THE CREATION OF PARTICLES FROM A VACUUM IN A NONSTEADY ISOTROPIC UNIVERSE

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The paper considers the creation of particles from a vacuum in a closed, open, and quasi-Euclidean Friedmann model. Finite general expressions are obtained for the density of the number of pairs created, and also new analytical estimates are given of the intensity of processes of creation at different stages of evolution of the universe.

The problem of pair creation in cosmology originated by the union of the theory of elementary particles and the general theory of relativity. In the papers by D. D. Ivanenko and A. A. Sokolov [1-3], interaction processes between quanta of a gravitational field and elementary particles were studied. In particular, D. D. Ivanenko and A. A. Sokolov [1-3] found the interaction cross section of two gravitons with the formation of an electron—positron pair.

Here we shall consider the effect of pair creation by a nonsteady classical gravitational field. The creation of particles and antiparticles by a gravitational field can be considered by analogy with the well-studied corresponding phenomenon in electrodynamics [4-8]. However, the specific properties of a gravitational field, in comparison with an electromagnetic field, are related with the necessity for distinguishing "true" and "virtual" gravitational fields and relating their special features to some or other specific solution of the problem.

Pair creation from a vacuum by a classical external field is a consequence of instability of the vacuum state. When the magnitude of the external field reaches a certain critical value, the work performed by the field at the distance of a Compton wavelength of the particle, becomes equal to $2mc^2$. In this case, virtual pairs, which are always present in the vacuum, escape to a mass surface and may be observed as real particles. On the other hand, the phenomenon of pair creation from a vacuum by external fields can be conceived also on a more rigorous basis without the inclusion of a model of the vacuum as a sea of virtual particles. It is well-known that the operator of a number of particles does not commute with the operators of the field, the current density, the particle density, etc. In consequence of this, there is always a so-called "zero" field in the vacuum, which manifests itself in zero vacuum fluctuations. The interaction of an external classical field with a zero field of particles and antiparticles can lead to the appearance of a form of parametric perturbation in the theory of oscillations. With this perturbation, the energy of the zero oscillations increases so strongly that the vacuum converts to a state containing a finite number of particles and antiparticles.

A study of the effect of particle creation from a vacuum by a gravitational field, is of great importance for cosmology. In particular, a point of view is possible according to which all observed matter was created from a vacuum at a certain characteristic time. Such a cosmology would be free from the difficulty associated with the infinite density of a substance in singularity. It is interesting also to consider the possibility of the role of the effect of pair creation during the relativistic collapse of a star. This has led to the appearance of a large number of papers devoted to the study of the effect of particle creation in cosmology (see, for example, K. P. Stanyukovich [9], L. Parker [10], Ya. B. Zel'dovich, and A. A. Starobinskii [11], and also [12, 13], etc.).

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Schwinger showed [4] that particle creation by an electromagnetic field is accompanied by polarization of the vacuum, which leads to the necessity for replacing Maxwell's equations by more general nonlinear equations [4]. If the processes of pair creation by a gravitational field are sufficiently intense, then Einstein's equations also, obviously, should be replaced by somewhat more general equations, which take into account the phenomenon of polarization of the vacuum.

Particle creation from a vacuum is considered in this paper, in homogeneous isotropic models of the universe. In this case, the effects of creation are not so intense that the reverse effect of the created particles on the starting metric should be taken into account and the question of the possible modification of Einstein's equations should be considered. Pair creation in the quasi-Euclidean and open Friedmann models had been considered previously [10, 12, 13]. Here we shall write the general expressions for the density of the created matter in open, closed, and quasi-Euclidean forms of the Friedmann model and also we shall give new numerical estimates for the number of particles created in unit volume of space at different stages of expansion of the universe.

The metric of homogeneous isotropic models in conformal-static form is written

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = a^2(\eta)\,[d\eta^2 - dr^2 - f^2(r)(d\theta^2 + \sin^2\theta\, d\varphi^2)], \tag{1}$$

where

$$\text{where } f(r) = \begin{cases} \operatorname{sh} r \\ r \\ \sin r \end{cases} \quad \text{for } \varkappa = \begin{cases} -1 \\ 0. \\ +1 \end{cases}$$

The quantity $\varkappa$ is a constant [when $a(\eta) = 1$] of the curvature of three-dimensional space and the time coordinate η is determined by the relation $dt = a(\eta)d\eta$, where t is the time in the synchronous frame of reference, c = 1.

The Klein—Fock equation for the quantum complex scalar field in the metric (1) is

$$\frac{1}{\sqrt{-g}}\,\frac{\partial}{\partial x^\mu}\left[\sqrt{-g}\, g^{\mu\nu}\frac{\partial\varphi}{\partial x^\nu}\right] + \left(m^2 + \frac{R}{6}\right)\varphi = 0, \tag{2}$$

where R is the scalar curvature (the tensor of the curvature is determined the same as in [14], $\hbar = 1$).

Equation (2) corresponds to the Lagrangian

$$L = \sqrt{-g}\left[g^{\mu\nu}\frac{\partial\varphi}{\partial x^\mu}\,\frac{\partial\varphi^*}{\partial x^\nu} - \left(m^2 + \frac{R}{6}\right)\varphi\varphi^*\right]. \tag{3}$$

The scheme of a quantum scalar field in isotropic Riemann space was developed in [15]. The general solution of Eq. (2) is written in the following way:

$$\varphi(\eta, \boldsymbol{x}) = \frac{1}{\sqrt{2}\, a(\eta)}\int dI\, [\psi_I(\boldsymbol{x})\, u_I^*(\eta)\, a_I^{(-)} + \psi_I^*(\boldsymbol{x})\, u_I(\eta)\, a_I^{(+)}], \tag{4}$$

where $\int dI$ denotes summing and integration with respect to the quantum numbers of one-particle states; $\psi_I(x)$ are eigenfunctions of the Laplace operator in the space part of the metric (1), with eigenvalues λ_I^2 and $u_I(\eta)$ is the solution of the equation

$$\ddot{u}_I(\eta) + \kappa_0^2(\eta)\, u_I(\eta) = 0, \quad \kappa_0^2(\eta) \equiv m^2 a^2(\eta) + \varkappa - \lambda_I^2 \tag{5}$$

with the starting conditions

$$u_I(\eta_0) = \frac{1}{\sqrt{\kappa_0(\eta_0)}}, \quad \dot{u}_I(\eta_0) = i\sqrt{\kappa_0(\eta_0)}.$$

The creation—annihilation operators of particles and antiparticles are denoted in accordance with [16].

Pair creation from a vacuum by a classical gravitational field is manifested mathematically in that the metric Hamiltonian [15] of the field (4) is diagonal with respect to the creation operator at a certain initial instant η_0 and when $\eta > \eta_0$, it becomes a nondiagonal quadratic form of the type [12, 13]:

$$H(\eta) = \frac{1}{2}\int dI\, \{E_I(\eta)\,(\overset{*}{a}{}_I^{(+)} a_I^{(-)} + a_{\bar{I}}^{(+)} \overset{*}{a}{}_{\bar{I}}^{(-)}) + F_I(\eta)\, \overset{*}{a}{}_I^{(+)} a_{\bar{I}}^{(+)} + F_I^*(\eta)\, \overset{*}{a}{}_{\bar{I}}^{(-)} a_I^{(-)}\}, \tag{6}$$

where

$$E_I(\eta) \equiv |\dot{u}_I(\eta)|^2 + \kappa_0^2(\eta)|u_I(\eta)|^2, \quad F_I(\eta) \equiv \pm [\dot{u}_I^2(\eta) + \kappa_0^2(\eta) u_I^2(\eta)],$$

$\tilde{I}$ is a set of quantum numbers for the complex-conjugate single-particle state.

The Hamiltonian (6) is written as Lagrangian, differing from Eq. (3) by the addition of the counterterm $(I\partial/a^2\partial\eta)$ $(a\dot{a}\varphi\varphi^*)$. The introduction of this counterterm, without changing the field equation (2), enables finite expressions for the density of the number of particles to be obtained.

The nondiagonal Hamiltonian (6) can be diagonalized by a Bogolyubov transformation of the creation—annihilation operators, of the type:

$$a_I^{(+)} = \alpha_I(\eta)\, b_I^{(+)}(\eta) + \beta_I^*(\eta)\, b_{\tilde{I}}^{(-)}(\eta),$$

$$a_I^{(-)} = \alpha_I^*(\eta)\, b_I^{(-)}(\eta) + \beta_I(\eta)\, b_{\tilde{I}}^{(+)}(\eta), \tag{7}$$

where $|\alpha_I|^2 - |\beta_I|^2 = 1$; the transformations for the operators $a_I^{*(\pm)}$ are obtained from Eq. (7) with a Hermitian conjugate.

The requirement for diagonality of Eq. (6) with respect to the operators $b_I^{(\pm)}$ and $b_I^{*(\pm)}$ reduces to the equation

$$|\beta_I(\eta)|^2 = \frac{|F_I(\eta)|^2}{4\kappa_0(\eta)\,[E_I(\eta) + 2\kappa_0(\eta)]}. \tag{8}$$

The vacuum state $|O_\eta>$, which is the state with the least energy in the sense of the Hamiltonian $H(\eta)$, is determined by

$$b_I^{(-)}(\eta)\,|O_\eta> = \overset{*}{b}{}_I^{(-)}(\eta)\,|O_\eta> = 0.$$

The matrix element of the density operator of the number of particles (antiparticles), determined at the instant η_0, with respect to the vacuum $|O_\eta>$ gives the expression for the density of the created pairs

$$n(\eta) = \frac{1}{a^3(\eta)} \int dI\, |\beta_I(\eta)|^2. \tag{9}$$

Numerical calculations of $n(\eta)$ from Eq. (9) and (8) are given in [12] for the case of the quasi-Euclidean Friedmann model. In this case, the function obtained by means of the first iteration of the Volterra integral equation equivalent to Eq. (5) was used for solving Eq. (5). Here we shall calculate $n(\eta)$ at an early and a recent stage of expansion of the universe, by means of the corresponding exact solutions of Eq. (5).

Let us consider first of all the case $\dot{a}(\eta)/a(\eta) \ll 1$, which corresponds to the recent stage of evolution of the universe. In this case

$$a(\eta) = a_0\,[1 + Ha_0(\eta - \eta_0)], \tag{10}$$

where $H \approx 10^{-28}$ per cm is Hubble's constant.

Taking Eq. (10) into account, Eq. (5) assumes the form

$$\frac{d^2 u_I(\xi)}{d\xi^2} + \omega_0^2(1 + h\xi)\, u_I(\xi) = 0, \tag{11}$$

where

$$\omega_0 \equiv \eta_0 \kappa_0(\eta_0), \quad h \equiv \frac{2m^2 a_0^3 \eta_0^3 H}{\omega_0^2}, \quad \xi \equiv \frac{\eta - \eta_0}{\eta_0}.$$

The exact solution of Eq. (11), with the initial conditions of Eq. (5), can be expressed in terms of an Airy function. Its asymptotic, for the condition $h\xi \ll 1$ [this is valid if $H(t - t_0) \ll 1$] is:

$$u_I(\xi) \approx \left(\frac{\eta_0}{\omega_0}\right)^{1/2} \left[\left(1 - \frac{h\xi}{4}\right) e^{i\omega_0\xi} + \frac{h}{4\omega_0}\sin\omega_0\xi. \right. \tag{12}$$

Substituting Eq. (12) in Eq. (8), we obtain after transformations

$$|\beta_I|^2 \approx \frac{h^2}{16\omega_0^2}\sin^2\omega_0\xi. \tag{13}$$

The expression can be obtained from Eq. (9) and (13) for the number of particles created per unit volume of space. For example, in the case of the open Friedmann model, we have

$$n \approx \frac{mH^2}{8\pi^2}\left(\frac{a_0}{a(\eta)}\right)^3 \int\limits_0^\infty \frac{x^2 dx}{(x^2+1)^3} \sin^2[\sqrt{x^2+1}\, ma_0(\eta-\eta_0)]. \tag{14}$$

With the natural condition $m(t-t_0) \gg 1$, Eq. (14) gives

$$n \approx \frac{mH^2}{256\pi} \sim 10^{-47}\frac{1}{\text{cm}^3} \tag{15}$$

for π mesons, which coincides with the calculation in [12]. From qualitative considerations, a result which coincides with Eq. (15) to an order of magnitude was obtained by K. P. Stanyukovich [9].

Let us now consider the case $a(\eta) = a_0\eta/\eta_0$, which corresponds to evolution of a universe filled with radiation, in the proximity of singularity.

In this case, the equation written in (5), in Weber's equation

$$\frac{d^2 u_l(\tau)}{d\tau^2} + q^2(\tau)\, u_l(\tau) = 0, \tag{16}$$

where

$$\tau \equiv \mu\frac{\eta}{\eta_0} = \sqrt{2ma_0\eta_0}\,\frac{\eta}{\eta_0}, \quad q^2(\tau) \equiv p^2 + \frac{\tau^2}{4} \equiv \frac{\eta_0^2(\varkappa-\lambda_l^2)}{\mu^2} + \frac{\tau^2}{4}.$$

The solution of Eq. (16), with the starting conditions of Eq. (5), is:

$$u_l(\tau) = \frac{i}{2}\left[\frac{\mu q(\mu)}{\eta_0}\right]^{-\frac{1}{2}} \left\{\left[\frac{\partial}{\partial\tau}E^*(-p^2,\tau)\Big|_{\tau=\mu}\right.\right.$$

$$\left. - iq(\mu)E^*(-p^2,\mu)\right]E(-p^2,\tau) + \left.\left[iq(\mu)E(-p^2,\mu) - \frac{\partial}{\partial\tau}E(-p^2,\tau)\Big|_{\tau=\mu}\right]E^*(-p^2,\tau)\right\}, \tag{17}$$

where

$$E(a,x) = \sqrt{2}\exp\left[\frac{\pi a}{4} + i\left(\frac{\alpha}{2}+\frac{\pi}{8}\right)\right]D_{-ia-\frac{1}{2}}\left(xe^{-i\frac{\pi}{4}}\right),$$

$\alpha \equiv \arg\Gamma(1/2 + ia)$ and $D_\nu(z)$ is the function of a parabolic cylinder [17].

Introducing the momentum variable $\kappa^2 = \varkappa - \lambda_l^2$ and calculating the asymptotic of Eq. (17) for the condition $\kappa/m \gg \eta/\eta_0$, we obtain from Eq. (8).

$$|\beta_l|^2 \approx \frac{m^4 a_0^4}{16\kappa^6\eta_0^2}\left[\left(\frac{\eta-\eta_0}{\eta_0}\right)^2 + \frac{4\eta}{\eta_0}\sin^2\kappa(\eta-\eta_0)\right]. \tag{18}$$

If we denote by κ_m the minimum value of momentum at which the condition of applicability of the asymptotic used is still satisfied, then it follows from Eqs. (9) and (18) that

$$n_{\kappa>\kappa_m} \approx \frac{3m^4}{128\pi^2\kappa_m^3}\,\frac{t-t_0}{t_0^3}. \tag{19}$$

Relation (19) coincides with [12] but gives a lower estimate for the density of pairs created in the initial stage of expansion of the universe. We note that the magnitudes of Eqs. (15) and (19) are small in comparison with the density of the matter specified by the metric (1). This result, as already mentioned above, allows the inverse effect of created particles in the starting metric to be neglected in the case of homogeneous isotropic spaces.

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