WEAK FOUR-FERMION INTERACTION IN TERMS OF SU(3) INVARIANTS. I

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The four-fermion weak interaction is formulated in terms of invariants of the groups $G_2 = L \otimes SU(2)$ and $G_3 = L \otimes SU(3)$. The four-fermion Hamiltonian is constructed as a four-fermion invariant of the group G_3 on the basis of the spin tensors $\psi^{lmn}_{\alpha\beta\gamma}$, where $\alpha, \beta, \gamma = 1, 2, 3, 4$ are the spinor indices and $l, m, n = 1, 2, 3$ are the unitary indices of the representation of G_3. It is shown that in the case of one multiplet $\psi^{lmn}_{\alpha\beta\gamma}$ one can construct one nontrivial invariant and in the case of two multiplets $\psi^{lmn}_{\alpha\beta\gamma}$ and $\varphi^{lmn}_{\alpha\beta\gamma}$, nine nontrivial invariants. Of these, only in the case of two of the invariants, which contain two multiplets, are the lepton and baryon numbers conserved independently. One of these invariants is considered in detail. In the case of SU(2) there is no fundamental difficulty and a number of relations are obtained for the constants of the weak interactions and the probabilities of processes. In the case of SU(3) it is shown that the known breakings of SU(3) in weak interactions can be localized in the lepton octet by choosing it in a special way.

INTRODUCTION

It is not difficult to formulate weak four-fermion interactions in terms of invariants in the case of the symmetry $G_2 = L \otimes SU(2)$, where L is the Lorentz group [1]. That is not the case for $G_3 = L \otimes SU(3)$. Because neither the hypercharge Y nor the third component I_3 of the isospin are conserved in weak interactions, to say nothing about the absence of a lepton octet, the very formulation of the question of describing a four-fermion interaction in terms of invariants of the group $G_3 = L \otimes SU(3)$ would appear to be devoid of meaning. Nevertheless, if the question is posed "sensibly," it can be given a positive answer, as we shall show below.

The problem is solved as follows: 1) It is first assumed that there exists a lepton octet as a representation of SU(3) [i.e., it transforms in accordance with an octet representation of SU(3)] and in weak four-fermion interactions there is no breaking of SU(3) symmetry. One constructs the formal mathematical apparatus of the four-fermion weak interactions in terms of the invariants of G_3 and obtains final results in terms of the baryon and lepton octets. 2) The lepton octet is constructed as a general 3 × 3 matrix. Since the electric charge is conserved in weak interactions, in the lepton octet the positions of the charged and the neutral particles are fixed. The lepton octet is constructed as an SU(3) representation by analogy with the baryon octet, the following substitution being made for the charged particles (with arbitrary coefficients): $p^+ \to e^+$, $\Sigma^+ \to e^{+\prime}$, $\Xi^- \to \mu^-$, $\Sigma^- \to \mu^{-1}$, and the neutral particles are replaced by a linear sum (with arbitrary coefficients) of the corresponding neutrinos: $\nu \equiv \nu_e$, $w \equiv \nu_\mu$, $\nu' \equiv \nu_e'$, $w \equiv \nu_\mu'$. Thus, the lepton matrix contains four new particles ($e^{+\prime}$, $\mu^{-\prime}$, ν', w') and 24 arbitrary parameters. If these 24 parameters are determined suitably, the matrix is a representation of SU(3), and the formalism is invariant under the group $G_3 = L \otimes SU(3)$. 3) The breaking of SU(3) symmetry is now expressed by the fact that in the resulting lepton matrix one assumes $e^{+\prime} = e^+$, $\mu^{-\prime} = \mu^-$, $\nu' = \nu$, $w' = w$. In addition, the remaining 14 constant parameters in the lepton matrix remain arbitrary, and these are to be determined appropriately. The normalization condition and the requirement that the lepton matrix be an octet yield seven equations. The remaining conditions arise from the requirement that in baryon decays there be no crossed terms of electron and muon neutrinos (of the type $(\bar{w}\gamma_\mu e^+)$, $(\bar{\mu}^-\gamma_\mu\nu)\ldots$). As a result of these

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TABLE 1a. Parameters of Leptonic Decay of Baryons

m^l_n	$\bar{A}\ B$	$16\,a^l_{0m}$, $8b^l_{0m}$, λ^l_{0m}	$(a_6\,c_6)^m_l$, $(a_5\,c_5)^m_l$, c^m_{6l}, c^m_{5l}	For the solution (28): c^m_{6l}, c^m_{5l}	
m^1_2 $\lvert\Delta\Theta\rvert = 1$ $\Delta s = 0$	1. $\bar{p}n$	2 —10 —0,4.	$-9(\delta\varepsilon_2 + \xi_4\varkappa_2) - 45\beta\varepsilon_2$ $9\alpha\varepsilon_1 - 45(\gamma\varepsilon_1 + \xi_1\varkappa_1)$ $9(\varepsilon_2\delta + \xi_4\varkappa_2) - 27\beta\varepsilon_2$ $9\alpha\varepsilon_1 - 27(\gamma\varepsilon_1 + \xi_1\varkappa_1)$	$9(c_4\,c_7\sin^2\Theta - c_2\,c_6\cos^2\Theta)\cos\Theta$ $27(c_5\cos^2\Theta -$ $-c_2\,c_3\,c_7\sin^2\Theta)\,c^1\cos\Theta$	$3\,a^m_{5l} = 5,\ a^m_{6l} = -1$
	2. $\bar{\Xi}^0\,\Xi^-$	—4 2 —4			
	3. $\bar{\Lambda}\,\Sigma^-$	$\sqrt{6}$ 2 $\sqrt{6}$—1			
	4. $\bar{\Sigma}^0\,\Sigma^-$	$-\sqrt{\frac{2}{3}}$ $-4\sqrt{2}$ 2			
m^1_3 $\lvert\Delta\Theta\rvert = 1$ $\lvert\Delta s\rvert = 1$	1. $\bar{p}\,\Sigma^0$	$2\sqrt{2}$ $-\sqrt{2}$ —4	$-9(\varepsilon_2\xi_2 + \sigma\varkappa_2) - 45\beta\varkappa_2$ $-45(\varepsilon_1\xi_3 + \rho\sigma) - 9\alpha\varkappa_1$ $9(\varepsilon_2\xi_2 + \sigma\varkappa_2) - 27\beta\varkappa_2$ $9\alpha\varkappa_1 - 27(\varepsilon_1\xi_2 + \rho\varkappa_1)$	$9(c_2\,c_4\sin^2\Theta - c_5\,c_7\cos^2\Theta)\sin\Theta$ $27(c_3\sin^2\Theta -$ $-c_2\,c_5\,c_7\cos^2\Theta)\,c_1\sin\Theta$	$3a^m_{5l} = 5,\ a^m_{6l} = -1$
	2. $\bar{n}\ \Sigma^-$	4 —2 —4			
	3. $\bar{p}\ \Lambda$	0 $3\sqrt{6}$ 0			
	4. $\bar{\Sigma}^+\,\Sigma^0$	2 —10 —0,4			
	5. $\bar{\Lambda}\ \Xi^-$	$\sqrt{6}$ $\sqrt{6}$ 2			
	6. $\bar{\Sigma}^0\ \Xi^-$	$-\sqrt{2}$ $5\sqrt{2}$—0,4			

general requirements, the lepton octet is determined in terms of e^+, μ^-, ν, w to within one arbitrary constant. 4) It remains to show that the lepton octet constructed in this way takes on, to the necessary extent, all breakings of SU(3) in weak four-fermion interactions. The proof is by direct calculation of the probabilities of leptonic decays of baryons of the octet in the formalism when the constructed lepton octet is used.

It is well known that to obtain experimental values of the probability of leptonic decays of baryons of the octet it is also necessary to introduce form factors. However, to avoid the introduction of new free parameters, which is necessary when form factors are taken into account, the probabilities of processes are calculated in the present work at a small momentum transfer, $q^2_\mu = 0$. The results are compared with the results of Cabibbo theory (for which it is known that the theory can be made to agree with the experiment if the form factors are suitably chosen) under the same conditions $q^2_\mu = 0$. Comparison shows that the results of these two formalisms in the case of leptonic decay of baryons of the octet are very similar. At the same time, Cabibbo theory (for $q^2_\mu = 0$) depends on at least three parameters (Θ, the Cabibbo angle and the parameters F and D). In the formalism developed below, there is only one parameter (the analog of Θ; F and D are determined by the specification of the symmetry).

In our work we calculate the probabilities (for $q^2_\mu = 0$) of processes that correspond in the usual terminology to "neutral currents," these being entirely absent in Cabibbo theory.

We also consider lepton—lepton interaction in SU(3) using our lepton octet and we determine the ratios of the weak coupling constants of different lepton—lepton interactions.

§1. General Form of the Hamiltonian of the Four-Fermion Interactions

1. Four-Fermion Invariants. In this section, we construct the formal mathematical apparatus of four-fermion weak interactions in terms of four-fermion invariants of the group $G_3 = L \otimes SU(3)$, assuming at the same time that in weak four-fermion interactions there is no breaking of SU(3). We shall describe the baryons by the octet $\psi^{[l\,m]n}_{[\alpha\beta]\gamma}$ and decuplet $\psi^{(l\,mn)}_{(\alpha\beta\gamma)}$, and leptons by the octet $\varphi^{[l\,m]n}_{[\alpha\beta]\gamma}$. Here, α, β, γ are the spinor indices and l, m, n the unitary indices. The square brackets denote antisymmetry and the round brackets symmetry. All these multiplets are symmetric with respect to the composite indices (ABC), and it is therefore sufficient to construct a four-fermion invariant from $\psi_{(ABC)}$ and $\varphi_{(ABC)}$ and their conjugate functions, which contain as irreducible representations the octet and decuplet in which we are interested.

TABLE 1b. Parameters of Leptonic Decay of Baryons

$\mathfrak{m}^l_m$	$\overline{A}B$	$16a^l_{0m}$	$8b^l_{0m}$	λ^l_{0m}		For the solution (28)
$\mathfrak{m}^2_3$ $\Delta\Theta = 0$ $\lvert\Delta s\rvert = 1$	1. $\overline{\Sigma}^- \ \Xi^-$	-2	10	$-0{,}4$	$c^m_{4l} = 9[\xi_4(\sigma-3\delta)+\xi_2(\delta-3\sigma)]$, $c^m_{2l} = 9[\xi_3(\rho-3\gamma)+\xi_1(\gamma-3\rho)]$, $c^m_{1l} = 9\varkappa_1\varepsilon_1$, $c^m_{3l} = -27\varkappa_2\varepsilon_2$	$c^m_{4l} = 18c_2c_7\sin 2\Theta$, $\alpha^m_{4l} = 1$; $2c^m_{3l} = -27c_4c_6\sin 2\Theta$, $\alpha^m_{3l} = 5/3$; $c^m_{2l} = -18c_2c_7\sin 2\Theta$, $\alpha^m_{2l} = 1$; $2c^m_{1l} = 9c_3c_5\sin 2\Theta$, $\alpha^m_{1l} = -1$
	2. $\overline{\Sigma}^0 \ \Xi^0$	$-\sqrt{2}$	$5\sqrt{2}$	$-0{,}4$		
	3. $\overline{\Lambda} \ \Xi^0$	$-\sqrt{6}$	$-3\sqrt{2}$	2		
	4. $\overline{n} \ \Lambda$	0	$3\sqrt{6}$	0		
	5. $\overline{p}^+ \ \Sigma^+$	4	-2	-4		
	6. $\overline{n} \ \Sigma^0$	$-2\sqrt{2}$	$\sqrt{2}$	-4		
$\mathfrak{m}^1_1$ $\Delta\Theta = 0$ $\Delta s = 0$	1. $\overline{n}\,n$	-4	2	-4	$2c^m_{2l} = 2c^m_{4l} = -57$, $c^m_{1l} = -30$, $c^m_{3l} = 0$,	$2c^m_{2l} = 2c^m_{4l} = -57$, $c^m_{1l} = -30$, $c^m_{3l} = 0$. $5\alpha^m_{1l} = 3$, $\alpha^m_{3l} = 0$, $57\alpha^m_{2l} = 57\alpha^m_{4l} = -9$.
	2. $\overline{\Xi}^0 \ \Xi^0$	-4	2	-4		
	3. $\overline{\Lambda} \ \Lambda$	-3	0	0		
	4. $\overline{\Sigma}^+ \ \Sigma^+$	-2	-8	$0{,}5$		
	5. $\overline{p}^+ \ p^+$	-2	-8	$0{,}5$		
	6. $\overline{\Sigma}^0 \ \Sigma_0$	-1	-4	$0{,}5$		
	7. $(\overline{\Sigma}^0\Lambda + \overline{\Lambda}\Sigma^0)$	$\sqrt{3}$	$-2\sqrt{3}$	-1		

From one multiplet $\psi(ABC)$ or $\varphi(ABC)$ one can construct one nontrivial four-fermion invariant [1]:

$$H = \frac{G_1}{\sqrt{2}} \mathfrak{m}^B_A(\overline{N}, N)\,\mathfrak{m}^A_B(\overline{N}, N), \quad \mathfrak{m}^B_A(\overline{N}, N) \equiv \overline{\psi}^{BCD}\psi_{ACD}, \tag{1}$$

$$A = (\alpha, l), \quad B = (\beta, m), \quad C = (\gamma, n), \quad D = (\delta, \kappa),$$
$$\alpha, \beta, \gamma, \delta = 1, 2, 3, 4; \quad l, m, n = 1, 2, 3. \tag{2}$$

From the two multiplets $\psi(ABC)$ and $\varphi(ABC)$ one can construct ten nontrivial invariants [(1), $\mathfrak{m}^B_A(\overline{N}, L)\,\mathfrak{m}^A_B(\overline{N}, L)$, $\mathfrak{m}^B_A(\overline{N}, N)\,\mathfrak{m}^A_B(\overline{N}, L)$, $\mathfrak{m}^B_A(\overline{N}, N)\,\mathfrak{m}^B_A(\overline{L}, N)$, and a further four invariants obtained from these by the substitution N → L, L → N, and the two invariants (3)]. If at the same time one restricts oneself to the requirement that the baryon and the lepton number be conserved separately, instead of (1) there remains only the two invariants

$$H = \frac{G_2}{\sqrt{2}}\{\mathfrak{m}^B_A(\overline{N}, N)\,\mathfrak{m}^A_B(\overline{L}, L) + \beta\mathfrak{m}^B_A(\overline{N}, L)\,\mathfrak{m}^A_B(\overline{L}, N)\}. \tag{3}$$

Here N are the baryons, L the leptons, and G_2 and β are two arbitrary constants.

If we restrict ourselves to only the baryon octet $\psi^{[lm]n}_{[\alpha\beta]\gamma}$, formulate the corresponding conditions on the spin and isospin, make $\psi(ABC)$ satisfy the Dirac equation in the form of the Bargmann–Wigner equation, and use the actual form of $\psi^{[lm]n}_{[\alpha\beta]\gamma}$ in terms of antisymmetric Dirac matrices (c, $c\gamma_5$, $c\gamma_5\gamma_\mu$, where c is the charge conjugation matrix) and the corresponding functions $N^{[lm]n}$, $N^{[lm]n}_\mu$, we find from (1)

$$\mathfrak{m}^{\alpha l}_{\beta m}(\overline{b}, b) = \delta_{\alpha\beta}\left\{\left[-\frac{1}{4}\delta_{lm}(\overline{b}b)^n_n + \frac{1}{8}(\overline{b}b)^l_m\right] + \frac{1}{4}(\overline{b}^n_m b^l_n)\right\} + (\gamma_\mu)_{\beta\alpha}\left\{\left[-\frac{1}{4}\delta_{lm}(\overline{b}_\mu b + b\overline{b}_\mu)^n_n\right.\right.$$
$$\left.\left. + \frac{1}{8}(\overline{b}_\mu b + \overline{b}\,b_\mu)^l_m\right] + \frac{1}{4}(\overline{b}^n_{\mu m} b^l_n + \overline{b}^n_m b^l_{\mu n})\right\} + \frac{1}{2}[\gamma_\nu\gamma_\mu]_{-\beta\alpha}\left\{\left[-\frac{1}{4}\delta_{lm}(\overline{b}_\mu b_\nu)^n_n + \frac{1}{8}(\overline{b}_\mu b_\nu)^l_m\right]\right.$$
$$\left. + \frac{1}{4}(\overline{b}^n_{\mu m} b^l_{\nu n})\right\} + \frac{1}{4}\{[\delta_{lm}(\overline{b}^\alpha b_\beta + \overline{b}^\alpha_\mu b_{\mu\beta})^n_n - 5(\overline{b}^\alpha b_\beta + \overline{b}^\alpha_\mu b_{\mu\beta})^l_m] - (\overline{b}^n_m b^l_n + \overline{b}^n_{\mu m} b^l_{\mu n})\}, \tag{4}$$

where $(\overline{b}b)^n_l \equiv \overline{b}^n_k b^k_l$. At the same time

$$m_0 = 0, \quad b \equiv L, \quad L_\mu = \frac{1}{4}\gamma_\mu L, \quad \overline{L}_\mu = \frac{1}{4}L\gamma_\mu, \tag{5}$$

TABLE 1c. Parameters of Leptonic Decay of Baryons

$(c_4\alpha_4)_l^m$	$-18\sin^2\theta$	$-27\cos^2\theta$
$2(c_3\alpha_3)_l^m$	$9(1-5\cos^2\theta)$	$9\cos^2\theta$
$(c_2\alpha_2)_l^m$	$-18\cos^2\theta$	$-27\sin^2\theta$
$2(c_1\alpha_1)_l^m$	$9\sin^2\theta$	$9\cos^2\theta$
$2c_{4l}^m$	$-44\sin^2\theta$	$-44\cos^2\theta$
$2c_{3l}^m$	$19\cos^2\theta-44$	$19\sin^2\theta-44$
$2c_{2l}^m$	$-44\cos^2\theta$	$-44\sin^2\theta$
$2c_{1l}^m$	$-25\cos^2\theta$	$-25\sin^2\theta$
For the solution (28)		
$(2c_4\alpha_4)_l^m$	$9(1-\xi_4^2-6\delta^2-5\xi_2^2)$	$9(1-\xi_2^2-\gamma^2-5\xi_4^2-5\sigma^2)$
$2(c_3\alpha_3)_l^m$	$9(1-5\xi_2^2)$	$9(1-5\varkappa_2^2)$
$2(c_2\alpha_2)_l^m$	$9(1-\xi_3^2-6\gamma^2-5\xi_1^2)$	$9(1-\xi_1^2-\delta^2-5\xi_3^2-5\rho^2)$
$2(c_1\alpha_1)_l^m$	$9(1-\xi_1^2)$	$9(1-\varkappa_1^2)$
$2c_{4l}^m$	$6\delta^2-25(1-\xi_4^2)-19\xi_2^2$	$6\sigma^2-25(1-\xi_2^2)-19\xi_4^2$
$2c_{3l}^m$	$-25-19\varepsilon_1^2$	$-25-19\varkappa_2^2$
$2c_{2l}^m$	$6\gamma^2-25(1-\xi_3^2)-19\xi_1^2$	$6\rho^2-25(1-\xi_1^2)-19\xi_3^2$
$2c_{1l}^m$	$-25(1-\varepsilon_1^2)$	$-25(1-\varkappa_1^2)$
$16\lambda_{0m}^l$	−64 −64 8 8 8 −3 −16	−64 −64 −64 8 8 0
$4b_{0m}^l$	1 1 −4 −4 −2 0 $\sqrt{3}$	1 1 1 −4 −4 −3
$16a_{0m}^l$	−4 −4 −4 −2 −1 −3 $\sqrt{3}$	−4 −4 −4 −2 −2 0
$\bar{A}B$	1. $\bar{p}^+p^+$ 2. $\bar{\Xi}^-\Xi^-$ 3. $\bar{\Sigma}^-\Xi^-$ 4. $\bar{n}n$ 5. $\bar{\Sigma}^0\Sigma^0$ 6. $\bar{\Lambda}\Lambda$ 7. $(\bar{\Lambda}\Sigma^0+\bar{\Sigma}^0\Lambda)$	1. $\bar{\Sigma}^+\Sigma^+$ 2. $\bar{\Sigma}^-\Sigma^-$ 3. $\bar{\Sigma}^0\Sigma^0$ 4. $\bar{\Xi}^-\Xi^-$ 5. $\bar{\Xi}^0\Xi^0$ 6. $\bar{\Lambda}\Lambda$
$\mathfrak{m}_m^l$	$\mathfrak{m}_2^2$ $\Delta\theta=0$ $\Delta s=0$	$\mathfrak{m}_3^3$ $\Delta\theta=0$ $\Delta s=0$

$$m_0 \neq 0, \quad b \equiv N, \quad N_\mu = q_\mu N, \quad \bar{N}_\mu = \bar{q}_\mu \bar{N},$$

$$q_\mu \equiv -\frac{i}{m_0} p_\mu, \quad \bar{q}_\mu \equiv -\frac{i}{m_0} p'_\mu, \quad p_\mu = (p_n, iE), \quad p'_\mu = (p'_n, iE'), \quad p_\mu^2 = -m_0^2. \tag{6}$$

In (4), the terms in the curly brackets coincide with the corresponding expressions in SU(2) (when the octet b_n^l is replaced by the doublet b^l). The terms outside the curly brackets are peculiar to SU(3). We can obtain $\mathfrak{m}_A^B(\bar{N}, L)$ and $\mathfrak{m}_A^B(\bar{L}, N)$ from (3) by the simple substitution $\bar{b} \to \bar{N}$, $b \to L$ and $\bar{b} \to \bar{L}$, $b \to N$, respectively.

In this paper we restrict ourselves to considering the following case: a) $m_L = 0$ (m_L is the lepton mass) and b) $\beta = 0$ in (3). Thus, from (3) we consider only the single invariant

$$H = \frac{G_2}{\sqrt{2}} \mathfrak{m}_{\beta m}^{\alpha l}(\bar{N}, N)\, \mathfrak{m}_{\alpha l}^{\beta m}(\bar{L}, L). \tag{7}$$

2. The Case SU(2). If for a lepton we substitute into (1)

$$L^l = \begin{pmatrix} e^+ \\ \nu \end{pmatrix} = \frac{1}{2}(1-\gamma_5)\begin{pmatrix} e^+ \\ \nu \end{pmatrix}, \quad L_\mu^l = \frac{1}{4}\gamma_\mu L^l, \quad \bar{L}_\mu^l = \frac{1}{4}\bar{L}^l \gamma_\mu, \tag{8}$$

we obtain

$$H = \frac{G_1}{\sqrt{2}} \frac{81}{16\cdot 16}\left\{(\bar{l}\Gamma_\mu l)(\bar{\nu}\Gamma_\mu \nu) + \frac{17}{81}((\bar{l}\Gamma_\mu l)^2 + (\bar{\nu}\Gamma_\mu \nu)^2)\right\}, \tag{9}$$

$$\Gamma_\mu^\pm \equiv \gamma_\mu(1 \pm \gamma_5).$$

TABLE 2. Values of the Parameters a_{jl}^{m} and b_{jl}^{m}

m_l^m	$a_j \neq 0$	$b_j \neq 0$
m_1^2	$a_5 = \gamma\varepsilon_1 + \xi_1 \varkappa_1 + 2\alpha\varepsilon_1$, $a_6 = 2\varepsilon_2 \delta + 2\varkappa_2 \xi_2 + \beta\varepsilon_2$, $a_7 = \delta\varepsilon_1 + \xi_2 \varkappa_1 + 2\beta\varepsilon_2$, $a_8 = 2\gamma\varepsilon_2 + 2\xi_3 \varkappa_2 + \alpha\varepsilon_2$.	$b_5 = -5(\gamma\varepsilon_1 + \xi_1\varkappa_1) - \alpha\varepsilon_1$, $b_6 = -\delta\varepsilon_2 - \xi_4\varkappa_2 - 5\beta\varepsilon_2$, $b_7 = -5(\delta\varepsilon_1 + \xi_2\varkappa_1) - \beta\varepsilon_1$, $b_8 = -\gamma\varepsilon_2 - \xi_3\varkappa_2 - 5\alpha\varepsilon_2$.
m_1^3	$a_5 = \varepsilon_1\xi_3 + \rho\varkappa_1 + 2\alpha\varkappa_1$, $a_6 = 2\varepsilon_2\xi_2 + 2\sigma\varkappa_2 + \beta\varkappa_2$, $a_7 = \varepsilon_1\xi_4 + \sigma\varkappa_1 + 2\beta\varkappa_1$, $a_8 = 2\varepsilon_2\xi_1 + 2\rho\varkappa_2 + \alpha\varkappa_2$.	$b_5 = -5(\varepsilon_1\xi_3 + \rho\varkappa_1) - \alpha\varkappa_1$, $b_6 = -\varepsilon_2\xi_2 - \sigma\varkappa_2 - 5\beta\varkappa_2$, $b_7 = -5(\varepsilon_1\xi_4 + \sigma\varkappa_1) - \beta\varkappa_1$, $b_8 = -\varepsilon_2\xi_1 - \rho\varkappa_2 - 5\alpha\varkappa_2$.
m_2^3	$a_1 = 2\varkappa_1\varepsilon_1,\ a_3 = \varkappa_2\varepsilon_2$, $a_2 = (\gamma + 2\rho)\xi_3 + (\rho + 2\gamma)\xi_1$, $a_4 = (\delta + 2\sigma)\xi_4 + (\sigma + 2\delta)\xi_2$, $a_9 = (\delta + 2\sigma)\xi_3 + (\rho + 2\gamma)\xi_4$, $a_{10} = (\gamma + 2\rho)\xi_4 + (\sigma + 2\delta)\xi_2$.	$b_1 = -\varepsilon_1\varkappa_1,\ b_3 = -5\varepsilon_2\varkappa_2$, $b_2 = +(5\rho + \gamma)\xi_1 - (5\gamma + \rho)\xi_3$, $b_4 = (5\sigma + \delta)\xi_2 - (5\delta + \sigma)\xi_4$, $-b_9 = (5\delta + \sigma)\xi_3 + (5\rho + \gamma)\xi_1$, $-b_{10} = (5\gamma + \rho)\xi_4 + (5\sigma + \delta)\xi_2$.
m_1^1	$a_1 = -2,\ a_2 = 2(3\alpha^2 - 2)$, $a_4 = 2(3\beta^2 - 2),\ a_9 = 6\alpha\beta$, $a_{10} = 6\alpha\beta$.	$b_1 = -2,\ 2b_2 = 1 - 6\alpha^2$, $2b_4 = 1 - 6\beta^2,\ b_9 = -3\alpha\beta$, $b_{10} = b_9$.
m_2^2	$a_1 = 2(\varepsilon_1^2 - 1),\ a_3 = \varepsilon_2^2 - 2$, $a_2 = 2\xi_3^2 + \xi_1^2 + 3\gamma^2 - 2$, $a_4 = 2\xi_4^2 + \xi_2^2 + 3\delta^2 - 2$, $a_9 = 3\gamma\delta + \xi_1\xi_2 + 2\xi_3\xi_4$, $a_{10} = 3\gamma\delta + \xi_1\xi_2 + 2\xi_3\xi_4$.	$2b_1 = 1 - \varepsilon_1^2,\ 2b_3 = 1 - 5\varepsilon_2^2$, $2b_2 = 1 - 6\gamma^2 - 5\xi_1^2 - \xi_3^2$, $2b_4 = 1 - 6\delta^2 - 5\xi_2^2 - \xi_4^2$, $-2b_9 = 6\gamma\delta + 5\xi_1\xi_2 + \xi_3\xi_4$, $b_{10} = b_9$.
m_3^3	$a_1 = 2(\varkappa_1^2 - 1),\ a_3 = \varkappa_2^2 - 2$, $a_2 = 2\xi_1^2 + \xi_3^2 + 3\rho^2 - 2$, $a_4 = 2\xi_2^2 + \xi_4^2 + 3\sigma^2 - 2$, $a_9 = 3\rho\sigma + \xi_3\xi_4 + 2\xi_1\xi_2$ $a_{10} = a_9$.	$2b_1 = 1 - \varkappa_1^2,\ 2b_3 = 1 - 5\varkappa_2^2$, $2b_2 = 1 - 5\xi_3^2 - 6\rho^2 - \xi_1^2$, $2b_4 = 1 - 5\xi_4^2 - 6\sigma^2 - \xi_2^2$, $2b_9 = 6\rho\sigma + 5\xi_3\xi_4 + \xi_1\xi_2$, $b_{10} = b_9$.
	For the solution (1.28) $\sin\Theta = 0$	
m_1^2	$a_5 = c_1 c_5,\ a_6 = -5c_2 c_6$.	$b_5 = -5c_1 c_5,\ b_6 = c_2 c_6$.
m_1^1	$a_1 = -2,\ a_2 = a_4 = -4$.	$b_1 = 2,\ b_2 = b_4 = 0{,}5$.
m_2^2	$a_1 = a_3 = -2,\ a_2 = -1$.	$b_1 = b_3 = 0{,}5,\ b_2 = -2$.
m_3^3	$a_3 = a_4 = -1$.	$b_3 = b_4 = -2$.

Similarly, if into (7) for the nucleons we substitute

$$N^l = \begin{pmatrix} p \\ n \end{pmatrix}, \quad N_\mu^l = q_\mu N^l, \quad \overline{N}_\mu^l = \overline{q}_\mu \overline{N}^l, \quad q_\mu = -\frac{i}{m_n} p_\mu^{(n)}, \quad \overline{q}_\mu = -\frac{i}{m_p} p_\mu^{(p)}, \qquad (8')$$

and for the leptons we use the forms (8), we obtain

$$H = \frac{G_2}{\sqrt{2}} \cdot \frac{81}{16 \cdot 16} \cdot \frac{15}{9} \Big\{ \Big[F \Big(\overline{p} \gamma_\mu \Big(1 - \frac{5}{3}\gamma_5 \Big) n \Big) - 0{,}4 P_\mu (\overline{p} n) \Big] (\overline{\nu} \Gamma_\mu^- e^+) + [F (\overline{n} \gamma_\mu (1 - 1{,}133\gamma_5) n)$$

$$+ 0{,}80 P_\mu (\overline{n} n)] (\overline{\nu} \Gamma_\mu^- \nu) + \frac{8}{15} [F (\overline{p} \gamma_\mu \gamma_5 p) + 2{,}25 P_\mu (\overline{p} p)] (\overline{\nu} \Gamma_\mu^- \nu) + \cdots \Big\}. \qquad (9')$$

Here we have omitted the terms corresponding to the reactions $e + p \to e + p$ and $e + n \to e + n$, which can be obtained from those above by the substitution $n \to p$, $\nu \to e$ and $p \to n$, $\nu \to e$, respectively: $2F = 1 + q_\mu \overline{q}_\mu$, $2P_\mu = q_\mu + \overline{q}_\mu$. In the limit of a small momentum transfer, $F \to 1$, $P_\mu \to q_\mu$.

3. "Current on Current" Formalism. With respect to the spinor indices (α, β) the tensor $\mathfrak{m}_{\beta m}^{\alpha l}$ can be decomposed with respect to the vectors of the algebra of Dirac matrices. In the general case, the expansion contains all five vectors (scalar, pseudoscalar, vector, pseudovector, and antisymmetric tensor). However, in the case of leptons, when $m_L = 0$, under the assumption that all the leptons of the multiplet (octet) have the same helicity, only the vector and the pseudovector remain in the expansion. Then the Hamiltonian of the four-fermion interaction of the two multiplets (7) can be represented in the form

$$H=\frac{G_2}{\sqrt{2}}\{I_\mu(\bar{N}, N)_m^l J_\mu(\bar{L}, L)_l^m - I_{\mu 5}(\bar{N}, N)_m^l J_{\mu 5}(\bar{L}, L)_l^m\}, \tag{10}$$

where I_μ and $I_{\mu 5}$ are the baryon and J_μ and $J_{\mu 5}$ the lepton currents and pseudocurrents. In particular,

$$\begin{gathered}
I_{\mu m}^l=\bar{\varphi}^i \varphi_j\left(\prod_\mu^{ij}\right)_m^l, \quad I_{\mu 5 m}^l=\bar{\varphi}^i \varphi_j\left(\prod_{\mu 5}^{ij}\right)_m^l, \\
\prod_\mu^{ij} \equiv \frac{1}{4}(q_\mu+\bar{q}_\mu) \Omega_{ij}+\frac{1}{16}(1+q_\nu \bar{q}_\nu) \gamma_\nu \omega_{ij}, \\
\prod_{\mu 5}^{ij} \equiv \frac{1}{16}(1+q_\nu \bar{q}_\nu) \gamma_\mu \gamma_5 \omega_{ij}, \quad \Omega_{ijm}^l=\delta_{ij} \delta_{lm}+\frac{1}{2}(-i f_{ij\kappa}+3 d_{ij\kappa}) \lambda_{\kappa m}^l, \\
\omega_{ijm}^l=-2 \delta_{ij} \delta_{lm}+4\left(i f_{ij\kappa}+\frac{3}{2} d_{ij\kappa}\right) \lambda_{\kappa m}^l, \\
\bar{b}_m^l=\bar{\varphi}^i \lambda_{im}^l, \quad b_m^l=\varphi^i \lambda_{im}^l ; \quad \kappa, i, j=1,2,3,4,5,6,7,8 ;\ l, m=1,2,3.
\end{gathered} \tag{11}$$

f_{ijk}, d_{ijk} are the known structure coefficients of the Gell–Mann matrices. It is not difficult to show that decays of baryons of the octet with $|\Delta Q| = 1$, $|\Delta S| = 0$ and $|\Delta Q| = 1$, $|\Delta S| = 1$ (Q is the electric charge and S is the strangeness) are contained in the tensors $\mathfrak{m}_{\beta 2}^{\alpha 1}(\bar{N}, N)$, $\mathfrak{m}_{\beta 3}^{\alpha 1}(\bar{N}, N)$, respectively. At the same time, one can show that

$$\begin{gathered}
\Omega_2^1 \sim(\lambda_1-i \lambda_2), \quad \Omega_3^1 \sim(\lambda_4-i \lambda_5), \\
\omega_2^1 \sim(\lambda_1-i \lambda_2), \quad \omega_3^1 \sim(\lambda_4-i \lambda_5).
\end{gathered} \tag{12}$$

Therefore, when

$$\begin{gathered}
|\Delta Q|=1, \quad|\Delta S|=0, \quad H \sim \mathfrak{m}_2^1 \sim I_{\mu 2}^1 \sim(\lambda_1-i \lambda_2), \\
|\Delta Q|=1, \quad|\Delta S|=1, \quad H \sim \mathfrak{m}_3^1 \sim I_{\mu 3}^1 \sim(\lambda_4-i \lambda_5).
\end{gathered} \tag{12'}$$

Thus, the main assumption of Cabibbo theory in this formalism arises as a consequence of the general structure of the invariant, and moreover quite independently of the assumption about the structure of the lepton octet.

4. General Structure of the Baryon Tensor $\mathfrak{m}_{\beta m}^{\alpha l}(\bar{N}, N)$. By analogy with the formalism of the foregoing subsection, the Hamiltonian (7) can be regarded as the "tensor on tensor" formalism. The baryon tensor $\mathfrak{m}_{\beta m}^{\alpha l}(\bar{N}, N)$, if the solution (6) is substituted into it and it is written out in full, can be represented as a sum of terms of the following general form:

$$\begin{gathered}
\mathfrak{m}_{\beta m}^{\alpha l}(\bar{A} B)=a_{0m}^l D_{\beta\alpha}(\bar{A} B)+b_{0m}^l(1+q_\nu \bar{q}_\nu)(\bar{A}^\alpha B_\beta), \\
D_{\beta\alpha} \equiv \delta_{\beta\alpha}+(\gamma_\mu)_{\beta\alpha}(q_\mu+\bar{q}_\mu)+\frac{1}{2}[\gamma_\nu \gamma_\mu]_{-\beta\alpha} q_\mu \bar{q}_\nu, \\
a_{0m}^l,\ b_{0m}^l - \text{are constant numbers.}
\end{gathered} \tag{13'}$$

For example,

$$\begin{gathered}
\mathfrak{m}_{\beta 2}^{\alpha 1}(\bar{N}, N)=D_{\beta\alpha} \frac{1}{8}\left\{(\bar{p} n)-2(\bar{\Xi}^0 \Xi^-)+\frac{3}{\sqrt{6}}(\bar{\Lambda} \Sigma^-)-\frac{1}{\sqrt{2}}(\bar{\Sigma}^0 \Sigma^-)\right\} \\
-\frac{1}{4}(1+q_\nu \bar{q}_\nu)\{5 \bar{p}^\alpha p_\beta-\bar{\Xi}_0^\alpha \Xi_\beta^-+\sqrt{6}\, \bar{\Lambda}^\alpha \Sigma_\beta^-+2 \sqrt{2}\, \bar{\Sigma}_0^\alpha \Sigma_\beta^-\}.
\end{gathered} \tag{13}$$

Thus, the baryon part of the Hamiltonian (10) of the reaction $B \to A + l_1 + \bar{l}_2$, where A and B are baryons and l_1 and l_2 are leptons, can be characterized by a set of two constants (a_0, b_0) or (b_0, λ_0). The values of these parameters are given in Table 1.

5. General Structure of the Lepton Tensor $\mathfrak{m}_{\alpha l}^{\beta m}(\bar{L}, L)$. In this paper we operate with a lepton matrix of the general structure

$$L_m^l = \begin{pmatrix} \alpha\nu + \beta w & \varepsilon_1 e^+ & \varkappa_1 e^+ \\ \varepsilon_2 \mu^- & \gamma\nu + \delta w & \xi_1\nu + \xi_2 w \\ \varkappa_2 \mu^- & \xi_3\nu + \xi_4 w & \rho\nu + \sigma w \end{pmatrix}, \tag{14}$$

where $\alpha, \beta, \gamma, \delta, \rho, \sigma, \varepsilon_1, \varepsilon_2, \varkappa_1, \varkappa_2, \xi_1, \xi_2, \xi_3, \xi_4$ are arbitrary constants.

Substituting (14) into the lepton tensor $m_{\alpha l}^{\beta m}(\bar{L}, L)$, we obtain

$$m_{\alpha l}^{\beta m}(\bar{L}, L) = \sum_{j=1}^{10} \frac{1}{4}\left\{\frac{1}{4}(\gamma_\mu)_{\alpha\beta}(a_j)_m^l J_\mu^j + (b_j)_l^m J_{\beta\alpha}^j\right\},$$

$$J_\mu^{(1)} \equiv J_\mu^{(e)} \equiv (\bar{e}\gamma_\mu e), \quad J_\mu^{(2)} \equiv J_\mu^{(\nu)} \equiv (\bar{\nu}\gamma_\mu \nu), \quad J_\mu^{(3)} \equiv J_\mu^{(\mu)} \equiv (\bar{\mu}\gamma_\mu \mu), \tag{15}$$

$$J_\mu^{(4)} \equiv J_\mu^{(w)} \equiv (\bar{w}\gamma_\mu w), \quad J_\mu^{(5)} \equiv (\bar{\nu}\gamma_\mu e), \quad J_\mu^{(6)} \equiv (\bar{\mu}\gamma_\mu w),$$

$$J_\mu^{(7)} \equiv (\bar{w}\gamma_\mu e^+), \quad J_\mu^{(8)} \equiv (\bar{\mu}\gamma_\mu \nu), \quad J_\mu^{(9} \equiv (\bar{\nu}\gamma_\mu \nu), \quad J_\mu^{(10)} \equiv (\bar{w}\gamma_\mu \nu);$$

$J_{\beta\alpha}^j$ has a similar structure:

$$J_{\alpha\beta}^{(1)} = (\bar{e}^\alpha e_\beta + \bar{e}_\mu^\alpha e_{\mu\beta}), \quad J_\mu^{(2)} = (\bar{\nu}^\alpha \nu_\beta + \bar{\nu}_\mu^\alpha \nu_{\mu\beta}), \ldots J_{\alpha\beta}^{(10)} = (\bar{w}^\alpha \nu_\beta + \bar{w}_\mu^\alpha \nu_{\mu\beta}). \tag{16}$$

The values of the parameters $(a_1)_m^l, \ldots (a_{10})_m^l, (b_1)_m^l \ldots (b_{10})_m^l$ are given in Table 2.

6. General Form of the Hamiltonian of the Four-Fermion Process. If (13') and (15) are substituted into (7), then after multiplication we obtain the Hamiltonian of the four-fermion process $A \to B + l_1 + l_2$ in the form

$$H(\bar{A}B) = \frac{G_2}{\sqrt{2}}\frac{b_0}{64} C_j (\bar{A} D_\mu^j B) J_\mu^j, \quad C_j \equiv 8a_j + 7b_j, \quad \lambda_0 = 4\frac{a_0}{b_0},$$

$$\alpha_j = \frac{9b_j}{C_j}, \quad D_\mu^j = \lambda_0 \frac{1}{2}(q_\mu + \bar{q}_\mu) + \frac{1}{2}(1 + q_\nu \bar{q}_\nu)\gamma_\mu(1 - \alpha_j \gamma_5). \tag{17}$$

All the quantities necessary for determining H $(\bar{A}B)$ from (17) are given in Table 1.

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