{int}} | I, i > |^2 \frac{L_0^2 d\kappa_f^{\parallel}}{(2\pi)^2} \delta(E_i - E_f - \hbar c \kappa_0^{(0)}) \equiv$$

$$\equiv dW^{(t)} + dW^{(l)}; \tag{10}$$

$$dW^{(t)} = \frac{\pi c^2}{2\hbar \kappa_0^{(0)} \varkappa_0^2 L_0^2} \left(\frac{2J_F + 1}{2J_I + 1} \right)^2 \Gamma_\gamma (|A(\varkappa)|^2 - |\hat{K} A(\varkappa)|^2) \times$$

$$\times | < \alpha'; R | e^{iKR} | \alpha; R > |^2 d\kappa_f^{\parallel} \delta(E_i - E_f - \hbar c \kappa_0^{(0)}); \tag{11}$$

$$dW^{(l)} = \frac{8\pi^2}{\hbar (K^{\parallel})^2 L_0^2} \frac{2J_F + 1}{2J_I + 1} \sum_L |N_{I \to F}(Q; CL)|^2 \cdot |B(iK^{\perp})|^2 \times$$

$$\times | < \alpha'; R | e^{iQR} | \alpha; R > |^2 d\kappa_f^{\perp} \delta(E_i - E_f - \hbar c \kappa_0^{(0)}). \tag{12}$$

Here $\Gamma_\gamma = \hbar W_\gamma$, where W_γ is the probability of a γ-transition of the quantum system $|F> \to |I>$; the presented matrix elements $N_{I \to F}$ are defined in [5]; J_I, J_F are the total spins of the corresponding states; and $\hat{K} = K / K$.

As follows from (10), the excitation probability of a quantum system dW is determined by two components, one of which ($dW^{(t)}$) can provisionally be considered long-acting while the other ($dW^{(l)}$) is short-lived. The first component is due to exchange by real vector photons and the corresponding interaction damps out at distances on the order of $\hbar c / \Gamma$. Interaction in the second component is realized at distances $\lesssim (\kappa_0^{(0)})^{-1}$. In a real situation

it is possible to consider $\Gamma \ll \hbar c \kappa_0^{(0)}$. Since the wave function of the particles and the quantum system we took in our model are localized in different domains of space the contribution of the short-range term to the total probability can be considered negligible. Let us note that analogous deductions about the exponential smallness of $dW^{(1)}$ were made in the monograph [2]; however, the effects due to the exchange by real vector photons were not considered in the work mentioned.

We limit ourselves henceforth to consideration of $dW^{(t)}$. By using the vector $A(x)$ it is possible to express $dW^{(t)}$ in terms of the probability per unit time $dw(\kappa, \kappa_f)$ of the spontaneous electromagnetic radiation of a charged particle bound in the potential $V(x)$ ($\hbar$ is the momentum of an emitted photon). Using wave functions in the form (2) and taking account of the requirement $\hbar\omega \ll E_i$ (ω is the radiation frequency), we obtain

$$dW^{(t)} = \frac{\pi^2}{\hbar\kappa_0^{(0)}\varkappa_0 L_0^3}\left(\frac{2J_F+1}{2J_I+1}\right)^2 \Gamma_\gamma (dw(\kappa, \kappa_f)/d\omega)_{\kappa=K} \times$$
$$\times |<\alpha'; R|e^{iKR}|\alpha; R_i^0>|^2 \equiv dW_0^{(t)} \cdot |<\alpha'; R|e^{iKR}|\alpha; R>|^2. \quad (13)$$

which yields the excitation probability for an isolated quantum system. If there is an ensemble of N quantum systems and the excitation of a given quantum state is considered in at least one of them, where the final state of the motion of the system center of gravity is not fixed, then the appropriate probability can be found in the following manner:

$$dW^{(t)} = dW_0^{(t)} \cdot \sum_{R_j}\sum_{\alpha'} |<\alpha'; R_j|e^{iKR_j}|\alpha; R_j>|^2. \quad (14)$$

For definiteness, we will consider excitation of quantum systems located at the sites of a crystalline lattice with coordinates $R_j^{(0)}$. Using the approximation of completeness for execution of summation over α' and considering the mean deviation of the atoms from the equilibrium position to be considerably less than the interstitial distance, we obtain

$$dW^{(t)} = dW_0^{(t)} \cdot \sum_{R_j^{(0)}} |e^{iKR_j^{(0)}}|^2. \quad (15)$$

Summing over $R_j^{(0)}$ with the periodicity condition taken into account, we find

$$dW^{(t)} = \frac{2\pi^2\Gamma_\gamma}{\hbar\kappa_0^{(0)}\varkappa_0 a^2}\left(\frac{2J_F+1}{2J_I+1}\right)^2 \frac{e^{-\delta d}}{1-e^{-2\delta a}} (dw(\kappa, \kappa_f)/d\omega)_{\kappa=K}, \quad (16)$$

where a is the lattice constant.

As is seen from (16), the maximal value of the probability $dW^{(t)}$ will be reached when the condition $\delta d \ll 1$ is satisfied. Then

$$dW^{(t)} = \frac{\pi^2 c}{(\kappa_0^{(0)})^2 a^3}\left(\frac{2J_F+1}{2J_I+1}\right)^2 \frac{\Gamma_\gamma}{\Gamma} (dw(\kappa, \kappa_f)/d\omega)_{\kappa=K}. \quad (17)$$

The expression obtained for $dW^{(t)}$ has a simple physical meaning. Introducing the photon pathlength $\ell = c\hbar/\Gamma = cT$ and the resonance absorption section $\sigma_r = 4\pi/(\kappa_0^{(0)})^2$, it is possible to represent $dW^{(t)}$ in the following form (to the accuracy of a nonessential factor):

$$dW^{(t)} \approx \sigma_r \cdot n_0 l \cdot P_\gamma, \quad (18)$$

where $n_0 = a^{-3}$ is the density of the quantum systems per unit volume, and $P_\gamma = \hbar^{-1}\Gamma_\gamma(dw/d\omega)_{K=K}$. Such a form of $dW^{(t)}$ permits treatment of the process under consideration in the form of the following sequence of elementary processes: The charged particle emits a photon with the probability P_γ in a spectrum band of width Γ_γ which is then absorbed by the crystal in the pathlength ℓ. Let us note that the expression for $dW^{(t)}$ (16) remains valid even for gaseous, liquid, or amorphous media, except in this case a is the mean spacing between particles of the medium ($a \ll L_0$, understandably).

4. Let us estimate the total probability $W^{(t)}$. As an illustration, we consider excitation of nuclei during passage of a beam of channeled positrons with 150 MeV energy through a monocrystal. We select the following values for the parameters $\hbar c \kappa_0^{(0)} \approx 1$, $\Gamma_\gamma \approx 10^{-3}$ MeV/eV (these values are characteristic for the transitions $2_1^+ \to 0^+$ in even–even atomic nuclei with the mass number $\gtrsim 100$); $d \approx a \approx 4\cdot 10^{-8}$ cm, $\Gamma \approx \Gamma_\gamma$. Computation of $(dw/d\omega)_{K=K}$ performed

by formulas from [6] yields the value $5 \cdot 10^{-8}$. Finally, we obtain the value $2.5 \cdot 10^{6}$ sec^{-1} for the probability $E^{(t)}$. We find the absolute excitation probability of at least one nucleus by a positron during planar channeling proceeding through a monocrystal of thickness $L_0 = 1$ mm: $P^{(t)} \approx W^{(t)}L_0/c \approx 8 \cdot 10^{-6}$. Using this value, we obtain for the magnitude of the effective section of quadrupole excitation of an atomic nucleus $\sigma = P^{(t)}/n_0L_0 \approx 5 \cdot 10^{-27}$ cm^2. Let us note that such a section exceeds the characteristic excitation sections of low-lying nuclear levels by free relativistic electrons by several orders [7]. We considered the excitation of an atomic nucleus. Estimation of the quantity $W^{(t)}$ in the case of atom excitation can be performed in an analogous manner [8].

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INVERSE SCATTERING PROBLEM IN THE RELATIVISTIC QUASICLASSICAL APPROXIMATION

I. L. Solovtsov and Yu. D. Chernichenko

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We apply the quasipotential approach of quantum field theory to solve the inverse scattering problem in the relativistic quasiclassical approximation. We obtain expressions for reconstructing the quasipotential from the phase shifts and consider both nonrelativistic and ultrarelativistic cases.

Today there exists a number of approaches to the problem of determining the interaction potential of elementary particles. The phenomenological approach, wherein the potentials are chosen are to contain variational parameters and are then fit to experimental data, has gained wide acceptance (see, for example, review article [1]). Another method of reconstructing the interaction potential is based on the solving the inverse problem (IP). The literature on solving the various formulations of the IP is quite extensive, as are the applications of the results (see, for example, [2-5]).

A distinguishing feature of these and other solutions of the IP is that, notwithstanding the diversity of approaches, they are all based on the nonrelativistic Schrödinger equation. At the same time our analysis indicates that relativistic effects could play a major role in a number of cases.

The inverse problem was considered by Malyarov and coworkers [8] within the relativistic quasipotential approach [6] by using the differential quasipotential equation proposed by

Gomel' Polytechnical Institute.

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