$$\tilde{S}^c(x, x') = (\gamma^\mu P_\mu + m)\tilde{G}^c(x, x'),\ \tilde{G}^c(x, x') = \int_{\Gamma^c} g(x, x', \tau)\,d\tau; \tag{23}$$

$$\tilde{\tilde{S}}(x, x') = (\gamma^\mu P_\mu + m)\tilde{\tilde{G}}(x, x'),\ \tilde{\tilde{G}}(x, x') = \theta(u^0 - u'^0)\int_{\Gamma^c+\Gamma_2+\Gamma_3} g(x, x', \tau)\,d\tau + \theta(u'^0 - u^0)\int_{\Gamma^c+\Gamma_4} g(x, x', \tau)\,d\tau. \tag{24}$$

The contours Γ_3 and Γ_4 are displayed in Fig. 1, respectively, for $u^0 > u'^0$ and

$$u^0 < u'^0 \left(a_3 = a_4 = \frac{1}{2}\ln\frac{\tilde{A}_3(u'^0) - \tilde{A}_3(-\infty)}{\tilde{A}_3(u^0) - \tilde{A}_3(-\infty)} - i\pi\right).$$

In the particular case as $\tilde{A}_3'(u^0) \to$ const we obtain representations for $\tilde{S}$, $\tilde{S}^c$, and $\tilde{\tilde{S}}$ in a constant electrical field and in a plane wave field from (22), (23), and (24), and which agree with those found earlier in [3, 6].

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GREEN'S ELECTRON FUNCTION IN A QUANTIZED PLANE WAVE FIELD

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It is shown that the Green's function of an electron that interacts with a quantized plane wave can be expressed in terms of the corresponding Green's function of a scalar particle. By using the known expression for the Green's function of a scalar particle, an integral representation is found with respect to the intrinsic time for the Green's electron function in a quantized plane wave of arbitrary form.

1. The problem of electron motion in the field of a free quantized monochromatic electromagnetic wave with which the interaction is taken into account exactly, was first solved in [1]. The solutions found in that paper were then extended to more complex cases by different authors [2-6]. The limits of applicability of such a model were then established in [7] within the framework of exact quantum electrodynamics.

The Green's function of a scalar particle interacting with a quantized plane electromagnetic wave field was found in [8], where no constraints were imposed on the wave. A relation between the electron and the scalar particle Green's function in a quantized electromagnetic field of arbitrary form is established in this paper and the Green's electron function is calculated. Exactly as in [8], the Green's function is represented in the form of a contour integral over the intrinsic time.

2. We select the x^3 axis along the direction of plane wave propagation. The 4-potential of the wave will then depend on the space—time variables in the combination x^0-x^3. It is convenient to introduce the new coordinates

$$u^0 = x^0 - x^3,\ \ u^1 = x^1,\ \ u^2 = x^2,\ \ u^3 = x^0 + x^3. \tag{1}$$

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We shall denote the components of any vector A relative to the new coordinate system by $\tilde{A}^\mu$ in contrast to the Cartesian A^μ.

The Green's function of an electron interacting with a quantized plane wave field will satisfy the equation

$$(\tilde{\gamma}^\mu \tilde{P}_\mu - m)\, G(x, x'; \xi, \xi') = -\delta^{(4)}(x - x')\,\delta(\xi - \xi'), \tag{2}$$

where

$$\delta(\xi - \xi') = \prod_{i,\lambda} \delta(\xi_{i\lambda} - \xi'_{i\lambda}),\ \tilde{P}_\mu = i\partial_\mu - e\tilde{A}_\mu(x, \xi),\ \partial_\mu = \partial/\partial u^\mu,\ \hbar = c = 1;$$

$$\tilde{A}^\mu(x, \xi) = \sum_{i,\lambda} \left(\frac{2\pi}{V\varkappa_i}\right)^{1/2} \tilde{e}^\mu_\lambda \left(c_{i\lambda} e^{-i\varkappa_i u^0} + \overset{+}{c}_{i\lambda} e^{i\varkappa_i u^0}\right) \tag{3}$$

is the operator-potential of the quantized wave field, V is the normalizing volume of the field

$$\overset{+}{c}_{i\lambda} = 2^{-1/2}\left(\xi_{i\lambda} - \frac{\partial}{\partial \xi_{i\lambda}}\right),\ c_{i\lambda} = 2^{-1/2}\left(\xi_{i\lambda} + \frac{\partial}{\partial \xi_{i\lambda}}\right)$$

are photon generation and annihilation operators in a coordinate representation with the frequency $\varkappa_i$ and polarization $\lambda = 1, 2$, $\xi_{i\lambda}$ is the field variable, and $\tilde{e}^\mu_\lambda = (0, \tilde{e}^1_\lambda, \tilde{e}^2_\lambda, 0)$ is the 4-vector of linear photon polarization.

The solution of (2) can be found by the inverse operator method [9]

$$G(x, x'; \xi, \xi') = -\frac{\tilde{\gamma}^\mu \tilde{P}_\mu + m}{(\tilde{\gamma}^\mu \tilde{P}_\mu)^2 - m^2 + i\varepsilon}\,\delta^{(4)}(x - x')\,\delta(\xi - \xi)' =$$

$$= (\tilde{\gamma}^\mu \tilde{P}_\mu + m)\, i \int_0^{+\infty} ds\, e^{is[(\tilde{\gamma}^\mu \tilde{P}_\mu)^2 - m^2 + i\varepsilon]}\,\delta^{(4)}(x - x')\,\delta(\xi - \xi'). \tag{4}$$

However, by starting from (2) it is simpler to express G in terms of the scalar particle Green's function, and then to determine G by using the known expression for it.

To this end, we introduce the projection operators $p_{(-)}$ and $p_{(+)}$

$$p_{(-)} = 1/4\, \tilde{\gamma}^3\tilde{\gamma}^0,\ p_{(+)} = 1/4\, \tilde{\gamma}^0\tilde{\gamma}^3,\ p_{(-)} + p_{(+)} = 1,$$
$$p_{(\pm)}p_{(\pm)} = p_{(\pm)},\ p_{(\pm)}p_{(\mp)} = 0. \tag{5}$$

By using $p_{(-)}$ and $p_{(+)}$ we write G in the form

$$G = G_{(+)} + G_{(-)},\ G_{(\pm)} = p_{(\pm)}G. \tag{6}$$

Multiplying (2) on the left by the matrices $\tilde{\gamma}^0$ and $\tilde{\gamma}^3$, we obtain a system of equations in the functions $G_{(+)}$ and $G_{(-)}$:

$$4i\partial_0 G_{(-)}(x, x'; \xi, \xi') = -\tilde{\gamma}^3\left[(\tilde{\gamma}^j \tilde{P}_j - m)\, G_{(+)}(x, x'; \xi, \xi') + \delta^{(4)}(x - x')\,\delta(\xi - \xi')\right], \tag{7}$$

$$4i\partial_3 G_{(+)}(x, x'; \xi, \xi') = -\tilde{\gamma}^0\left[(\tilde{\gamma}^j \tilde{P}_j - m)\, G_{(-)}(x, x'; \xi, \xi') + \delta^{(4)}(x - x')\,\delta(\xi - \xi')\right], \tag{8}$$

$$j = 1,\ 2.$$

The last equation can be solved formally for $G_{(+)}$:

$$G_{(+)}(x, x'; \xi, \xi') = -\tilde{\gamma}^0 (4i\partial_3)^{-1}\left[(\tilde{\gamma}^j \tilde{P}_j - m)\, G_{(-)}(x, x'; \xi, \xi') + \delta^{(4)}(x - x')\,\delta(\xi - \xi')\right], \tag{9}$$

where $(i\partial_3)^{-1}$ is the inverse operator to $i\partial_3$. To determine the operator $(i\partial_3)^{-1}$, we note that the expansion of the scalar particle Green's function D in the field (3) in a Fourier integral in the variable u^3 has the form

$$D(x, x'; \xi, \xi') = \int_{-\infty}^{+\infty}{}' e^{-i\kappa u^3} D'(u^0, u^1, u^2, \kappa, x'; \xi, \xi') d\kappa, \tag{10}$$

where the prime on the integration symbol means that the integral of the function D' that has the singular point $\kappa = 0$ is understood to be improper (D' is the Fourier transform of D in the variable u^3). This is easy to see if the integral representation is used for the function D that is found in [8]. It is natural to assume that an analogous expansion holds for the function G also. Taking this assumption into account, we find the explicit form of the operator $(i\partial^3)^{-1}$.

$$(i\partial_3)^{-1} f(x) = \int_{-\infty}^{+\infty}{}' \frac{1}{\kappa} e^{-i\kappa u^3} f'(u^0, u^1, u^2, \kappa)\, d\kappa. \tag{11}$$

Substituting (9) into (7), we find the equation in $G_{(-)}$

$$(\widetilde{P}_\mu \widetilde{P}^\mu - m^2)(i\partial_3)^{-1} G_{(-)}(x, x'; \xi, \xi') = -[(\widetilde{\gamma}^j \widetilde{P}_j + m)(i\partial_3)^{-1} p_{(-)} + \widetilde{\gamma}^3]\, \delta^{(4)}(x - x')\, \delta(\xi - \xi'). \tag{12}$$

In deriving (12) it was taken into account that $(\tilde{\gamma}^j P_j)^2 = \tilde{P}_j \tilde{P}^j$. In the following it is convenient to convert the right side of (12) to a form such that the operator acting on the product of δ functions would commutate with the operators in the left side of (12). To do this we note that

$$\widetilde{A}^\mu(x, \xi)\, \delta^{(4)}(x - x')\, \delta(\xi - \xi') = \widetilde{A}^\mu(-x', \xi')\, \delta^{(4)}(x - x')\, \delta(\xi - \xi'). \tag{13}$$

Taking (13) into account, we have

$$[(\widetilde{\gamma}^j \widetilde{P}_j + m)(i\partial_3)^{-1} p_{(-)} + \widetilde{\gamma}^3]\, \delta^{(4)}(x - x')\, \delta(\xi - \xi') = [(\widetilde{\gamma}^j \widetilde{P}'_j + m) p_{(-)} + \widetilde{\gamma}^3 i\partial_3] \cdot (i\partial_3)^{-1} \delta^{(4)}(x - x')\, \delta(\xi - \xi'), \tag{14}$$

where $\tilde{P}'_j = i\partial_j - e\tilde{A}_j(-x', \xi')$. Taking account of (14) and the representation for the inverse operator in the form (4), we write the solution of (12) in the form

$$G_{(-)}(x, x'; \xi, \xi') = [(\widetilde{\gamma}^j \widetilde{P}'_j + m) p_{(-)} + \widetilde{\gamma}^3 i\partial_3]\, D(x, x'; \xi, \xi'), \tag{15}$$

where

$$D(x, x'; \xi, \xi') = i \int_0^{+\infty} ds\, e^{is(\widetilde{P}_\mu \widetilde{P}^\mu - m^2 + i\varepsilon)}\, \delta^{(4)}(x - x')\, \delta(\xi - \xi')$$

is the scalar particle Green's function in the field (3) that satisfies the equation

$$(\widetilde{P}_\mu \widetilde{P}^\mu - m^2)\, D(x, x'; \xi, \xi') = -\delta^{(4)}(x - x')\, \delta(\xi - \xi'). \tag{16}$$

Substituting (9) and (15) into (6) and taking account of (16), we obtain the following expression for G:

$$G(x, x'; \xi, \xi') = (\widetilde{\gamma}^\mu \widetilde{P}_\mu + m)\, \widetilde{G}(x, x'; \xi, \xi'),$$

$$\widetilde{G}(x, x'; \xi, \xi') = [1 + 1/4\, \widetilde{\gamma}^0 \widetilde{\gamma}^j e (\widetilde{A}_j(x, \xi) - \widetilde{A}_j(-x', \xi'))(i\partial_3)^{-1}]\, D(x, x'; \xi, \xi'). \tag{17}$$

The relationship (17) permits determination of the function G by means of the known function D. Let us note that a relationship between the electron and the scalar particle Green's functions can be established in an analogous manner in a nonquantized plane wave field. If the c-numerical potentials of the nonquantized field are denoted by $\tilde{a}^\mu(x)$, then this relation is obtained from (17) by formal replacement of $\tilde{A}^\mu(x, \xi)$ by $\tilde{a}^\mu(x)$ and $\tilde{A}^\mu(-x', \xi')$ by $\tilde{a}^\mu(x')$. The latter replacement of the potentials follows from comparing the transformation (13) for $\tilde{A}^\mu$ with the analogous transformation

$$\widetilde{a}^\mu(x)\, \delta^{(4)}(x - x') = \widetilde{a}^\mu(x')\, \delta^{(4)}(x - x') \tag{18}$$

for $\tilde{a}^\mu$.

Let us transform (17) to a more convenient form. To do this, we write D in the form

$$D(x, x'; \xi, \xi') = U(x, \xi)\overset{+}{U}(x', \xi')\int_0^{+\infty} ds\tilde{f}(x, x'; \xi, \xi'; s), \qquad (19)$$

where

$$U(x, \xi) = \exp\Big(iu^0 \sum_{i,\lambda} \varkappa_i \overset{+}{c}_{i\lambda} c_{i\lambda}\Big).$$

As already noted, the function $\tilde{f}$ is determined in [8]. Because of the tedium in writing the explicit form of the function $\tilde{f}$, we do not present it. Substituting (19) into (17) and taking into account that $\tilde{f}$ depends on u^3 in terms of the factor

$$\exp[-i/4 s(u^0 - u'^0)(u^3 - u'^3)],$$

as well as the relationships

$$\overset{+}{U}(x, \xi)\tilde{A}_\mu(x, \xi)U(x, \xi) = \tilde{A}_\mu(0, \xi);$$

$$U(x', \xi')\tilde{A}_\mu(-x', \xi')\overset{+}{U}(x', \xi') = \tilde{A}_\mu(0, \xi'),$$

we finally obtain

$$G(x, x'; \xi, \xi') = U(x, \xi)\overset{+}{U}(x', \xi')(\tilde{\gamma}^\mu\tilde{Q}_\mu + m)\tilde{\tilde{G}}(x, x'; \xi, \xi'), \qquad (20)$$

$$\tilde{\tilde{G}}(x, x'; \xi, \xi') = \int_0^{+\infty} ds\left[1 + e\tilde{\gamma}^0\tilde{\gamma}^j \frac{\tilde{A}_j(0, \xi) - \tilde{A}_j(0, \xi')}{u^0 - u'^0} s\right]\tilde{f}(x, x'; \xi, \xi'; s),$$

where

$$\tilde{Q}_\mu = \Big(i\partial_0 - \sum_{i,\lambda}\varkappa_i \overset{+}{c}_{i\lambda}c_{i\lambda},\ i\partial_1 - e\tilde{A}_1(0, \xi),\ i\partial_2 - e\tilde{A}_2(0, \xi),\ i\partial_3\Big).$$

In conclusion, we note that if electron interaction with all photons except one is neglected in (20) (the monochromatic linearly polarized wave approximation), then we obtain an expression for the Green's electron function in a one photon field. This expression agrees with that found earlier in [10].

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