LITERATURE CITED

1. P. V. Strokov and G. I. Flesher, Izv. Vyssh. Uchebn. Zaved., Fiz., No. 5, 26 (1987).

ASYMPTOTIC BEHAVIOR OF THE EFFECTIVE LAGRANGIAN OF QUANTUM ELECTRODYNAMICS FOR PARTICLES WITH ANOMALOUS MOMENTS

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The asymptotic behavior of the effective Lagrangian of quantum electrodynamics (QED) is studied in the framework of a phenomenological treatment of the anomalous moments of the electron and their dependence on an external field.

The present paper is a direct continuation of [1]* and is devoted to a further study of the effective Lagrangian of QED in an arbitrary constant electromagnetic field with the inclusion of the anomalous magnetic (AMM) and electric (AEM) moments of the electron.

As was shown in [1], the effective Lagrangian of QED in the single-loop approximation can be written in the form

$$L[f,\ g]=-\frac{1}{8\pi^2}\int_0^\infty\frac{ds}{s^3}\exp\{-(m^2+2\mu^2\Phi^2)s\}\Big[(es)^2 g\cos(2\mu^2\Phi^*\Phi s)\times$$
$$\times\frac{\operatorname{Re}\operatorname{ch}(exs)}{\operatorname{Im}\operatorname{ch}(exs)}-1-\frac{2}{3}(es)^2 f-(\mu^2 s)^2\Phi^*\Phi-(es)^2 g\sin(2\mu^2\Phi^*\Phi s)\Big]. \tag{1}$$

For a weak field ($E \ll E_0$, $H \ll H_0$, E_0, $H_0 = m^2/|e|$) (1) can be written as an asymptotic expansion in the parameters E/E_0, H/H_0. In the first approximation

$$L_1[f,\ g]=\frac{e^4}{360\pi^2 m^4}(4f^2+7g^2)+\frac{e^2}{8\pi^2 m^2}g\,[(\mu_1^2-\mu_2^2)g-2\mu_1\mu_2 f] \tag{2}$$

the field invariants f and g enter quadratically. The first term in (2) is the well-known Heisenberg–Euler Lagrangian [2], while the second term is a correction to this Lagrangian due to the presence of the AMM and AEM of the electron. Because the electric moment μ_2 has not been observed experimentally, we have, at least for weak fields $\mu_1 \gg \mu_2$. Using this and the fact that in weak fields $\mu_1 = \mu_0\alpha/2\pi$, $\mu_0 = e/2m$, we obtain from (2) the modified Heisenberg–Euler Lagrangian

$$L_1[f,\ g]=\frac{e^4}{360\pi^2 m^4}\left[4f^2+\left(7+20\frac{\mu_1^2}{\mu_0^2}\right)g^2\right], \tag{3}$$

due to the AMM of the electron.

If the field invariants f and g are not both zero, then it is possible to transform to a Lorentz frame in which $\boldsymbol{E}\parallel\boldsymbol{H}$. Then (1) can be rewritten in the form

$$L[E,\ H]=-\frac{1}{8\pi^2}\int_0^\infty\frac{ds}{s^3}\exp\{-(m^2+2\mu^2\Phi^2)s\}\Big[(es)^2 EH\cos(2\mu^2\Phi^*\Phi s)\times \tag{4}$$

*The units and notations are taken from [1].

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$$-\operatorname{cth}(eHs)\operatorname{ctg}(eEs)-1-\frac{1}{3}(es)^2(H^2-E^2)+(\mu^2 s)^2\Phi^*\Phi-(es)^2 EH\quad\sin(2\mu^2\Phi^*\Phi s)\Big].\tag{4}$$

In a constant electric field ($H \to 0$) the effective Lagrangian (4) takes the form ($\mu_2 = 0$):

$$L[E]=-\frac{e^2E^2}{8\pi^2}\int_0^\infty\frac{dx}{x^3}\exp\left\{-\frac{m^2}{|e|E}\left(1-\frac{\mu_1^2E^2}{m^2}\right)x\right\}\left[x\operatorname{ctg}x-1+\frac{1}{3}x^2\right].\tag{5}$$

where $x = |e|Es$. For $E \gg E_0$ we obtain for the real part of $L[E]$, to logarithmic accuracy [3]

$$\operatorname{Re}L_1[E]=-\frac{e^2E^2}{24\pi^2}\ln\left(\frac{|e|E}{m^2}\right),\tag{6}$$

if $\mu_1 = 0$. Equation (6) leads to a negative correction to the energy density of the field $\varepsilon_0 = E^2/8\pi$. For supercritical fields ($E \gg E_0$) the total energy density ε of the fields is

$$\varepsilon=\frac{E^2}{8\pi}-\frac{e^2E^2}{24\pi^2}\ln\left(\frac{|e|E}{m^2}\right).$$

It goes to zero when

$$E=E_{\max}=E_0\exp(3\pi/\alpha)\sim 10^{560}E_0.$$

This was the basis of the assertion in [4] that E_{max} is the limiting possible value of the electric field strength. But in our opinion this conclusion must be regarded as unfounded in the framework of QED because of the existence of the well-known situation of "zero charge." As was shown in [5], the strong external field asymptotic limit in QED can be established in the interval

$$1\ll E/E_0\ll\exp(3\pi/\alpha)\tag{7}$$

and therefore any conclusion about the asymptotic behavior of the physical quantities outside of this interval is incorrect.

We note that (5) is meaningful if the following condition is satisfied:

$$1-\frac{\mu_1^2E^2}{m^2}>0.\tag{8}$$

We consider the condition (8) for supercritical fields $E \gg E_0$ and take into account the dependence of μ_1 on E [6]:

$$\mu_1=\mu_0\frac{\alpha\Gamma(1/3)}{81\chi^{2/3}},\quad\chi=\frac{E}{E_0}\left(1+\frac{p_\perp^2}{m^2}\right)^{1/2},\quad p_\perp\neq 0.\tag{9}$$

This leads to the conclusion that the condition (8) can be violated if

$$E=E_{\max}=E_0\left(\frac{162}{\alpha\Gamma(1/3)}\right)^4\left(1+\frac{p_\perp^2}{m^2}\right)\sim 10^{12}E_0\left(1+\frac{p_\perp^2}{m^2}\right).$$

When $p_\perp = 0$ and for $E \gg E_0$, the AMM is equal to twice the Schwinger quantity $\mu_1 = \mu_0\alpha/\pi$ and hence the condition (8) is violated for

$$E=E_{\max}=E_0\frac{2\pi}{\alpha}\sim 10^3E_0.\tag{10}$$

In an electric field the asymptotic behavior of the effective Lagrangian, with the AMM of the electron taken into account, can be established in the interval

$$1\ll E/E_0<\frac{2\pi}{\alpha}\ll\exp(3\pi/\alpha).\tag{11}$$

The electric field strength $E_{max} = 2\pi E_0/\alpha$ must be considered as a limiting value, such that for this field strength the consistency of the phenomenological treatment of the AMM of the electron breaks down in QED.

When (11) is satisfied, and for supercritical fields, we obtain for the real part of the effective Lagrangian (5), to logarithmic accuracy,

$$\operatorname{Re} L_2[E] = -\frac{e^2E^2}{24\pi^2}\left[\ln\left(\frac{|e|E}{m^2}\right) + \ln\left(1 - \frac{\mu_1^2 E^2}{m^2}\right)^{-1}\right].$$

We note that the condition $\mu_2 = 0$ used in [1] and here in the study of the asymptotic properties of the effective Lagrangian in a magnetic field ($E \to 0$) and in an electric field ($H \to 0$) does not restrict the generality of the treatment if the AEM of the electron is due solely to radiative corrections in the external field. Indeed, in this case $\mu_2 = 0$ from the fact that $g = 0$ [6].

For large values of the parameter $\chi >> 1$

$$\chi = \left[\left(\frac{H}{H_0}\right)^2 \frac{p_\perp^2}{m^2} + \left(\frac{E}{E_0}\right)^2 \left(1 + \frac{p_\perp^2}{m^2}\right)\right]^{1/2}. \tag{12}$$

The AEM of the electron behaves as [6]:

$$\mu_2 = \mu_0 \frac{5\alpha}{3\pi}\left(\frac{EH}{E_0H_0}\right)\chi^{-2}\ln\left(\frac{\chi}{\sqrt{3}\gamma}\right), \quad \ln\gamma = C, \tag{13}$$

where C is the Euler constant, $C = 0.577$. It follows from (12) and (13) that when $E, H >> m^2/|e|$, the AEM can be significantly different from zero only if $H \sim E$. Hence we consider the case when $E = H$. From (4) we then obtain

$$L[E = H] = -\frac{e^2E^2}{8\pi^2}\int_0^\infty \frac{dx}{x^3}\exp(-Ax)\,[x^2\cos(Bx)\operatorname{cth} x \operatorname{ctg} x - 1 + B^2x^2 - x^2\sin(Bx)], \tag{14}$$

where we have introduced the notation

$$A = \frac{E_0}{E}\left[1 + \frac{\mu_1\mu_2}{\mu_0^2}\left(\frac{E}{E_0}\right)^2\right], \quad B = 2\frac{E}{E_0}\left[\left(\frac{\mu_1}{\mu_0}\right)^2 - \left(\frac{\mu_2}{\mu_0}\right)^2\right]. \tag{15}$$

It follows from (9) and (13) that for all values of E/E_0 inside the interval (11), the parameters A, $|B| << 1$. Therefore the asymptotic behavior of the effective Lagrangian for supercritical fields $E >> E_0$ which satisfy (11) is given by (14) for small values of the parameters A and B. In particular, when $\mu_1 = \mu_2 = 0$ we obtain from (14) the Schwinger asymptotic form of the real part of the effective Lagrangian for $E = H$:

$$\operatorname{Re} L_1[E = H] = \frac{m^4}{16\pi^2}\ln\left(\frac{E|e|}{m^2}\right).$$

In the general case the asymptotic form of the real part of (14), to logarithmic accuracy, is

$$\operatorname{Re} L_2[E = H] = \frac{e^2E^2}{16\pi^2}(A^2 - B^2)\ln\left(\frac{1}{A}\right),$$

where the quantities A and B are given by (15), while μ_1 and μ_2 are given by (9) and (13), respectively, with χ given by (12) with $E = H$.

LITERATURE CITED

1. P. M. Lavrov, Izv. Vyssh. Uchebn. Zaved., Fiz., No. 5, 3 (1987).
2. W. Heisenberg and H. Euler, Z. Phys., 38, 714 (1936).
3. A. I. Akhiezer and V. B. Berestetskii, Quantum Electrodynamics [in Russian], Nauka, Moscow (1969).
4. M. Greenman and F. Rohrlich, Phys. Rev., D8, 1103 (1973).
5. B. L. Voronov and G. Yu. Kryuchkov, Teor. Mat. Fiz., 41, 40 (1979).
6. V. I. Ritus, Zh. Éksp. Teor. Fiz., 75, 1560 (1978).