quantity $(\varphi^{-1}(p) + \alpha)^{-1}$ has a complex pole and $r^{-1}\sin(\kappa r + d)$ if it has a real pole.

Screening will also exist in a system of three-color quarks and antiquarks. In this case the term describing the quark–quark interaction in the functional (3) is replaeed by

$$\varphi(x-y)(\bar{\psi}_a(x) I_m{}^a{}_b \psi^b(x))(\bar{\psi}_c(y) I^{mc}{}_d \psi^d(y)),$$

where a, b, c, d are "color" indices, and the Lagrangians $L_{\psi\sigma}$ and L_ψ in (4) are replaced by

$$L_{\psi\sigma} = -i\bar{\psi}_a(x) I_m{}^a{}_b \psi^b(x)\sigma^m(x),\ L_\sigma = -\frac{1}{2}\sigma^m(x)\varphi^{-1}(x-y)\sigma_m(y).$$

Hence the polarization term in (6) is multiplied by the quantity $l = I_m{}^a{}_b I_m{}^b{}_a$ (no summation with respect to m). We have $l = 3$ for a system of quarks and $l = 6$ for a system of quarks and antiquarks.

We have calculated the screening of the potential at T = 0. Screening also takes place when the temperature is nonzero. In the approximation $p \ll \sqrt{2mT}$, $\rho \ll mT^2/e^2$ (ρ is the density) se can obtain an expression analogous to (7) with $\alpha = 8\pi\alpha\rho T^{-1}$.

Hence in a many-quark system the quark–quark interaction may not provide the formation of a condensate of the necessary type. In several models the quark condensate is constructed from the same type of four-fermion interaction ψ^4 as the Cooper pair condensate [4]. However the question arises as to the nature of this interaction. In the case of a Cooper condensate it is produced by interactions between electrons due to phonons in the lattice of ions. Therefore to model the Higgs field as a quark condensate it is necessary to assume the existence of an analogous "phonon" interaction between quarks, and also a "medium" in which these phonons arise. The role of such a medium could be played either by the gluon vacuum (condensate), since in this case it is not required to have flavor, or any type of pre-Onnes vacuum.

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ENERGY SPECTRUM OF THE DIRAC EQUATION FOR THE SCHARZSCHILD AND KERR FIELDS

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We consider the effect of relativistic corrections and rotation of the central body on the structure of the energy spectrum of a particle with spin in the Schwarzchild and Kerr fields. A splitting of levels is obtained, which corresponds to the classical shift of the perihelion of the orbit and precession of the plane of the gravitational spin-orbit interaction and several nonlinear spin effects are calculated.

The discrete spectrum of resonance states in the case of finite motion ($E < \mu\cdot c^2$) of spinless and spinning particles in the Schwarzschild and Kerr fields was considered in [1, 2]. In the case of particles with spin one-half, if

$$R_G/\lambda_c = 2G\mu M/\hbar c \ll 1,\ a \equiv \frac{I}{Mc} \leqslant \frac{GM}{c^2} \qquad (1)$$

then these states are characterized by the nonrelativistic hydrogenic spectrum [2]. However it is obvious that relativistic and spin effects, and also the rotation of the central body, must lead to a more complicated spectrum. This question is considered in the present paper.

Although separation of variables can be done exactly for the Dirac equation in the Kerr field [3], the nonlinear effects that arise when $a \to GM/c^2$, are difficult to take into ac-

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count. Therefore we consider the case of slow rotation ($a \ll DM/c^2$), when the Kerr metric can be linearized. In the region

$$r \geqslant r_+ = GM/c^2 + \sqrt{G^2M^2/c^4 - a^2} \approx 2GM/c^2 \tag{2}$$

it has the form

$$ds^2 = e^\nu dt^2 - e^\lambda dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) + \frac{4Ma}{r}\sin^2\theta d\varphi dt, \tag{3}$$

where

$$e^\nu = e^{-\lambda} = 1 - R_G/r \equiv Y^2;\ I = Man_z;\ R_G = 2M. \tag{4}$$

(Here and below we use a system of units in which c = ħ = G = 1).

We consider the Dirac equation in this metric:

$$\gamma^\mu \nabla_\mu \psi + i\mu\psi = 0, \tag{5}$$

where γ^μ are the generalized Dirac matrices, which satisfy the relations

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}. \tag{6}$$

The covariant derivatives ∇_μ of the spinor is determined with the help of the Rock—Ivanenko coefficients [4]:

$$\nabla_\mu = \partial_\mu - \Gamma_\mu,\ \Gamma_\mu = -\frac{1}{4}\gamma^\nu(\gamma_{\nu,\mu} - \gamma_\rho\Gamma_\nu{}^\rho{}_{,\mu}). \tag{7}$$

1. In the case of the Schwarzschild metric the variables can be separated with the help of the usual spherical spinors (see [5], [6]). It is necessary to choose the matrices in the form

$$\begin{aligned}
&\gamma^t = e^{-\nu/2}\tilde{\gamma}^0 = e^{-\nu/2}\tilde{\beta};\quad \gamma^\varphi = r^{-1}\sin^{-1}\theta(\sin\varphi\tilde{\gamma}^1 - \cos\theta\tilde{\gamma}^2),\\
&\gamma^r = e^{-\lambda/2}(\sin\theta\cos\varphi\tilde{\gamma}^1 + \sin\theta\sin\varphi\tilde{\gamma}^2 + \cos\theta\tilde{\gamma}^3),\\
&\gamma^\theta = r^{-1}(\cos\theta\cos\varphi\tilde{\gamma}^1 + \cos\theta\sin\varphi\tilde{\gamma}^2 - \sin\theta\tilde{\gamma}^3).
\end{aligned} \tag{8}$$

Then

$$\Psi = r^{-1}e^{\nu/4}e^{-i\omega t}\begin{bmatrix} F(r)\, Y_l^{m_j(j)}(\theta,\varphi) \\ -iG(r)(\boldsymbol{n},\boldsymbol{\sigma})\, Y_l^{m_j(j)}(\theta,\varphi)\end{bmatrix}, \tag{9}$$

where

$$Y_l^{m_j(j=l+1/2)}(\theta,\varphi) = (-1)^\kappa i\begin{bmatrix} \sqrt{\frac{j+m_j}{2j}}\, Y_l^{m_j-1/2}(\theta,\varphi) \\ -iG(r)(\boldsymbol{n},\boldsymbol{\sigma})\, Y_l^{m_j(j)}(\theta,\varphi)\end{bmatrix}, \tag{10.1, 10.2}$$

$$Y_l^{m_j(j=l-1/2)}(\theta,\varphi) = \begin{bmatrix} \sqrt{\frac{j-m_j+1}{2j+2}}\, Y_l^{m_j-1/2}(\theta,\varphi) \\ \sqrt{\frac{j+m_j+1}{2j+2}}\, Y_l^{m_j+1/2}(\theta,\varphi)\end{bmatrix}, \tag{11}$$

and σ_i are the Pauli matrices; $(\boldsymbol{n},\boldsymbol{\sigma}) = \sin\theta\cos\varphi\sigma_1 + \sin\theta\sin\varphi\sigma_2 + \cos\theta\sigma_3$,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{12}$$

The radial function solutions (9) satisfy the equations

$$e^{-\lambda/2}F' + \frac{\kappa}{r}F = [e^{-\nu/2}\omega + \mu]G, \tag{13}$$

$$e^{-\lambda/2}G' - \frac{\kappa}{r}G = [-e^{-\nu/2}\omega + \mu]F, \tag{14}$$

where the number κ runs through positive and negative integers

$$l = \left|\kappa + \frac{1}{2}\right| - \frac{1}{2} = \begin{cases} |\kappa|, & \kappa > 0, \\ |\kappa| - 1, & \kappa < 0. \end{cases} \tag{15}$$

The number m_j is the projection of the total angular momentum of the particle in the z-direction ($\theta = 0$), l is the orbital quantum number, $j = |\kappa| - 1/2$ is the total angular momentum of the particle. The numbers κ, m_j, j are constants of the motion, as in any central field. From the definition of the current density four-vector

$$J^{\mu} = \bar{\psi}\gamma^{\mu}\psi, \quad \bar{\psi} = \psi^{*}\tilde{\beta} \tag{16}$$

we obtain the normalization integral for the stationary states

$$\int\limits_{r \geq R_G} \bar{\psi}\psi \sqrt{-g}\, e^{\nu/2} d^3x = \int\limits_{R_G}^{\infty} (|F|^2 + |G|^2)\, dr. \tag{17}$$

ombining the equations (13) and (14), we find for the function F

$$F'' + \left\{\frac{\omega^2}{Y^4} - \frac{1}{Y^2}\left[\mu^2 + \frac{\kappa(\kappa+1)}{r^2}\right]\right\} F = \hat{Q}_{\kappa} F, \tag{18}$$

where

$$\hat{Q}_{\kappa} = \frac{\omega + \mu Y}{Y^2}\left(\frac{Y^2}{\omega + \mu Y}\right)' \frac{d}{dr} + \frac{\kappa}{r}\, \frac{\omega + \mu Y}{Y^5}\left(\frac{Y^2}{\omega + \mu Y}\right)'. \tag{19}$$

If we use the fact that $\kappa(\kappa + 1) = l(l + 1)$, it is not difficult to show that if we formally put $\hat{Q}_{\kappa} = 0$, we obtain an equation for the radial functions of a spinless particle in the Schwarzschild field. The equation for the function G can be obtained from (18) with the help of the substitutions $\omega \to -\omega$, $\kappa \to -\kappa$. It will be shown below that the operator $\hat{Q}_{\kappa}$ contains a gravitational spin-orbit interaction and other nonlinear spin effects.

In the nonrelativistic (Pauli) approximation, the formation G can be assumed to be equal to zero, and the normalized function F for bound states can be expressed in terms of Laguerre polynomials [7]. Introducing $\omega = \mu + \varepsilon$, where $|\varepsilon| \ll \mu$, we obtain

$$\frac{\omega_n}{\mu} \equiv 1 + \frac{\varepsilon}{\mu} = 1 - \frac{\mu^2 M^2}{2n^2}, \quad n = 1 + l + n_r = 1, 2, 3, \ldots, \tag{20}$$

which agrees with the result of [2].

In the next (weakly relativistic) approximation we carry out the necessary expansions in (18) and (19) to terms of order v^2/c^2, and introducing the coordinate $\rho = r - R_G$, we obtain

$$\left[\frac{1}{2\mu}\frac{d^2}{d\rho^2} + \varepsilon - V_M - \frac{l(l+1)}{\rho^2}\right] F = (V^{(1)} + V^{(2)} + V^{(3)} + V^{(4)})\, F, \tag{21}$$

where

$$V_M = \frac{\mu M}{\rho}, \tag{22}$$

while the right-hand side is a sum of perturbing potentials, which will be given below. In this approximation we have, according to (13)

$$G = \frac{1}{2\mu}\left(F' + \frac{\kappa}{\rho} F\right), \tag{23}$$

hence the normalization condition takes the form

$$\int\limits_{\rho=0}^{\infty} (|F|^2 + |G|^2)\, d\rho = \int\limits_{\rho=0}^{\infty} |F|^2 \left(1 + \frac{\varepsilon - V_M}{2\rho}\right) d\rho = 1. \tag{24}$$

We consider separately each of the terms of the perturbation. The term $V^{(1)}$ results from the renormalization of the energy of the particle in the gravitational field and does not remove the degeneracy with respect to the orbital quantum number l:

$$\frac{1}{\mu} < V^{(1)} > = \frac{\Delta\varepsilon_n}{\mu} = \frac{15}{8} \frac{\mu^4 M^4}{n^4}. \tag{25}$$

The interaction $V^{(2)}$ corresponds to the classical effect of a cumulative shift in the perihelion of the orbit of the spinless particle. It removes the degeneracy with respect to the orbital quantum number l:

$$\frac{\Delta\varepsilon_{nl}}{\mu} = \frac{1}{\mu} < V^{(2)} > = -\frac{3(\mu M)^4}{n^3\left(l+\frac{1}{2}\right)}\left(1 - \frac{1}{3}\delta_{l_0}\right). \tag{26}$$

The additional perturbation $V^{(3)}$ can be interpreted as a gravitational spin–orbit interaction. It can be written in the form

$$V^{(3)} = \frac{(L\sigma)}{4\mu^2\rho}\frac{dV_M}{d\rho}, \tag{27}$$

where $(L\sigma) = -(1+\kappa)$.

This interaction gives the following contribution to the energy of the particle

$$\frac{\Delta\varepsilon_{n\kappa l}}{\mu} = -\frac{\mu^4 M^4(1-\delta_{l_0})}{4n^3\kappa(l+1/2)}. \tag{28}$$

This contribution is positive for $\kappa < 0$, i.e., $j = l + 1/2$ (spin parallel to the orbital angular momentum) and negative for $\kappa > 0$, i.e., $j = l - 1/2$. The effect of the gravitational spin–orbit interaction was discussed by Mitskevich in the linear approximation in the gravitational constant and in the chronometrically invariant formulation of the Dirac equation. We note that in our case equation (27) for the energy of the gravitational spin–orbit interaction is six times smaller than that obtained in [8]. The term $V^{(4)}$ is also due to the spin of the particle:

$$\frac{1}{\mu} < V^{(4)} > = \left\langle -\frac{M}{2\mu\rho(\rho+r_g)}\frac{d}{d\rho}\right\rangle = -\frac{\mu^4 M^4(1-\delta_{l_0})}{2n^3 l\left(l+\frac{1}{2}\right)(l+1)}. \tag{29}$$

The final expression for the energy of a Dirac particle in the Schwarzschild field for $2\mu M \ll l + 1/2$ has the form

$$\frac{\omega_{n\kappa l}}{\mu} = 1 - \frac{\mu^2 M^2}{2n^2} - \frac{3\mu^4 M^4}{n^4}\left\{\frac{n}{l+\frac{1}{2}}\left[\left(1-\frac{1}{3}\delta_{l_0}\right) + \frac{1}{12}(1-\delta_{l_0})\left(\frac{1}{\kappa} + \frac{2}{l(l+1)}\right)\right] - \frac{5}{8}\right\}. \tag{30}$$

We now discuss the fact that inclusion of relativistic and spin effects removes the degeneracy with respect to both the total angular momentum and the orbital angular momentum, in contrast to the Coulomb electric field in flat space, in which it is well-known that the energies of the bound states of an electron depend only on the numbers n and j. For a given value of l the binding energy of the particle is smaller for $\kappa < 0\left(j = l + \frac{1}{2}\right)$ than when $\kappa > 0\left(j = l - \frac{1}{2}\right)$. The contribution of nonlinear spin effects, as is evident from (30), is comparable to the contribution of the spin–orbit interaction only for the lowest levels.

2. We determine the γ-matrices for the linearized Kerr metric (3) in terms of the γ-matrices corresponding to the Scharzschild geometry:

$$\boldsymbol{\gamma}_t = \gamma^t,\ \boldsymbol{\gamma}_t = \gamma_t + g_{t\varphi}\gamma^\varphi,\ \boldsymbol{\gamma}^\varphi = \gamma^\varphi + g^{t\varphi}\gamma_t,\ \boldsymbol{\gamma}_\varphi = \gamma_\varphi, \tag{31}$$

where the boldface quantities correspond to the Kerr metric. Using the definition (8) of the γ-matrices and the fact that $e^{-\lambda/2} \approx 1$ in the region of interest here, we can write the Hamiltonian of the system in the form

$$H = \hat{H}_s + \hat{H}_1 + \hat{H}_2 = \hat{H}_s + \frac{2(I \cdot L)}{r^3} + \frac{1}{2}(\Omega\sigma), \tag{32}$$

where H_S is the Hamiltonian of a particle in the Schwarzschild metric, and Ω is the angular velocity of the Lenz–Thirring precession:

$$\Omega = r^{-3}\{3(\boldsymbol{I}\cdot\boldsymbol{n})\,\boldsymbol{n} - \boldsymbol{I}\}. \tag{33}$$

Before discussing the results of the calculation we note that the interaction $\hat{H}_1$ is of the same form as the interaction between the dipole moment of the central body and the orbital angular momentum of a spinless particle. However, in the case of a particle with spin the operator $\hat{L}_z$ no longer has a definite value in states with given n, l, and j. This leads to to a different value for the splitting in energy. Comparing H_1 with magnitude of the spin-orbit interaction $V^{(3)}$ characteristic of fine structure, it is not difficult to see that when $\mu a \ll 1$ it will lead to a hyperfine structure of the levels. Considering $H_1 + H_2$ as a perturbation, we write the Dirac equation in the form

$$\left[\varepsilon - V_M + \frac{1}{2\mu}\frac{d^2}{d\rho^2} - \frac{l(l+1)}{\rho^2}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = (V^{(1)} + V^{(2)} + V^{(3)} + V^{(4)} + H_1 + H_2)\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}, \tag{34}$$

where

$$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix} = F_{nl}(\rho)\, Y_l^{m_j(j)}(\theta, \varphi). \tag{35}$$

The calculation of the matrix elements is not difficult and leads to

$$\frac{1}{\mu} < H_1 > = \frac{4m_j}{2l+1}\,\frac{\mu a(\mu M)^4|\kappa|}{n^3 l\left(l+\frac{1}{2}\right)(l+1)}, \tag{36}$$

and also

$$\frac{1}{\mu} < nl_j|H_2|nl_j > = \left\langle\frac{Ma}{2r^3}\right\rangle_r 4m_j\,\frac{\pm l(l+1) + 3\left(l+\frac{1}{2}\right) \mp 3\left(m_j^2 + \frac{1}{4}\right)}{(2l-1)(2l+1)(2l+3)}. \tag{37}$$

Therefore all of the electron levels (except the ground state) are split into 2j + 1 sublevels, corresponding to the number of possible projections of the angular momentum onto the rotation axis of the central body:

$$\frac{\Delta\omega_{nl_j}^{m_j}(a)}{\mu} = \frac{2m_j}{2l+1}\,\frac{\mu a(\mu M)^4}{n^3 l\left(l+\frac{1}{2}\right)(l+1)} \times \begin{cases} 2|\kappa| - \dfrac{3\left(m_j^2+\frac{1}{4}\right) - l(l+1) - 3\left(l+\frac{1}{2}\right)}{(2l-1)(2l+3)}, \\[2ex] 2|\kappa| + \dfrac{3\left(m_j^2+\frac{1}{4}\right) - l(l+1) + 3\left(l+\frac{1}{2}\right)}{(2l-1)(2l+3)}. \end{cases} \tag{38}$$

The upper expression inside the brackets (corresponding to the upper sign in (37)) is chosen when $j = l + \frac{1}{2}$, $\kappa < 0$, and the lower expression is chosen when $j = l - \frac{1}{2}$, $\kappa > 0$.

3. We have shown that relativistic gravitational effects, the rotation of the central body, and spin, lead to a complicated structure of the spectrum of quasibound states of a Dirac particle in the Schwarzschild and Kerr fields. These factors create fine and hyperfine splitting of the levels of the nonrelativistic hydrogenic spectrum if the condition (1) is satisfied. A specific relativistic effect is the splitting of the 2p and 2s levels in the Schwarzschild field, and, as a result of the gravitational spin-orbit interaction, there are two possible transitions:

$$\frac{E_{2P1/2} - E_{2S1/2}}{\mu c^2} = \frac{28}{96}\left(\frac{G\mu M}{\hbar c}\right)^4; \quad \frac{E_{2P3/2} - E_{2S1/2}}{\mu c^2} = \frac{31}{96}\left(\frac{G\mu M}{\hbar c}\right)^4. \tag{39}$$

In this way the Schwarzschild gravitational field is different from the Coulomb electric field in flat space. Another difference is the absence of a contact interaction in the s-state, and also the presence of nonlinear spin effects.

The Hamiltonian of the dipole-orbital and dipole-spin interactions (32), due to the rotation of the central body, resembles in form the Hamiltonian of the hyperfine structure of a hydrogenic atom, due to the interaction between the magnetic moment of the nucleus and that of the electron [9]. However, in our case, as before, the contact interaction is absent. In addition, the effective gravitational g-factor of an electron is equal to one, whereas in the electromagnetic case it is equal to two.* The splitting of the $2p_{1/2}$ level into two sublevels and the splitting of the $2p_{3/2}$ level into four sublevels is specific to the interaction considered here. In particular:

*The gravitational g-factor of the electron was discussed earlier in [10].

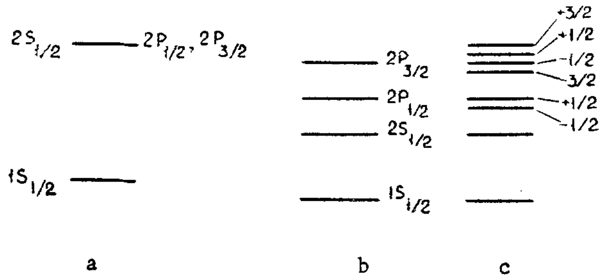

Fig. 1. Energy level diagram of the lowest levels of a Dirac particle in the Schwarzschild and Kerr fields: a) degenerate nonrelativistic spectrum in the Scharzschild field (hydrogenic); b) same field, but with the inclusion of relativistic and spin effects. Here the degeneracy with respect to the orbital and internal quantum numbers is removed; c) removal of the spatial degeneracy of the spectrum in the Kerr field. The numbers to the right are the values of the projections of the total angular momentum onto the rotation axis of the central body.

$$\Delta E_{2P_{1/2}}^{m_j=\pm 1/2} = \pm \frac{1}{15}\mu c^2 \left(\frac{\mu a c}{\hbar}\right)\left(\frac{G\mu M}{\hbar c}\right)^4, \tag{40.1}$$

$$\Delta E_{2P_{3/2}}^{m_j=\pm 1/2} = \frac{1}{24}\mu c^2 \left(\frac{\mu a c}{\hbar}\right)\left(\frac{G\mu M}{\hbar c}\right)^4, \tag{40.2}$$

$$\Delta E_{2P_{3/2}}^{m_j=\pm 3/2} = \frac{3}{40}\mu c^2 \left(\frac{\mu a c}{\hbar}\right)\left(\frac{G\mu M}{\hbar c}\right)^4. \tag{40.3}$$

An energy-level diagram for the lowest levels of a Dirac particle is shown in Fig. 1. In conclusion, we note that spin effects and the gravitational spin-orbit interaction in particular become significant when $\mu M \sim M_{PL}^2$. An estimate of the wave length of radiation emitted by the electron in the transitions (39) is:

$$\lambda = \begin{Bmatrix} 1.14 \\ 1.02 \end{Bmatrix} 10^{-9}\left(\frac{3.85\cdot 10^{-12}\,\text{cm}}{R_G}\right)^4 \text{cm} \quad \begin{matrix}, & \text{for } \Delta j = 0, \\ , & \text{for } \Delta j = 1, \end{matrix} \tag{41}$$

and so fine structure effects could appear in principle in the radiation and interaction of a "gravitational atom" with radiation. The role of such an atom could be played by a primary microscopic black hole ($R_G \lesssim 10^{-12}$ cm) with filled levels and in a state of quantum thermal vaporization.

If $\mu ac \sim \hbar$, then the interaction of the dipole moment of the central body with the spin of the particle also becomes significant. When $a \to GM/c^2$ and $\mu M/M_{PL}^2 \sim m_j$ the structure of the spectrum will be determined to a large degree by nonlinear effects. Study of these problems may be considered in the future.

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ASYMPTOTIC BEHAVIOR OF THE EFFECTIVE POTENTIAL OF COMPOSITE FIELDS IN A CURVED SPACE-TIME

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Renormalization group equations are obtained, which allow studying the asymptotic behavior of the effective potential of composite fields in an external gravitational field. For asymptotically free theories, the behavior of the effective potential is found for large values of composite fields and of the space-time curvature.

1. In this paper we continue studying the asymptotic behavior of the effective action in an external gravitational field, starting in [1]. There we considered the behavior of the effective potential of elementary fields in the limit of large curvature and/or large values of scalar fields. Now we shall consider the asymptotic behavior of the effective potential of composite fields. The interest towards studying composite fields arose because they provide a possibility of dynamical generation of particle masses, in analogy to the Bardeen–Cooper–Schrieffer model in the theory of superconductivity (see, e.g., [2, 3]), and recently, due to the studying of the gluon condensate (see, e.g., [4]). Also notice that composite fields proved to be useful in studying fluctuations in the expanding Universe model [5].

The asymptotic behavior of the effective potential of composite fields in flat space was studied in [6] using the renormalization group equations. It was established that the condition of stability of the theory in the limit of large composite fields poses constraints on the multiplet content of the theory.

In the present work we shall show how one can obtain various renormalization group equations for the effective potential of composite fields in an external gravitational field. These equations allow us to study the behavior of the effective potential not only in the limit of large composite fields (as in the case of a flat space), but also in the limit of large curvature, or in both limits.

2. Consider an arbitrary theory containing the scalars φ^i, ψ^K and gauge fields A_μ, both of which we shall denote as ω. Let us also define the composite fields $\sigma \equiv \{\varphi^i\varphi^i, A_\mu^a A^{a\mu}\}$. Let us denote $\Omega \equiv \{\omega, \sigma\}$. The generating functional for the Green functions of fields Ω is:

$$e^{iW_0(I, K)} = \int d\omega e^{i\left[S + \int d^4x \sqrt{-g}(I\omega + K\cdot\sigma)\right]}.$$

Here S is the renormalized action of the theory in an external gravitational field (taking the ghosts and the gauge conditions into account, the integration over the ghosts being included into $d\omega$); I and K are the sources of the fields ω and σ, respectively. Denote $N \equiv \{I, K\}$. The functional $W_0(I, K)$ leads to a finite theory for K = 0. The Green functions containing composite field insertions, are divergent. In order to find out the structure of these divergences, let us consider the $K\sigma$ terms as additional vertices in the Lagrangian. Then, computing the index of a divergence, it is easy to see that the diagrams containing K in the first and second power are the only divergent ones. Hence the generating functional W_R, which leads to finite Green functions of both elementary and composite fields (as well as the vacuum energy, due to the presence of vacuum counterterms [7, 9] in S), has the form

$$e^{iW_R} = \int d\omega e^{i\left[S + \int d^4x \sqrt{-g}(I\omega + KZ_{K\sigma}\sigma + KZ_{K\omega}\omega + Z^{(1)}K + KZ^{(2)}K + Z^{(3)}KR)\right]}. \quad (1)$$

Here $Z_{K\sigma}$, $Z_{K\omega}$, $Z^{(1)}$, $Z^{(2)}$, $Z^{(3)}$ are the renormalization constants, of which only $Z^{(1)}$, $Z_{K\omega}$ are dimensional; R is the scalar curvature. The constants $Z_{K\sigma}$, $Z_{K\omega}$, $Z^{(1)}$, $Z^{(2)}$ are the same as in the case of a flat space [6], while $Z^{(3)}$ is a new constant which has no flat-space analog. The functional W_R depends on the sources N, the ensemble of charges f, and the renormaliza-

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