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A Stochastic Approach to Optimum Decomposition of Cyclic Patterns in Sedimentary Processes[1]

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This paper examines the issues of sedimentary cycles by means of reversible Markov chains. Two types of cyclic patterns in sedimentary processes are considered in terms of symmetric cycles (ABCDCBA) and asymmetric cycles (ABCDABCD). By introducing concepts of reversibility and unidirectionality, a general solution is given for decomposing all possible cyclic patterns of these two types existing in a sedimentary sequence. On the basis of two new measures f_R and f_U, the most probable trends in a sequence can be identified in an optimum way. Effective and reliable use of the technique proposed here is demonstrated by a case study.

KEY WORDS: Markov chain, reversible process, unidirectional process, cyclic pattern, sedimentary sequence.

INTRODUCTION

The concept of cyclic properties in sedimentary processes has been long accepted and widely examined (Gingerich, 1969; Schwarzacher, 1969; Merriam, 1970; Hattori, 1976; Wang, 1981). Sedimentary cycles are so-called chiefly because of the occurrence of many lithologies in a sedimentary succession which seems to be cyclic to some extent. The study of cyclic properties in a sedimentary section is far from trivial because it often can show to a certain extent associations between changes of lithological facies and sedimentary environments favorable to deposition of mineral resources.

Several stochastic models have been proposed and employed in the past, serving to identify the cyclic patterns in a sedimentary sequence (Hattori, 1976; Wang, 1981; Pan, 1987). A common feature among these methodologies is that only one prevailing cyclic pattern can be discriminated from a lithological sequence by means of somewhat ambiguous criterion although cycles of asymmetric type can be recognized partially by Pan's method. Cyclic patterns ex-

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hibited in actual lithological successions, however, are much more complicated mainly due to complex mechanisms of sedimentary processes, superimposed geological structures, as well as other unobservable stochastic effects. Multi-patterns of sedimentary cycles can, therefore, be observed in many sedimentary sections even though a dominant trend might be essential to formation of a lithological succession. Thus, it would be worthwhile to identify all probably existing cyclic patterns from which some dominant sedimentary trends could be distinguished by some optimum criteria.

Applications of Markov chains or Markov processes to sedimentary problems have been demonstrated during the past several decades. Recent work regarding this study can be found in the ideal granite model (Vistelius, 1976, 1981; Vistelius and Harbaugh, 1980). The study of reversibility in a sedimentary process, however, has not appeared, even though the theory of mathematical aspects about reversible Markov chains has been classified (Qian and Hou, 1977).

Theoretically, the following two cyclic types are generally considered: a symmetric pattern (*ABCDCBA*) and an asymmetric pattern (*ABCDABCD*) (Hattori, 1976). In most actual cases, however, the two typical cycles are not completely separated; instead, they are mixed to a certain extent in some complicated ways. Moreover, multipatterns of the same type often can be observed in an actual sequence. Therefore, a practical issue is how to identify these two cyclic types and separate different patterns of the same type in an actual sedimentary section. The goal of this paper is to explore a feasible solution to the problem on the basis of the reversible concept.

BASIC CONCEPTS AND EXAMPLES

In this section, some basic definitions are presented, and two typical examples are given to demonstrate the basic ideas that will be developed theoretically later and that will help to understand some essential features of cyclic patterns existing in a homogeneous sedimentary process. (Note: T in A^T represents the transition of vector or matrix A; I represents the unit matrix.)

Let E denote the limit set of stratum states identified in a sedimentary process $X(n)$ $(n > 0)$ which is assumed to be homogeneous Markov chain in the state space E with initial distribution $u^T = (u_1, u_2, \cdots, u_N)$ (N is the number of states in E). The matrix of upward transitional probabilities is denoted by $P = (p_{ij})$, and the corresponding information matrix for process $X(n)$ is $C = (u_i p_{ij})$.

Definition 1. $Q = (q_{ij})$ is called a reversible cyclic matrix (RCM), if $q_{ij} = q_{ji}$ (q_{ij}, elements of Q; $i, j \in E$); $R = (r_{ij})$ is called a unidirectional cyclic matrix (UCM), if $r_{ij} r_{ji} = 0$ and $\Sigma_i r_{ij} = \Sigma_i r_{ji}$ $(i, j \in E)$, (assume q_{ij}, $r_{ij} \geq 0$). A sedimentary process is called a reversible process if $C = \Sigma_i Q_i$; a sedimentary process is called a unidirectional process if $C = \Sigma_i R_i$ (Q_i is RCM; R_i is UCM).

The following example shows that the information matrix C may be decomposed into the sum of a RCM and a UCM; the general result is presented in a later section.

Example 1. Suppose that the state space $E = (1, 2, 3)$ and the matrix of upward transitional probabilities is

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = (P_1, P_2, P_3)$$

By definition

$$u_j = u^T P_j \qquad j \in E$$

where $u^T = (u_1, u_2, u_3)$ is the initial distribution, so that

$$u_1 p_{11} + u_2 p_{21} + u_3 p_{31} = u_1$$
$$u_1 p_{12} + u_2 p_{22} + u_3 p_{32} = u_2$$
$$u_1 p_{13} + u_2 p_{23} + u_3 p_{33} = u_3$$

that is

$$u_2 p_{21} + u_3 p_{31} = u_1(p_{12} + p_{13})$$
$$u_1 p_{12} + u_3 p_{32} = u_2(p_{21} + p_{23})$$
$$u_1 p_{13} + u_2 p_{23} = u_3(p_{31} + p_{32})$$

Hence

$$u_1 p_{12} - u_2 p_{21} = u_2 p_{23} - u_3 p_{32} = u_3 p_{31} - u_1 p_{13}$$

If we also assume that

$$u_1 p_{12} - u_2 p_{21} = a$$

the following result can be obtained

$$u_i p_{i,i+1} = u_{i+1} p_{i+1,i} + a \qquad i \in E \qquad i + 1 = 1 \qquad \text{when} \qquad i = 3$$

Suppose $a > 0$, and denotes

$$d_1 = u_2 p_{21} \qquad d_2 = u_3 p_{32} \qquad d_3 = u_1 p_{13} \qquad \text{then}$$

$$C = (u_i p_{ij}) = \begin{bmatrix} u_1 p_{11} & d_1 + a & d_3 \\ d_1 & u_2 p_{22} & d_2 + a \\ d_3 + a & d_2 & u_3 p_{33} \end{bmatrix}$$

$$= \begin{bmatrix} u_1p_{11} & d_1 & d_3 \\ d_1 & u_2p_{22} & d_2 \\ d_3 & d_2 & u_3p_{33} \end{bmatrix} + \begin{bmatrix} 0 & a & 0 \\ 0 & 0 & a \\ a & 0 & 0 \end{bmatrix}$$

$$= Q + R$$

Clearly, the first matrix in the decomposition is a RCM, whereas the second is a UCM.

Definition 2. Matrix $Q^{(l)} = (q_{ij}^{(l)})$ is called a reversible flow (RF) in sedimentary process $X(n)$ $(n > 0)$ if $i_1 \neq i_2 \neq, \cdots, \neq i_m$ and f_R exist such that

$$q_{ij}^{(l)} = q_{ji}^{(l)} = f_R \quad i = i_k \quad j = i_{k+1} \quad k = 1, 2, \cdots, m-1 \qquad i, j \in E$$

$$= 0 \qquad \text{otherwise}$$

where f_R is defined as the reversible degree of states $i_1, \cdots, i_m \in E$, $m > 2$.

Definition 3. Matrix $R^{(l)} = (r_{ij}^{(l)})$ is called unidirectional flow (UF) in sedimentary process $X(n)$ $(n > 0)$, if $i_1 \neq i_2 \neq, \cdots, \neq i_m$ and $f_U > 0$ exist such that

$$r_{ij}^{(l)} = f_U \qquad i = i_k \qquad j = i_{k+1} \qquad k = 1, 2, \cdots, m, \; i_{m+1} = i_1$$

$$= 0 \qquad \text{otherwise}$$

where f_U is defined as the unidirectional degree of states $i_1, i_2, \cdots, i_m \in F$, $m > 2$.

Definitions 2 and 3 correspond exactly to the symmetric type of cycles (*ABCDCBA*) and the asymmetric type of cycles (*ABCDABCD*) in a sedimentary sequence, respectively. These two types of cyclic patterns are assumed to be the only forms existing in sedimentary sections. This consideration obviously is an approximation to the real world. Magnitudes of f_R and f_U values characterize intensities of these two types of cyclic flows. The larger the values are, the stronger the corresponding cyclic trends will be.

Example 1 shows a special case because the process has a unique pattern of UF in three states so that the UCM is equal to the UF. Most practical sedimentary sequences, however, are much more complicated, especially when the number of stratum states increases. This statement is explained by the following example.

Example 2. Suppose that we have stratigraphic section from which 8 states are identified, and that the matrix of upward transitional probability observed is

$$P = (p_{ij}) = \begin{array}{c} \left[\begin{array}{cccccccc} 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.25 & 0.25 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0.25 & 0 & 0 & 0 & 0 & 0.25 & 0.5 & 0 \\ 0 & 0 & 0.67 & 0 & 0 & 0 & 0.33 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.67 & 0 & 0.33 \end{array}\right] \end{array} \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \end{array}$$

The initial (stationary) distribution obtained is

$$u^T = (u_1, u_2, \cdots, u_8)$$
$$= (1/11, 1/11, 2/11, 1/11, 1/11, 2/11, 3/22, 3/22)$$

Following the same procedure performed in example 1, decomposition of the information matrix is

$$C = (u_i p_{ij}) = (1/22)I + (1/22) \left[\begin{array}{cccccccc} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \end{array}\right]$$

$$= (1/22)I + R$$

Clearly, this process does not contain any RF; that is, the process is a purely unidirectional cyclic one. Let us look at the relationship between UCM (R) and UF $[R^{(l)}]$. Obviously, R can be decomposed in the following two ways

(1)

$$R = \frac{1}{22}\begin{bmatrix} 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix} + \frac{1}{11}\begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0 \end{bmatrix}$$

$$= R^{(1)} + R^{(2)}$$

(2)

$$R = (1/22)\begin{bmatrix} 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&0&0&0&1&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$+ (1/22)\begin{bmatrix} 0&1&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0 \end{bmatrix}$$

$$+ (1/22)\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$= R^{(3)} + R^{(4)} + R^{(5)}.$$

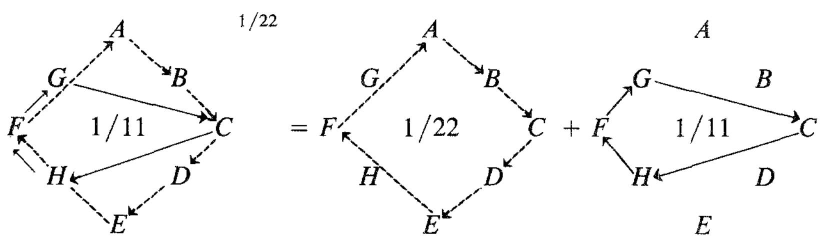
In the above decompositions, $R^{(l)}$ $(l = 1, \cdots, 5)$ are unidirectional flows. Therefore, in this case, a UCM is composed of several unidirectional flows. Another feature in this example is that decomposition of the UCM is not unique. In decomposition (1), the result shows that two asymmetric cycles exist in the process

$$R^{(1)}\colon A \longrightarrow B \longrightarrow C \longrightarrow D \longrightarrow E \longrightarrow F \longrightarrow A \qquad (f_U = 1/22)$$

and

$$R^{(2)}\colon C \longrightarrow H \longrightarrow F \longrightarrow G \longrightarrow C \qquad (f_U = 1/11)$$

These decompositions can be shown by the following



R $\qquad$ $R^{(1)}$ $\qquad$ $R^{(2)}$

Values of f_U indicate that the cyclic trend $R^{(2)}$ is stronger than $R^{(1)}$. Decomposition (2) indicates that three asymmetric cycles exist

$$R^{(3)}\colon C \longrightarrow D \longrightarrow E \longrightarrow F \longrightarrow G \longrightarrow C \qquad (f_U = 1/22)$$

$$R^{(4)}\colon A \longrightarrow B \longrightarrow C \longrightarrow H \longrightarrow F \longrightarrow A \qquad (f_U = 1/22) \qquad \text{and}$$

$$R^{(5)}\colon C \longrightarrow H \longrightarrow F \longrightarrow G \longrightarrow C \qquad (f_U = 1/22)$$

Examining these cycles carefully, however, we find that they are not independent. Clearly combination of $F \longrightarrow G \longrightarrow C$, a part of the transitional pattern in $R^{(3)}$, and $C \longrightarrow H \longrightarrow F$, a part of the transitional pattern in $R^{(4)}$, produces an asymmetric cycle $C \longrightarrow H \longrightarrow F \longrightarrow G \longrightarrow C$, which is exactly the same cycle as $R^{(5)}$. Consequently, we have only two independent patterns, among three, in decomposition (2). Moreover, $R^{(5)}$ is the same pattern as $R^{(2)}$, whereas the combination of $R^{(3)}$ and $R^{(4)}$ can produce pattern $R^{(1)}$.

A GENERAL DECOMPOSITION OF CYCLE TYPES FOR A HOMOGENEOUS SEDIMENTARY PROCESS

According to examples presented in the previous section, a homogeneous sedimentary process generally has multipatterns of cycles, each of which can be separated; the resultant patterns are independent although cyclic patterns can be decomposed in different ways. Example 1 indicates that the information matrix can be expressed uniquely by the addition of one RCM and one UCM, which means that a general homogeneous sedimentary process is a mixture of a reversible cyclic process and a unidirectional cyclic one, and if $d_1 = d_2 = d_3 = 0$, the process is a pure unidirectional one. In this section, these results are proved in more general terms. For the sake of simplicity, we can assume that all diagonal elements in information matrix (C), RCM, as well as UCM, are zeroes.

Theorem 1. Any homogeneous sedimentary process $X(n)$ $(n > 0)$ with information matrix $C = (u_i p_{ij})$ can be decomposed uniquely into the addition of a reversible cyclic process and a unidirectional cyclic process. That is

$$C = Q + R$$

where Q is the information matrix of a reversible cycle process, whereas R is the information matrix of a unidirectional cycle process. Proof for the theorem is given in the Appendix.

Theorem 2. Any homogeneous reversible sedimentary process can be decomposed uniquely into the sum of a limited number of independent reversible flows; and any homogeneous unidirectional process can be decomposed uniquely into the sum of a limited number of independent unidirectionary flows, that is

$$Q = \sum_{l=1}^{T} Q^{(l)} \qquad \text{and} \qquad R = \sum_{l=1}^{S} R^{(l)}$$

where $Q^{(l)}$ and $R^{(l)}$ are RF and UF, respectively; T and S are limiting positive integers.

The first part of this theorem seems straightforward, so proof is only given to the second part. The performance of the proof is important because it provides an explicit approach to identify the existing cyclic flows in a sedimentary sequence.

Naturally, $R \neq (0)$ is always assumed. This simply leads to a fact that $j_0(\in E)$ and $j_1(\in E)$ exist such that $r_{j_0 j_1} > 0$. Moreover, according to the definition of UCM, $j_2(\neq j_0)$ must exist in E such that $r_{j_1 j_2} > 0$. In a similar way, if we begin with $r_{j_1 j_2} > 0$, a state $j_3(\neq j_1)$ can be found such that $r_{j_2 j_3} > 0$. Analogously, a series of states can be identified in E

$$j_0, j_1, \cdots j_k \qquad k > 2$$

where k is a first number in the sequence such that $j_k \in \{ j_0, j_1, \cdots, j_{k-2} \}$.

In general, if $j_k = j_{k_0}$ where $k_0 \in \{0, 1, 2, \cdots, k-2\}$, we delete states $j_0, j_1, \cdots, j_{k_0 - 1}$, and denote

$$i_1 = j_{k_0} \qquad i_2 = j_{k_0+1}, \cdots, i_M = j_{k-1}, i_{M+1} = j_{k_0} = j_k = i_1$$

so that states $i_1, i_2, \cdots, i_M$ are different from each other, and the following inequalities are guaranteed

$$r_{i_1 i_2}, r_{i_2 i_3}, \cdots, r_{i_{M-1} i_M}, r_{i_M i_1} > 0$$

In the following step, let

$$f_U^{(1)} = \min_{1 \leq i \leq M} (r_{i_k i_{k+1}}) > 0$$

denote set

$$D = [(i_1, i_2), (i_2, i_3), \cdots, (i_{M-1}, i_M), (i_M, i_1)] \qquad \text{and}$$

$$\begin{aligned} r_{ij}^{(1)} &= f_U^{(1)} \quad (i, j) \text{ in } D \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

According to the definition, $R^{(1)} = [r_{ij}^{(1)}]$ is a unidirectional flow. Denote

$$R_1 = [r_{ij(1)}] = (r_{ij}) - (r_{ij}^{(1)})$$

then, R_1 can be shown to be also a UCM, and the number of nonzero elements in R_1 is less than that in R. If $R_1 = (0)$, the procedure would be terminated; otherwise, repeating the steps above, a second matrix of cyclic flow can be found: $R^{(2)} = [r_{ij}^{(2)}]$, and the corresponding cyclic measure is $f_U^{(2)} > 0$. Again denote

$$R_2 = [r_{ij(2)}] = [r_{ij(1)}] - [r_{ij}^{(2)}]$$

The same performance will not be terminated until a number S is found such that $[r_{ij}^{(s+1)}] = (0)$, that is

$$
\begin{aligned}
R = (r_{ij}) &= [r_{ij}^{(1)}] + R_1 \\
&= [r_{ij}^{(1)}] + [r_{ij}^{(2)}] + \cdots + [r_{ij}^{(s)}] \\
&= \sum_{l=1}^{S} R^{(l)}
\end{aligned}
$$

where $R^{(l)} = [r_{ij}^{(l)}]$ and the corresponding measures are $f_U^{(1)}, f_U^{(2)}, \cdots, f_U^{(s)}$.

Clearly the decomposition above is independent and unique. The same procedure can prove the reversible case in the theorem. From the two theorems above, the following corollary is obvious:

Corollary: Any homogeneous sedimentary process can be decomposed uniquely into the sum of T independent reversible flows and S independent unidirectional flows (T, $S < \infty$).

This result indicates that any sedimentary sequence contains a limited number of independent symmetric and asymmetric cycles among available stratum states, and these cycles can be identified and separated.

In summary, the following can be recognized on the basis of previous definitions and conclusions.

1. A homogeneous sedimentary process is a pure reversible one if and only if no UF exists; that is, $f_U = 0$.
2. A homogeneous sedimentary process is a pure unidirectional one if and only if only one UF exists and $Nf_U = 1$.
3. A homogeneous sedimentary process is a mixture of UF and RF, if and only if $1 \le S < \infty$ UFs, $1 \le T < \infty$ RFs, and $0 < f_U, f_R < 1/N$ exist.

Some important facts should be demonstrated also about initial distributions. A sedimentary process $X(n)$ will be stationary if no significant depositional discontinuity or structural nonconformity occurs in the sequence. Moreover, the universally assumed homogeneity indicates that the initial distribution should be equal to the stationary distribution which, in turn, is equal to the limited distribution in this situation. The statement can be proved mathematically. This result is substantial because it provides an alternative means to estimate initial distributions, instead of dealing directly with stationary distributions.

PROCEDURE FOR IDENTIFYING THE MOST PROBABLE CYCLE TYPES IN A SEDIMENTARY SEQUENCE

Most actual sedimentary processes probably are mixtures between pure reversible and pure unidirectional cyclic parts. Two types of cyclic patterns also can be observed in one actual sequence because of uncontrollable probabilistic effects even though the process does have only one type of cycle. Effective use of Markov analysis will help us identify the real cyclic patterns by straining ''white noises'' out of the process. Optimum use of stochastic techniques, however, will assist us to distinguish the most probable cyclic trends from a large number of possible patterns on the basis of some quantitative criteria. Basically, theoretical solutions to the issues have been established in previous sections, but the following algorithm is necessary for us to search economically and efficienctly for the solutions.

1. Estimate the matrix of upward transitional probabilities (P) from observed frequencies of upward transitions on an actual sedimentary section.
2. Calculate the limited distribution which can be regarded as an estimate of the stationary distribution (u).
3. Decompose information matrix $C = (u_i p_{ij})$ into the sum of a RCM(Q) and a UCM(R) on the basis of Theorem 1.
4. Search for possible cyclic patterns by following the description given below (taking UCM as an example).

First, suppose that the number of nonzero elements in R is N_0. Define a measure as a cutoff, for instance

$$c_r = \bar{r} + r_0 s_r, \qquad \text{where}$$

$$s_r^2 = N_0^{-1} \sum_{i,j \in E} (r_{ij} - \bar{r})^2 \qquad \bar{r} = N_0^{-1} \sum_{i,j \in E} r_{ij}$$

and r_0 is a nonnegative constant being determined, e.g., 0, 0.5, etc.

Second, simplifying matrix $R = (r_{ij})$ to $\bar{R} = (\bar{r}_{ij})$ by deleting those elements in R that are less than c_r. Arrange nonzero elements in $\bar{R}$ in order from large to small in a vector w

$$w^T = (\bar{r}_{i_1 j_1}, \bar{r}_{i_2 j_2}, \cdots, \bar{r}_{i_m j_m})$$

where $\bar{r}_{i_1 j_1} \geq \cdots \geq \bar{r}_{i_m j_m}$ and m is the number of nonzero elements in $\bar{R}$.

A criterion is necessary for ranking elements with equal values. When $\bar{r}_{ij} = \bar{r}_{lk}$, if $i + j > l + k$ or $i + j = l + k$ and $i > l$, the two elements will be arranged on the order $(\bar{r}_{lk}, \bar{r}_{ij})$; otherwise, they will be ordered to be $(\bar{r}_{ij}, \bar{r}_{lk})$.

Finally, beginning with w, find all possible UFs and calculate measures f_U by means of the technique described in the previous sections.

Obviously, the procedure can be used also to search for possible RFs in Q provided that some notations are changed.

5. Identify the most probable cyclic patterns in the sedimentary sequence, making use of the criteria of measures f_R and f_U.

A CASE STUDY

As an illustration of the algorithm and effect of the technique proposed, a case study is presented. Data recorded by drilling in Shanxi are employed (Wang, 1981).

The region is covered largely by Lower Jurassic sediments. River facies are extensively developed in most parts of the region whereas lake and marsh facies are found also on the alluvial plain. A striking feature of the Jurassic sedimentary formation is the occurrence of fine-sandy purple mudstone containing some perthitic lenses. Mineral content and structures permit five stratum states to be distinguished in the region as follows: coarse sandstone (A), fine sandstone (B), mudstone and clay siderite shale (C), sandy shale (D), carbonaceous shale and coal (E).

The total number of strata observed in a typical borehole is 89; the upward transitional frequencies recorded are

$$F = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 7 & 2 & 6 & 2 \\ 7 & 0 & 2 & 12 & 1 \\ 2 & 1 & 0 & 1 & 3 \\ 1 & 6 & 1 & 0 & 15 \\ 6 & 8 & 2 & 4 & 0 \end{bmatrix} \end{array} \begin{array}{c} \text{sum} \\ 17 \\ 22 \\ 7 \\ 23 \\ 20 \end{array}$$

The matrix of upward transitional probabilities therefore is estimated as

$$\hat{P} = \begin{bmatrix} 0.0 & 0.41 & 0.12 & 0.35 & 0.12 \\ 0.32 & 0.0 & 0.09 & 0.55 & 0.04 \\ 0.29 & 0.41 & 0.0 & 0.14 & 0.43 \\ 0.04 & 0.27 & 0.04 & 0.0 & 0.65 \\ 0.30 & 0.40 & 0.10 & 0.20 & 0.0 \end{bmatrix}$$

By definition, the limited distribution is

$$\hat{u}^T = \begin{array}{c} (0.18 \quad 0.25 \quad 0.08 \quad 0.26 \quad 0.23) \\ \;\; A \qquad\; B \qquad\; C \qquad\; D \qquad\; E \end{array}$$

The information matrix C is calculated to be

$$\hat{C} = (\hat{u}_i \hat{p}_{ij}) = \begin{bmatrix} 0.0 & 0.07 & 0.02 & 0.06 & 0.02 \\ 0.08 & 0.0 & 0.02 & 0.14 & 0.01 \\ 0.02 & 0.01 & 0.0 & 0.01 & 0.03 \\ 0.01 & 0.07 & 0.01 & 0.0 & 0.17 \\ 0.07 & 0.09 & 0.02 & 0.05 & 0.0 \end{bmatrix}$$

According to Theorem 1, $\hat{C}$ can be decomposed into the sum of a RCM(Q) and a UCM(R) as follows

$$Q = (q_{ij}) = \begin{bmatrix} 0 & 0.07 & 0.02 & 0.01 & 0.02 \\ & 0.0 & 0.01 & 0.07 & 0.01 \\ & \cdot & 0.0 & 0.01 & 0.02 \\ & \cdot & \cdot & 0.0 & 0.05 \\ & \cdot & \cdot & \cdot & 0.0 \end{bmatrix} \quad \text{and}$$

$$R = (r_{ij}) = \begin{bmatrix} 0.0 & 0. & 0.0 & 0.05 & 0.0 \\ 0.01 & 0.0 & 0.01 & 0.07 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.01 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.12 \\ 0.05 & 0.08 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

In the next step, possible cyclic patterns are recognized from matrices Q and R. For safety's sake, we take r_0 as zero. Therefore, calculations are needed only on average values $\bar{q}$ for Q and $\bar{r}$ for R. The results obtained are $\bar{q} = 0.03$ and $\bar{r} = 0.04$; the cutoffs, accordingly, are: $c_q = 0.03$ and $c_r = 0.04$ for matrices Q and R, respectively.

Replacing elements in Q which are less than the cutoff c_q by zeroes, Q is simplified to be

$$\overline{Q} = (\overline{q}_{ij}) = \begin{bmatrix} 0 & 0.07 & 0 & 0.0 & 0.0 \\ & 0.0 & 0 & 0.07 & 0.0 \\ & & 0 & 0.0 & 0.0 \\ & & & 0.0 & 0.05 \\ & & & & 0.0 \end{bmatrix}$$

Nonzero elements in $\overline{Q}$ are arranged into the vector

$$v_1^T = (\overline{q}_{12}, \overline{q}_{21}, \overline{q}_{24}, \overline{q}_{42}, \overline{q}_{45}, \overline{q}_{54})$$

Thus, the first RF is found to be

$$\mathrm{RF}(1)\colon A \longrightarrow B \longrightarrow D \longrightarrow B \longrightarrow A\ [\overline{Q}^{(1)}]$$

and the corresponding measure $f_R^{(1)}$ is

$$f_R^{(1)} = \min(\overline{q}_{12}, \overline{q}_{24}) = 0.07$$

Then, calculate

$$\overline{Q}_1 = \overline{Q} - \overline{Q}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0.0 \\ & 0 & 0 & 0 & 0.0 \\ & & 0 & 0 & 0.0 \\ & & & 0 & 0.05 \\ & & & & 0.0 \end{bmatrix}$$

$\overline{Q}_1$ now contains only two states. In stratigraphic analysis, this situation is ignored generally (only those cycles in which more than two states are involved are considered in this analysis). Accordingly, this sequence contains only one possible reversible cycle.

For decomposition of R, the same procedure is followed. By cutoff c_r, R is simplified to be

$$\overline{R} = (\overline{r}_{ij}) = \begin{bmatrix} 0.0 & 0.0 & 0 & 0.05 & 0.0 \\ 0.0 & 0.0 & 0 & 0.07 & 0.0 \\ 0.0 & 0.0 & 0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0 & 0.0 & 0.12 \\ 0.05 & 0.08 & 0 & 0.0 & 0.0 \end{bmatrix}$$

Vector w_1 is constructed as

$$w_1^T = (\bar{r}_{45}, \bar{r}_{52}, \bar{r}_{24}, \bar{r}_{14}, \bar{r}_{51})$$

Therefore, the first unidirectional flow is found to be

$$\text{UF(1)}: B \longrightarrow D \longrightarrow E \longrightarrow B \longrightarrow D \longrightarrow E\,[\bar{R}^{(1)}] \quad \text{and}$$

$$f_U^{(1)} = \min(\bar{r}_{45}, \bar{r}_{52}, \bar{r}_{24}) = 0.07$$

Matrix $\bar{R}_1$ next is calculated

$$\bar{R}_1 = \bar{R} - \bar{R}^{(1)} = \begin{bmatrix} 0.0 & 0.00 & 0 & 0.05 & 0.00 \\ 0.0 & 0.00 & 0 & 0.00 & 0.00 \\ 0.0 & 0.00 & 0 & 0.00 & 0.00 \\ 0.0 & 0.00 & 0 & 0.00 & 0.05 \\ 0.05 & 0.01 & 0 & 0.00 & 0.00 \end{bmatrix}$$

Furthermore, $\bar{R}_1$ is simplified by cutoff c_r to be

$$\bar{R}_2 = \begin{bmatrix} 0.00 & 0 & 0 & 0.05 & 0.00 \\ 0.00 & 0 & 0 & 0.00 & 0.00 \\ 0.00 & 0 & 0 & 0.00 & 0.00 \\ 0.00 & 0 & 0 & 0.00 & 0.05 \\ 0.05 & 0 & 0 & 0.00 & 0.00 \end{bmatrix}$$

The second UF can be obtained from the following vector

$$w_2^T = (\bar{r}_{14}, \bar{r}_{51}, \bar{r}_{45}).$$

Hence,

$$\text{UF(2)}: D \longrightarrow E \longrightarrow A \longrightarrow D \longrightarrow E \longrightarrow A\,[\bar{R}^{(2)}]$$

and the corresponding measure f_U is

$$f_U = \min(\bar{r}_{14}, \bar{r}_{51}, \bar{r}_{45}) = 0.05$$

Checking that

$$\bar{R}_2 = \bar{R}_1 - \bar{R}^{(2)} = (0)$$

we have two possible unidirectional flows in this sequence.

In light of cyclic measures f_R and f_U, patterns RF(1) and UF(1) should be regarded as major trends in sedimentation. This indicates that the process is a mixture of two types of cycles. Note that $\bar{Q}$ can be decomposed in other ways, but results will be the same as that given above so long as independence is guaranteed among the different separated flows.

Changes from coarse sandstone and fine sandstone to sandy shale are normal transitions of lithologic facies in sedimentary environments, such as rivers and lakes. So symmetric cycle RF(1) represents normal changes of facies. The transition of facies from sandy shales to carbonaceous shales also is natural in lake and marsh environments. Thus, asymmetric cycle UF(1) is another representative sedimentation pattern in this sequence. Moreover, this cycle might be more important because it is associated with carbonaceous shale and coal, one of the major interests of this stratigraphic study. Change from carbonaceous shale to coarse sandstone, however, is somewhat odd. Probably, UF(2) represents a disturbance of some unobservable factors during the normal sedimentation process.

In addition, sandy shale has the largest limited probability (0.26), which indicates that it is a dominant state in the sequence. Once its deposition terminates in a period, it would give priority to the carbonaceous shale for its successor. Analogically, carbonaceous shale acted as an "absorber" of the sandy shale.

CONCLUDING REMARKS

Strictly speaking, perfectly reversible sedimentary processes rarely exists in the real world. Introduction of the concept, however, enables us to discriminate among existing asymmetric cycles in a sedimentary sequence by measuring the departure of an actual sequence from the ideal situation. Similarly, few pure unidirectional processes can be observed in actual studies. Use of the concept, again, is beneficial because it allows identification of probable symmetric cycles in a sequence by gauging deviation of the actual process from the extreme case.

In general, a sedimentary process is a mixture of reversible and unidirectional ones. In many cases, however, one of these different types may play a dominant role in a sequence. These different situations can be clearly distinguished by measures f_R and f_U that mutually are comparable. If max $\{f_R\} \simeq$ max $\{f_U\} >$ max $\{c_r, c_q\}$, the sequence is a significant mixture of the two types; otherwise, if max $\{f_R\} \simeq$ max $\{f_U\} \leq$ min $\{c_r, c_q\}$, the sequence would not have significant cyclic patterns even though some weak trends might exist. If max $\{f_R\} \gg$ max $\{f_U\}$, the sequence is dominated by reversible cycles; otherwise, if max $\{f_U\} \gg$ max $\{f_R\}$, the major trend in the sequence would be unidirectional cycles.

Definition of cutoffs c_r and c_q are flexible. If they are zero, all possible RFs and UFs can be identified independently. But this treatment will increase calculations, especially when the number of states is large. In this case, use of cutoffs (c_r and c_q) will be effective not only to avoid a heavy burden of calculation, but also eliminate trivial or unreal cycles in a sequence.

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APPENDIX: PROOF FOR THEOREM 1

Given the initial distribution

$$u^T = (u_1, u_2, \cdots, u_N)$$

and the matrix of the upward transitional probabilities as

$$P = (p_{ij})$$

The information matrix is

$$C = (u_i p_{ij})$$

Let

$$u_i p_{ij}^{(l)} = \min \{u_i p_{ij}, u_j p_{ji}\} \qquad (i, j) \in E$$

Obviously

$$0 \le p_{ij}^{(l)} \le p_{ij}$$

$$u_i p_{ij}^{(l)} = u_j p_{ji}^{(l)} \quad (i, j) \in E$$

Again let

$$r_{ij} = u_i p_{ij} - u_i p_{ij}^{(l)}$$

Therefore

$$0 \le r_{ij} \le u_j p_{ij} \quad i \ne j$$

$$r_{ii} = u_i p_{ii} - u_i p_{ii}^{(l)} = 0$$

If $u_i p_{ij} = u_j p_{ji}$, $r_{ij} = r_{ji} = 0$; otherwise, if $u_i p_{ij} > u_j p_{ji}$, $u_i p_{ij}^{(l)} = u_j p_{ji}$. Thus

$$r_{ij} r_{ji} = [u_i p_{ij} - u_i p_{ij}^{(l)}][u_j p_{ji} - u_j p_{ji}^{(l)}] = 0$$

The similar argument can show that if $u_i p_{ij} < u_j p_{ji}$, $r_{ji} r_{ij} = 0$.
In summary, we have

$$r_{ij} r_{ji} = 0 \tag{1}$$

Moreover

$$\begin{aligned}\sum_{i \in E} r_{ij} &= \sum_{i \in E} u_i p_{ij} - \sum_{i \in E} u_i p_{ij}^{(l)} \\ &= \sum_{i \in E} u_i p_{ij} - \sum_{i \in E} u_i p_{ij} + \sum_{i \in E} r_{ji} \\ &= u_j - u_j + \sum_{i \in E} r_{ji} \\ &= \sum_{i \in E} r_{ji} \end{aligned} \tag{2}$$

If $R = (r_{ij}) \neq (0)$, combination of Eqs. (1) and (2) shows that R is a UCM.

In the next step, we let

$$q_{ij} = u_i p_{ij} - r_{ij}$$

Then

$$\begin{aligned} q_{ij} - q_{ji} &= u_i p_{ij} - r_{ij} - (u_j p_{ji} - r_{ji}) \\ &= u_i p_{ij}^{(l)} - u_j p_{ji}^{(l)} \\ &= \min \{u_i p_{ij}, u_j p_{ji}\} - \min \{u_j p_{ji}, u_i p_{ij}\} \\ &= 0 \end{aligned}$$

Hence

$$q_{ij} = q_{ji} \qquad (i, j) \in E \tag{3}$$

Equation (3) shows that if $Q = (q_{ij}) \neq (0)$, Q is a RCM. Finally

$$C = Q + R$$

This theorem is an extension of Qian and Hou's result (1977).

REFERENCES

Fudan University, 1981, Stochastic Process: Peoples Publishers, Beijing, 350 p.

Gingerich, W. D., 1969, Markov Analysis of Cyclic Alluvial Sediments: Sedman. Petro., v. 39, p. 330–332.

Hattori, Isamu, 1976, Entropy in Markov Chains and Discrimination of Cyclic Patterns in Lithologic Successions: Math. Geol. v. 8, p. 477–497.

Krumbein, W. C. and Dacey, M. F., 1969, Markov Chains and Embedded Markov Chains in Geology: Math. Geol., v. 1, p. 79–96.

Merriam, Daniel F., 1970, Comparision of British and American Carboniferous Cyclic Rock Sequence: Math. Geol., vol. 2, p. 241–264.

Pan, G. C., 1987, Decomposition of Unidirectionary Cyclic Patterns in Sedimentations: Geological Review (in Chinese), in press.

Qian, M. and Hou, Z., 1977, Inversible Markov Processes: Hunan Scientific Publishing House, Changsha, Hunan, China, 255p.

Schwarzacher, W., 1969, The Use of Markov Chains in the Study of Sedimentary Cycles: Math. Geol., vol. 1, p. 17–39.

Vistelius, Andrew B., 1976, A Degenerated Case of the Model of the Ideal Granite: Math. Geol., v. 8, p. 505–506.

Vistelius, Andrew B., 1981, Gravitational Stratification, *in* R. G. Graid, and M. L. Labovitz, (Eds.) Future Trends in Geomathematics, p. 154–158.

Vistelius, Andrew B. and Harbaugh, John W., 1980, Granite Rocks of Yosemite Valley and An Ideal Granite Model: Math. Geol. v. 12, p. 1–24.

Wang, B., et al, 1981, The Applications of Markov Chains to Sedimentary Cycles, *in* A Collection of Geomathematical Papers, Geological Publishing House, Beijing, China, p. 55–67.