

Equation (9) gives us the average value of the jump in the energy density, which gives rise to separation of the region D_0 .

From (9), we obtain the following estimates:

- 1) $\sigma \sim 10^{20} \text{ cm}^2$ (sun), $\langle \delta\rho \rangle = \langle \delta\varepsilon \rangle / c^2 \sim 10^7 \text{ g/cm}^3$;
- 2) $\sigma \sim 10^{12} \text{ cm}^2$ (neutron star), $\langle \delta\rho \rangle \sim 10^{15} \text{ g/cm}^3$;
- 3) $\sigma \sim 10^{-66} \text{ cm}^2$ (singularity), $\langle \delta\rho \rangle \sim 10^{93} \text{ g/cm}^3$.

Thus separation of small regions is inhibited by a strong potential barrier. Motion induced in space by a change in the topology of the space itself will require enormous expenditures of energy. The parameters of superdense configurations are close to those for separation from space. This confirms our conclusions, obtained in [1] for a closed model of the universe. Breakdown of connectedness is to be expected in gravitational collapse of massive stars because in this case singularities arise (based on Penrose's theorems [4], p. 242), which entail a singularity of the curvature. It is easy to see that the above picture of the breakdown of connectedness is in many ways similar to the process of gravitational self-closure accompanied by gravitational collapse of homogeneous spherically symmetrical configurations, analyzed in detail in [5] (p. 52). For this reason, it may be expected that singularities form due to breakdown of the connectedness of 3-space.

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THEORY OF SPATIALLY PERIODIC STRUCTURES.

BOSE EXCITATION GREEN'S FUNCTIONS

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We discuss Green's function techniques in the description of spatial ordering viewed as a Bose-Einstein condensation of the density wave of the ordering units.

1. Two approaches can be used in treating spatial ordering in quantum statistics [1, 2]. The first is based on the exclusion principle, according to which units forming the spatially periodic structure (the atoms of a crystallizing liquid or solid solution or the phase separations in a quasiperiodic macrostructure of dissociating alloys) cannot occupy the same spatial position \vec{r} . This allows the representation of the ordering process as a redistribution of fermions over the states \vec{r} . The corresponding Green's function formalism is identical in form to the techniques of Gor'kov in the theory of superconductivity, and has been discussed in [1].

In the second approach, the ordering process is thought of as a redistribution of the Bose density of the ordered structure over values of the wavevector \vec{k} . The condition that this method be applicable is that the Bose amplitudes C_k be statistically independent for different values of \vec{k} [3]. However it can easily be shown that if the total number of structural units is conserved, the C_k satisfy the relation*

$$\sum_{\kappa} \langle |C_{\kappa}|^2 \rangle = \text{const}, \quad (1)$$

*The proof of (1) is carried out in similar fashion to the case of an ordered solid solution [4], where $\text{const} = C(1 - C)N$, C is the concentration, and N is the total number of atoms.

where the angular brackets denote a Gibbs average. In practice, this condition is not important for temperatures much higher than the critical temperature T_c . But the C_k are strongly coupled close to T_c and even more strongly coupled for $T < T_c$. This can be seen in the well-known experimental fact that spatial ordering is always accompanied by the appearance of delta-function singularities (reflections) in the x-ray, electron, and thermal neutron scattering patterns. Since the intensity is proportional to $\langle |C_k|^2 \rangle$ we have a "condensation" of the Bose density at the points $\vec{k} = \vec{k}_g$ corresponding to reflections from the ordered structure. However a marked condensation of the Bose density for a given value of \vec{k} must be accompanied by a decrease in the Bose density for other values of \vec{k} , as follows from (1) and thus the C_k are not independent. Ignoring the fact that the C_k are coupled can lead to nonsensical results of the type $\langle |C_k|^2 \rangle < 0$ [2] for $T < T_c$ [2].

The condition (1) will be satisfied in a natural way if we consider the Bose density of ordered units as quasiparticles whose number is conserved [2]. This approach allows the use of the methods of quantum statistics where the behavior of the delta-function singularities for $T \leq T_c$ is represented as a Bose-Einstein condensation of quasiparticles corresponding to the Bose density. In order to avoid misunderstanding, we point out that the use of operator methods here does not in any sense imply direct quantization of the dynamical variables such as energy, momentum, and so on. Indeed, the problem is purely classical. But it is convenient to use here the machinery of second quantization or Green's functions in the intermediate steps of the calculation. The methods of second quantization have been discussed in the previous paper [2]; here we consider the Green's function formalism, which in addition to its simplicity and rigor allows one to obtain some significant new results, particularly the effect of ordering on the behavior of other quasiparticles in the system, such as electrons or phonons. In the present paper we consider a uniform system in which the Green's function will depend only on the coordinate differences. Then it will be convenient to use the spatial Fourier transform representation. This approach can be generalized easily to the nonuniform case, such as the case of an external field. In Sec. 2 we write out the basic relations; in Sec. 3 the case of zero temperature is considered, while in Sec. 4 we consider nonzero temperatures. Finally the results are discussed in Sec. 5.

2. We consider a solid solution where the ordering process occurs via a redistribution of the concentration $\delta C(\vec{r}) = C(\vec{r}) - C$, where $C(\vec{r}) = 1; 0$ is the occupation number of site \vec{r} by atoms of a given kind, and C is the concentration of these atoms. Following the methods of second quantization, we replace the classical quantity $\delta C(\vec{r})$ in the configurational Hamiltonian

$$H = \frac{1}{2} \sum_{r,r'} W(r-r') \delta C(r) \delta C(r'), \quad (2)$$

where $W(\vec{r} - \vec{r}')$ is the energy [4], by the Hermitian operator

$$\delta \hat{C}(r) = \frac{1}{2} \sqrt{C(1-C)} [\hat{\psi}(r) + \hat{\psi}^+(r)]. \quad (3)$$

The normalization is chosen such that the average of the quasiparticle number operator $\hat{N}(\vec{r}) = \hat{\psi}^+(\vec{r})\hat{\psi}(\vec{r})$ will satisfy the usual condition

$$\sum_r \bar{N}(r) \equiv \sum_r \langle \hat{\psi}^+(r) \hat{\psi}(r) \rangle = N, \quad (4a)$$

where N is the total number of atoms. The presence of Bose-Einstein condensation implies that the field operators $\hat{\psi}^+(\vec{r})$, $\hat{\psi}(\vec{r})^*$ are Bose operators. The field operators are written in the form

$$\hat{\psi}(r) = \sum_{\kappa} \psi_{\kappa}(r) \hat{a}_{\kappa}, \quad \hat{\psi}^+(r) = \sum_{\kappa} \psi_{\kappa}^*(r) \hat{a}_{\kappa}^{\dagger}, \quad (5)$$

*The exclusion principle with respect to the arrangement of atoms over sites \vec{r} referred to in the introduction rigorously leads to the Pauli permutability relation [2]. However in the approximate method of second quantization used here [5] these operators can be considered to be quasi-Bose operators.

where $\psi_{\mathbf{k}}(\vec{\mathbf{r}})$ is the wavefunction; in the expansion of $\delta C(\vec{\mathbf{r}})$ it is given by the plane wave $N^{-1/2} \exp(\pm i\vec{\mathbf{k}}\vec{\mathbf{r}})$; the operators $\hat{a}_{\mathbf{k}}^+$, $\hat{a}_{\mathbf{k}}$ are the quasi-Bose creation and annihilation operators of concentration waves such that they increase or decrease the occupation number in the state $\vec{\mathbf{k}}$ by unity. The concentration wave number operator $\hat{N}_{\mathbf{k}} = \hat{a}_{\mathbf{k}}^+ \hat{a}_{\mathbf{k}}$ satisfies the condition

$$\sum_{\mathbf{k}} \hat{N}_{\mathbf{k}} \equiv \sum_{\mathbf{k}} \langle \hat{a}_{\mathbf{k}}^+ \hat{a}_{\mathbf{k}} \rangle = N, \quad (4b)$$

and this reduces to (1) when the concentration wave amplitude $C_{\mathbf{k}} = N^{-1/2} \sum_r \delta C(\mathbf{r}) \exp(-i\mathbf{k}\mathbf{r})$ is replaced by the operator $\hat{C}_{\mathbf{k}} = V \overline{C(1-C)} \hat{a}_{\mathbf{k}}$.

Using (2) and (3) we can write the Hamiltonian $\hat{H}' = \hat{H} - \mu \hat{N}$ in the form [2]:

$$\begin{aligned} \hat{H}' = \frac{1}{2} \sum_{\mathbf{r}, \mathbf{r}'} U(\mathbf{r} - \mathbf{r}') [\hat{\phi}^+(\mathbf{r}) \hat{\phi}(\mathbf{r}') + \hat{\phi}(\mathbf{r}) \hat{\phi}^+(\mathbf{r}') + \hat{\phi}(\mathbf{r}) \hat{\phi}(\mathbf{r}') + \\ + \hat{\phi}^+(\mathbf{r}) \hat{\phi}^+(\mathbf{r}')] - \mu \sum_{\mathbf{r}} \hat{\phi}^+(\mathbf{r}) \hat{\phi}(\mathbf{r}), \end{aligned} \quad (6)$$

where $U(\vec{\mathbf{r}} - \vec{\mathbf{r}}') = C(1-C)W(\vec{\mathbf{r}} - \vec{\mathbf{r}}')/4$, and $\mu < 0$ is the chemical potential of the concentration wave determined by (4a). The above formalism reduces the problem of ordering of a solid solution to a degenerate Bose gas whose number of particles is conserved.

3. First we consider the case $T = 0$. We decompose the ψ -operators into parts for the condensate $\hat{\Xi}$ and supercondensate $\hat{\psi}'$ (see Sec. 26 of [6]):

$$\begin{aligned} \hat{\phi} = \hat{\Xi} + \hat{\psi}', \quad \hat{\phi}^+ = \hat{\Xi}^+ + \hat{\psi}'^+; \\ \hat{\Xi}(\mathbf{r}) = N^{-1/2} \hat{a}_{\mathbf{k}_s} e^{i\Phi_s(\mathbf{r})}, \quad \hat{\Xi}^+(\mathbf{r}) = N^{-1/2} \hat{a}_{\mathbf{k}_s}^+ e^{-i\Phi_s(\mathbf{r})}; \\ \Xi(\mathbf{r}) = \langle \hat{\phi}(\mathbf{r}) \rangle = \sqrt{N_s} e^{i\Phi_s(\mathbf{r})}, \quad \Xi^*(\mathbf{r}) = \langle \hat{\phi}^+(\mathbf{r}) \rangle = \sqrt{N_s} e^{-i\Phi_s(\mathbf{r})}, \end{aligned} \quad (7)$$

where N_s is the number of concentration waves in the condensate with wavevector $\vec{\mathbf{k}}_s$ and $\Phi_s = \vec{\mathbf{k}}_s \vec{\mathbf{r}}$ is the phase of the concentration wave. Then the Green's functions G and F , F^+ are given by*

$$\begin{aligned} iG(X_1, X_2) = iG(X_1 - X_2) = \langle N | T \hat{\Psi}'(X_1) \hat{\Psi}'^+(X_2) | N \rangle; \\ iF(X_1, X_2) = iF(X_1 - X_2) = \langle N - 2 | T \hat{\Psi}'(X_1) \hat{\Psi}'(X_2) | N \rangle; \\ iF^+(X_1, X_2) = iF^+(X_1 - X_2) = \langle N + 2 | T \hat{\Psi}'^+(X_1) \hat{\Psi}'^+(X_2) | N \rangle, \end{aligned} \quad (8)$$

where we use the standard notation of [6]. The Fourier transforms $G(\omega, \vec{\mathbf{k}})$ and $F^+(\omega, \vec{\mathbf{k}})$ satisfy the equations (see Sec. 41 of [6])

$$\begin{aligned} [\omega - (U(\mathbf{k}) - \mu)] G(\omega, \mathbf{k}) - U(\mathbf{k}) F^+(\omega, \mathbf{k}) = 1; \\ [\omega + (U(\mathbf{k}) - \mu)] F^+(\omega, \mathbf{k}) + U(\mathbf{k}) G(\omega, \mathbf{k}) = 0. \end{aligned} \quad (9)$$

From these equations and the boundary conditions it follows that (see Sec. 33 of [6]):

$$G(\omega, \mathbf{k}) = \frac{\omega + \varepsilon_0(\mathbf{k})}{\omega^2 - \varepsilon^2(\mathbf{k}) + i0}; \quad (10)$$

*Here we use the same notation as for fermions [1] though in the latter case the single-particle boson functions (8) appear as two-particle functions.

$$F^+(\omega, \kappa) = \frac{-U(\kappa)}{\omega^2 - \varepsilon^2(\kappa) + i0}; \quad (11)$$

$$\varepsilon_0(\kappa) = U(\kappa) - \mu; \quad (12)$$

$$\varepsilon(\kappa) = \sqrt{\mu^2 - C(1-C)\omega(\kappa)/2}. \quad (13)$$

According to (10), the Bose branch of the spectrum of elementary excitations is given by (13). The number of elementary excitations for $\vec{k} \neq \vec{k}_S$ is given by the distribution

$$\bar{N}_\kappa \equiv \langle \hat{a}_\kappa^+ \hat{a}_\kappa \rangle = iG(t = -0, \kappa) = -\frac{(\varepsilon(\kappa) + \mu)^2}{4\mu\varepsilon(\kappa)}. \quad (14)$$

Equation (9) can be written in terms of diagrams:

$$\begin{aligned} \leftarrow \rightleftarrows &= \leftarrow + \leftleftarrows \\ \leftleftarrows &= \rightarrow \bullet \end{aligned} \quad (15)$$

where the following graphical notation is introduced

$$\begin{aligned} iG(\omega, \kappa) & \quad iG_0(\omega, \kappa) & \quad iF(\omega, \kappa) & \quad iF^+(\omega, \kappa) - iU(\kappa) \\ \leftleftarrows, & \leftarrow & \leftleftarrows & \leftleftarrows, \bullet \\ -G^T(i^{\nu}_s, \kappa) & -G_0^T(i^{\nu}_s, \kappa) & -F^T(i^{\nu}_s, \kappa) & -\bar{F}^T(i^{\nu}_s, \kappa) - U(\kappa), \end{aligned} \quad (16)$$

Here the upper row corresponds to $T = 0$ and the lower row to $T \neq 0$ (see Sec. 4 below). The unperturbed Green's function is

$$G_0(\omega, \kappa) = (\omega - \varepsilon_0(\kappa) + i0)^{-1}. \quad (17)$$

Thus the quasiparticles are described by the dispersion law (12), and as can be seen from the Hamiltonian (6) in the \vec{k} -representation, correspond to traveling concentration waves with wavefunction $\psi_{\vec{k}}(\vec{r}) = N^{-1/2} \exp(i\vec{k}\vec{r})$. The elementary excitations given by dispersion law (13) correspond to standing concentration waves with wavevectors $\vec{k} \neq \vec{k}_S$. This follows from the fact that the standing waves are linear combinations of the traveling wave operators.

$$\hat{b}_\kappa = u_\kappa \hat{a}_{\kappa_S + \kappa} - v_\kappa \hat{a}_{\kappa_S - \kappa}, \quad \left. \begin{matrix} u_\kappa^2 \\ v_\kappa^2 \end{matrix} \right\} = -\frac{(\varepsilon(\kappa) \mp \mu)^2}{4\mu\varepsilon(\kappa)}, \quad (18)$$

which diagonalize the Hamiltonian (6) (see [2]).

4. The transition to nonzero temperature is carried out by using either retarded and advanced Green's functions or Matsubara Green's functions (see Chap. 4 of [6]). We use the first approach here. When $T \neq 0$, the imaginary part of the retarded Green's function $G^R(\omega, \vec{k})$ is analytic in the upper half of the ω -plane and is given by the imaginary part of (10). Hence

$$\text{Im } G(\omega, \kappa) = \text{cth} \frac{\omega}{2T} \text{Im } G^R(\omega, \kappa) = -\pi(1 + 2n_\kappa)(\omega + \varepsilon_0(\kappa))\delta(\omega^2 - \varepsilon^2(\kappa)),$$

where $n_\kappa = [\exp(\varepsilon(\vec{k})/T) - 1]^{-1}$ is the Bose distribution function of the elementary excitations. Thus for $T \neq 0$ we have

$$G(\omega, \kappa) = \frac{\omega + \varepsilon_0(\kappa)}{\omega^2 - \varepsilon^2(\kappa) + i0} + 2\pi i n_\kappa (\omega + \varepsilon_0(\kappa)) \delta(\omega^2 - \varepsilon^2(\kappa)). \quad (19a)$$

Similarly we obtain for the F^+ function

$$F^+(\omega, \kappa) = \frac{-U(\kappa)}{\omega^2 - \varepsilon^2(\kappa) + i0} + 2\pi i U(\kappa) n_\kappa \delta(\omega^2 - \varepsilon^2(\kappa)). \quad (19b)$$

The number distribution of concentration waves with respect to states $\vec{k} \neq \vec{k}_S$ is given by

$$\bar{N}_\kappa = iG(t = -0, \kappa) = -\frac{(\varepsilon(\kappa) + \mu)^2}{4\mu\varepsilon(\kappa)} - \frac{\varepsilon^2(\kappa) + \mu^2}{2\mu\varepsilon(\kappa)} n_\kappa. \quad (20)$$

The temperature dependent Green's functions are given by the relations*

$$\begin{aligned}
 G^T(\tau_1, \mathbf{R}_1; \tau_2, \mathbf{R}_2) &= G^T(\tau_1 - \tau_2, \mathbf{R}_1 - \mathbf{R}_2) = - \langle T_\tau \hat{\Psi}'^M(\tau_1, \mathbf{R}_1) \hat{\Psi}'^M(\tau_2, \mathbf{R}_2) \rangle; \\
 F^T(\tau_1, \mathbf{R}_1; \tau_2, \mathbf{R}_2) &= F^T(\tau_1 - \tau_2, \mathbf{R}_1 - \mathbf{R}_2) = - \langle T_\tau \hat{\Psi}'^M(\tau_1, \mathbf{R}_1) \hat{\Psi}^M(\tau_2, \mathbf{R}_2) \rangle; \\
 \bar{F}^T(\tau_1, \mathbf{R}_1; \tau_2, \mathbf{R}_2) &= \bar{F}^T(\tau_1 - \tau_2, \mathbf{R}_1 - \mathbf{R}_2) = - \langle T_\tau \hat{\Psi}'^M(\tau_1, \mathbf{R}_1) \hat{\Psi}'^M(\tau_2, \mathbf{R}_2) \rangle,
 \end{aligned}
 \tag{21}$$

where the notation of [6] is used. An analytical form of the equations for G^T , \bar{F}^T can be obtained from the equations for G and F^+ by replacing (it) with τ (in the Fourier representation we replaced ω by $i\zeta_s = 2\pi iTs$, where $s = 0, \pm 1, \pm 2, \dots$). The graphical representation of these equations is given by (15), as in the case $T = 0$ where the diagram elements (16) are now interpreted using the lower row. The solution of these equations

$$G^T(\zeta_s, \kappa) = - \frac{i\zeta_s + \varepsilon_0(\kappa)}{\zeta_s^2 + \varepsilon^2(\kappa)}, \quad \bar{F}^T(\zeta_s, \kappa) = \frac{w(\kappa)}{\zeta_s^2 + \varepsilon^2(\kappa)}
 \tag{22}$$

describes the ordering process over all temperatures. The dispersion law is given in the form (13) as for $T = 0$, but the chemical potential is now a function of temperature whose form is determined by (4).

The definition of the concentration wave distribution function

$$\bar{N}_\kappa = -G^T(\tau = -0, \kappa) = -T \lim_{\tau \rightarrow -0} \sum_{s=-\infty}^{\infty} G^T(\zeta_s, \kappa) e^{-i\zeta_s \tau}
 \tag{23}$$

leads to (20) which diverges for $T \leq T_c$ at the point $\vec{k} = \vec{k}_s$ corresponding to the Bose condensate. In comparison with the earlier definition, \bar{N}_κ as given by (23) has the advantage that not only can the concentration wave distribution (20) be determined, but also the explicit Bose distribution of the standing concentration waves n_κ .

5. The basic features of our results are the appearance of a band gap

$$\Delta = \sqrt{\mu^2 - \mu C(1-C)w(\kappa_s)^2}
 \tag{24}$$

in the Bose branch of the spectrum of elementary excitations (where $w(\vec{k})$ is the Fourier transform of the displacement energy) and a divergence in the concentration wave distribution function. These features are analyzed in detail in [2]; here we discuss only the basic results.

At high temperatures $\varepsilon(\kappa) \approx \varepsilon_0(\kappa)$, $\Delta \approx \varepsilon_0(\kappa_s)$, $\bar{N}_\kappa \approx n_\kappa \sim 1$ and standing and traveling concentration waves are equivalent, being distributed nearly uniformly over the Brillouin zone. This corresponds to the total absence of order. As the temperature is lowered, the gap width decreases, going to zero at the phase transition point

$$T_c = 0,5069 C(1-C)|w(\kappa_s)|.
 \tag{25}$$

The concentration wave distribution function increases near $\vec{k} = \vec{k}_s$ (since the total number of concentration waves is conserved, it must decrease for values of \vec{k} far from \vec{k}_s) and has the singular form $\bar{N}_\kappa \propto (\vec{k} - \vec{k}_s)^{-2}$ for $\vec{k} \rightarrow \vec{k}_s$. The dispersion law for $T \leq T_c$, $\vec{k} \rightarrow \vec{k}_s$ has the form $\varepsilon(\vec{k}) \propto |\vec{k} - \vec{k}_s|$.

In terms of the Goldstone approach, the acoustic (gapless) dispersion law corresponds to long-lived hydrodynamic modes which restore the symmetry of the disordered phase. It can then be shown that the Bose-Einstein condensation leads to a breaking of the symmetry of the system, with the appearance of long-range order in the concentration wave phase distribution, the phase playing the role of a restoring variable (the broken symmetry variable is the concentration wave amplitude). This picture was used here and earlier in [2] a posteriori. Study of this theorem and the correlation function formalism associated with it will require a separate publication.

*We point out the negative sign in front of the F functions (compare with (42.1) of [6]).

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RESONANCE EFFECTS WITH THE PHOTOPRODUCTION OF ELECTRON-POSITRON
PAIRS IN THE FIELD OF A PLANE ELECTROMAGNETIC WAVE

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The differential cross section is obtained for the formation of electron-positron pairs by a photon at a Coulomb center in the field of a plane electromagnetic wave whose intensity $\eta = eA'/m \ll 1$ and frequency $\omega'/m \ll 1$. Expressions are given for the resonance frequencies of the photon for which the emission of a virtual electron (positron) at a mass shell occurs. It is shown that resonance occurs only for electron or positron energies $E \approx m^2/\omega'$. The resonance differential cross section under interference conditions is obtained. It is shown that the resonance cross section can exceed by several orders of magnitude the ordinary cross section for the photoproduction of pairs without a field.

The resonance photoproduction of pairs at nuclei and electrons in an external electromagnetic field was examined in work [1] in the ultrarelativistic case for large-angle scattering of the electron and positron. The single-photon approximation was used for the external field, and it was assumed that the incident photon and the excitation-wave photon are moving counter to each other.

We shall examine the photoproduction of pairs at a Coulomb center in the field of a plane electromagnetic wave in the general relativistic case, which permits the most complete analysis of the resonance region, related to the emission of a virtual electron (positron) at a mass shell. The Feynmann diagrams for this process are presented in Fig. 1. The solid outer lines denote the wave functions of the electron and positron, whose 4-momenta are respectively p^- and p^+ , in the external electromagnetic field [2], the inner line denotes the Green function for the electron (positron) in the plane-wave field [3, 4], $k = \omega n = \omega(1, \mathbf{n})$ is the 4-momentum of the incident photon, q is the transferred momentum, and $k' = \omega' n' = \omega'(1, \mathbf{n}')$ is the 4-momentum of the excitation-wave photon.

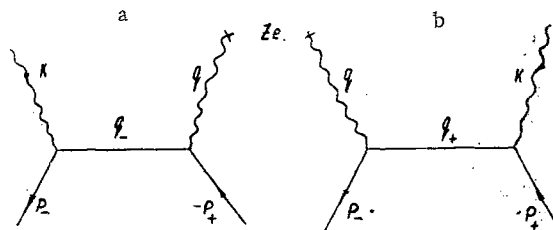


Fig. 1

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