THE COSMOLOGICAL TERM, COMPENSATION AND SINGULARITIES

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The necessity for the cosmological term in the Einstein and other theories of gravitation is emphasized; in these theories a "compensation" interpretation of the gravitational field as a calibration field is of fundamental importance and it leads, in particular, to torsion and nonlinearities for spinor and other fields. In conjunction with these ideas, the importance of allowing for exposed singularities is pointed out and their analogy with the latest elementary particle models is stressed.

1. The Cosmological Term

The latest experiments and astronomical discoveries, the analysis of the topology and basic principles of the theory of gravitation, and the recent tendencies to unify all forms of interactions and particles have produced a multitude of generalizations of Einstein's general theory of relativity. We refer the reader to the literature for the details and limit ourselves here to a list of the different types of gravidynamics [1] since this will be useful in what follows: 1) the geometricized catalog of the Twenties of generalizations of Riemannian geometry which aimed at a unification with electromagnetism and which despite their lack of success produced, as is now clear, a number of useful ideas (generalized connectivity, torsion, etc.); 2) the pragmatic catalog in the Seventies of "viable" theories which agreed with the observations in the post-Newtonian approximation (the scalar-tensor theory and artificially constructed variants with additional parameters, together with the general theory of relativity); 3) the group catalog of gravidynamics, constructed usually in the spirit of the relativistic field theory and the theory of elementary particles. Of prime importance among these is the compensation theory which gives a unified interpretation of gravitation, electromagnetism, and other fields and which leads to a torsional as well as a curved space and which supplements the spinor equation with certain nonlinear terms which form the basis of a promising unified particle theory. The different variants can, of course, also be considered from the point of view of each of the three catalogs.

We deal first, however, (in view of the continuing discussions caused to some extent by an erroneous neglect of the cosmological term) with the form of the equations of general relativity which include the cosmological term (introduced by Herglotz) first used by Einstein for a static model universe. The recent increasing acceptance of this term (in the monographs by Möller, Mavridges, Ellis, Petrov, Weinberg, and others) is in contrast to the repeated rejections by Landau and Lifshitz, Wheeler, Rice, and Ruffini, and others.

Under general covariance and the usual restrictions by minimal nonlinearities and the ranks of the derivatives we can give the following arguments in favor of the cosmological term:

1) The construction of equations from the condition that the covariant derivative of the tensor should vanish (Einstein, metric theories of Thorne) requires for generality a "dummy" cosmological term. 2) The energy tensor is defined to within the addition of a cosmological-type term (Fock). 3) The variational principle (in the spirit of Hilbert) requires an invariant which leads to a cosmological term. 4) The Friedman cosmology and its nonuniform, anisotropic variants do not forbid a cosmological term (despite what Einstein thought). 5) The exclusion of this term would require additional conditions, such as an

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allowance for symmetry in excluding the mass of the neutrino or photon (Ivanenko and Brodskii). 6) The de Sitter cosmology with a cosmological term retains the meaning of the limiting case (as the Finstein model does partially). 7) Empirical data require a cosmological term (McVittie). 8) The term leads to general cosmological solutions which can in principle be tested (Lemaître—Kardashev increase in scale length; expansion acceleration and the problem of helium as discussed by Ivanenko, Gorelik, Aman and others).

Other arguments are: 9) the cosmological term gives an interesting comparison with the energy of a vacuum (McVittie, Zel'dovich). 10) Averaging of the "microscopic" equations induces the presence of the term (Shirokov and Fisher). 11) Averaging of the quantum-operator equations leads to the cosmological term (Gorelik). 12) Quantization of gravitation produces a cosmological term (Ivanenko and Brodskii [2], de Britt and others). 13) Generalization of general relativity theory to a space with torsion leads to a cosmological term (Ponomarev). 14) The general compensation treatment of gravitation leads to this term (Frolov). 15) The cosmological term plays a critical role in theorems on the presence of singularities (Penrose—Hocking).

Similar arguments hold in other variants of gravidynamics. We might note that the replacement of a constant cosmological term by a variable (scalar, say) term would imply the introduction of a term somewhat similar to the Hoyle matter source which leads to definite predictions which could be tested experimentally.

2. Compensating Fields

Salam has recently given the following outline of the main developments in compensation theory: 1) 1918, the electromagnetic field regarded as compensating (Weyl); compensons = photons. 2) 1929, the covariant derivative is compensating (Weyl, Fock, Ivanenko); compensons = gravitons. 3) 1954, compensons are compared to internal symmetries, isospin and so on (Yang, Mills). 4) 1972, unification of weak and electromagnetic interactions (Salam et al.) [2]. To 2) we should add the work done on the treatment of the gravitational field as a compensating field, transfering from constant to variable parameters in the Lorentz—Poincaré group [Utiyama, Brodskii, Ivanenko, and others (1959-1962)]. In 3) we should note the vector meson theory of Sakurai (1961).

We now give a short summary of compensation theory, basing our treatment on the recent results of the Moscow group (Sardanashvili, Danilyuk, and others). The basic idea is that the action integral of the fields ψ should remain invariant under a transformation from constant parameters of some Lie group G to localized parameters which are functions of the coordinates; "calibration" fields are introduced to "compensate" the terms arising from the derivatives and as Frolov has shown, these are of two different types: $h_a{}^\mu$, Δ_a^m (m is the index of the field group, a is the layer index, and μ enumerates the coordinates). The compensating derivative of the field function

$$D_a \psi = h_a^\mu \partial_\mu \psi - \varepsilon \Delta_a^m I_m \psi$$

(I are the group generators; ε is the "coupling constant") can be considered as the covariant derivative in a layer of stratified space whose base is coordinate space; this introduces a geometric structure into the layer with a connectivity Δ. In the particular case where G is the Lorentz group the h_a^μ are the usual tetrads and the layer is the tangent Minkowski space. The metric is defined in terms of the bilinear form $g^{\mu\nu} = h_a^\mu h_b^\nu g^{ab}$. From the condition that the covariant derivative of h_a^μ should vanish, generalized connectivities appear in the base; i.e., in the space—time

$$\Gamma^\nu_{\sigma\mu} = h^\nu_a h^q_{\mu,\sigma} - \varepsilon \Delta^m_\sigma I^a_{mb} h^b_\mu h^\nu_a,$$

and the asymmetric part of these necessarily introduces the torsion tensor $Q{:}Q^\nu_{\sigma\mu} = \Gamma^\nu_{[\sigma\mu]}$. The covariant derivative of the metric is in general nonzero.

From variation over ψ of the Lagrangian (for the analysis we refer the reader to [3, 4]) we get equations of motion in the layer (of the type given by Dirac and others) with a compensating derivative which generalizes our coefficients (Fock, Ivanenko) to an arbitrary stratification of space. In the particular case of a Lorentz group the presence of torsion as well as curvature is taken into account. Second, variation over the generalized tetrads leads to equations which generalize the Einstein gravitational equations (with the cosmological term) in the layer [5]. In the particular case of a Lorentz group we get the Einstein—Cartan gravitation equations (in Trautman's terminology) allowing for torsion as well as curvature. Finally, in the absence of torsion we obtain the usual equations with an Einstein—Hilbert kinematic left side and the energy tensor on the right. In general, however, it is reasonable to add to the energy tensor the source terms derived from the left side which describe the spin, isospin, and other symmetries. There thus arises the attractive possibility of allowing, for example, for the effect on the gravitational isospin and so on of the overall number of baryons in the universe $\approx 10^{80}$, similar to Trautman's suggestion of the cosmological allowance for spin.

Finally, by analogy with the Palatini method, variation over the "connectivities" Δ (which in the particular case of a Lorentz group are the Ricci rotation coefficients) gives a relationship between the compensating field and the symmetry currents of the original fields ψ; for the special case of a Lorentz group we get an important relationship (Kibble—Frolov) between the torsion and spin

$$Q^{\mu}_{\alpha\beta} = \varkappa \left(S^{\mu}_{\alpha\beta} + \delta^{\mu}_{[\alpha} S^{\nu}_{\beta]\,\nu}\right).$$

From the point of view of particle theory in special relativity, it is proper to take the spin together with the energy and the approach is reminiscent of Kazimir invariance; on the other hand, general relativity is degenerate in this sense. Taking the most general, quadratic-type, Langrangians suggested by compensation theory we get a differential rather than an algebraic equation for the torsion and the quantization of this leads to torsions.

In addition to the conclusion about torsion, and the allowance for the effect of the asymmetric part and also for the contribution from any symmetries (including internal), one of the most important results of the compensation theory is the introduction of nonlinear generalizations in the field equations. This very general result contains as a particular case the theorem first obtained by Rodichev on the addition of a cubic nonlinearity to the Dirac equation for total torsional antisymmetry. It is in general quite clear that by substituting into the spinor equation the linear torsion term which arises in the connectivity and whose source is the spin density $\sim\psi^2$, we obtain a cubic nonlinearity. In the simplest case of a Lorentz group the nonlinearity in the Lagrangian is $(\bar{\psi}\gamma_{\mu}\gamma_5\psi)^2$, and in the case of the conformal group necessary for describing the general fermion—fermion interaction we get the combination of a vector and pseudovector nonlinearity which is so familiar from the theory of weak interactions and beta decay [3].

It is well known that the Dirac spinor equation with the addition of cubic nonlinear terms [first introduced by Ivanenko (1938)] occupies, as a description of pramatter, a basic position in the unified theory of matter; this theory, as developed by Heisenberg and Durra, or in a slightly different way by the Moscow group (Brodskii, Kurdgelaidze, Naumov, Nugen Hgok Zao and others), has already produced important results. It is natural to think of giving a geometric interpretation on the basis of compensation theory (or single torsion) to the Heisenberg self-action term which produces the possibility of excited states in pramatter in the form of actual baryons, mesons, and so on.

Without going into detail, we emphasize that if we also take unitary symmetry ("nonlinear quarks") into account we can, by means of a special propagator and the method of perturbations, obtain masses which are close to the empirical values for mesons and baryons [of the octet and even decuplet (Hugen Hgok Zao and Naumov) which were not calculated by Heisenberg] and also the fine structure constant $\alpha = 1/115$ (if a number of additional assumptions are made; 1/120 for Heisenberg). The use of the less sophisticated but rather obvious method of "matching" enables the magnetic moments of hadrons to be obtained in very close agreement with experiment (Kurdgelaidze and Bas'yuni). It has recently proved possible to connect our nonlinear theory with the model which unifies weak and electromagnetic interactions (Hugen Ngok Zao). Such results of the nonlinear spinor theory of matter, which partially overlaps with the compensation approach (by the introduction of torsion), are very satisfying and promising. The possibility of real effects from gravitational or geometric phenomena on the structure of elementary particles ($\approx 10^{-14}$ cm) is also suggested by the allowance for the contribution from "strong" gravitation (Salam; see Papyrin) produced by f-mesons; according to normal ideas such classical effects are completely negligible in the microworld but quantum gravitation, our discreteness of space and so on can only become apparent at distances $\approx 10^{-32}$ cm [6]; we refer the reader to the last sections of this article for a discussion of analogs of gravitational and quark "strings."

3. Torsion

We see that torsion, which has been suggested by several arguments as one of the "closest" generalizations of Riemannian geometry on the introduction of asymmetric connectivity (Cartan, 1922, and also Weyl, Palatini, Einstein, and Wiesenbeck) and also from the point of view of the tetrad formalism (Ivanenko, 1964, and Rodichev, 1961, and others), now arises necessarily from a compensation approach which has proved to be fruitful in electrodynamics and mesodynamics. We refer the reader to the literature (Hehl [7], Ponomarov [8]) and limit ourselves here to a few points. Expanding the third-rank torsion tensor into irreducible parts

$$Q^{\alpha}_{\beta\gamma} = 1/2\,(\Gamma^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\beta}) = \tilde{Q}^{\alpha}_{\beta\gamma} - \frac{1}{3}\,(\delta^{\alpha}_{\beta}\,Q_{\gamma} - \delta^{\alpha}_{\gamma}\,Q^{\beta}) + \sqrt{-g}\,\varepsilon_{\beta\gamma\delta\rho}\,Q'^{\delta\rho\alpha},$$

$\tilde{Q}^{\beta}_{\alpha\gamma}$ is the traceless part; Q_{α} is the trace; Q'_{α} the pseudotrace; we retain for this particular case only the time component of the pseudotrace

$$Q'_{\alpha} = (\lambda^{1/2}(t),\ 0,\ 0,\ 0),$$

and we arrive at the normal Einstein—Hilbert type equations in which the additions from torsion give (through the connectivities) a cosmological-type term which depends on time. If we put $\lambda(t) \sim a^{-6}$ (where a is the Friedman factor) we remove the cosmological singularity.

The cosmological term clearly removes the singularity in playing the role of the centrifugal force if the classical description of "spinning" matter is used, $Q^{\alpha}_{\beta\gamma} = u^{\alpha} S_{\beta\gamma}$ ($u^{\alpha} S_{\alpha\beta} = 0$). The allowance for torsion, which simulates nonlinearity, promises the emergence of other promising ideas for cosmological models.

An analysis of the effects produced by torsion in the equations of motions of planets, for example, has been carried out by Ponomarev, who has also noted a connection with the Kerr metric [8]. The disappearance of singularities does not contradict the Penrose—Hocking statement because the energy condition of their theorem is not satisfied in torsional spaces.

4. Remarks on Singularities

We now deal with certain problems of singularities in relation with the dependent problem of the cosmological term which, as we have shown above, can prevent the appearance of collapse. The theory of collapse is based on the Schwarzchild solution for a point mass with its nonsingular event horizon (i.e., a "dressed" internal singularity) (see, however, the comment by Polishchuka) which is reached by a rest observer after an infinite time; a small charge (Nordstrom—Reissner) and a relatively small angular momentum (Kerr) do not alter the situation; however, **a charge which is large relative to the mass or a significant angular momentum makes a qualitative change** in the picture and eliminates the horizon [9]. The collapse of a "Schwarzchildoid" might correspond to the real situation if small perturbations did not alter the characteristics of the singularities. In fact, though, a Schwarzchildoid is idealized and degenerate since any small perturbations and even a transition to gravitational theories more general than relativity theory can greatly change the situation. For example, in the scalar-tensor theory, which is one of the most widely studied "closest" generalizations of general relativity, a test particle incident towards the center reaches a real singularity (not hidden behind the horizon) in a finite time as reckoned by an external observer [10]. According to Gorelik (whose results are used below) the picture of collapse in the scalar-tensor theory is thus significantly altered (despite the opinion of Thorne); however, collapse does still occur here, in spite of the apparently incorrect conclusions of a number of other authors.

We will, however, use standard general relativity to consider the stability of the Schwarzchild solution. As the simplest perturbation we take a scalar field in the spherically symmetric case; we start from the Lagrangian (and for the time being we omit the R/6 term)

$$2\varkappa L = -R + \gamma\,(\varphi_{\alpha}\varphi^{\alpha} - \mu^{2}\varphi^{2}),\ (\varphi_{\alpha} = \partial_{\alpha}\varphi).$$

The static, spherically symmetric, massless solution is (Fisher et al.)

$$ds^{2} = e^{\nu}\,dt^{2} - e^{\lambda}\,dr^{2} - r^{2}\,d\Omega^{2},\quad N = \exp\,(\nu - \lambda)/2;$$

$$M = \exp\left(\frac{\nu+\lambda}{2}\right),\quad \Delta = r^{2} + 4\gamma A^{2},\quad \left(Nr + \frac{r_0 - \sqrt{\Delta}}{2}\right)^{1-\frac{r_0}{\sqrt{\Delta}}}$$

$$\times\left(Nr + \frac{r_0 + \sqrt{\Delta}}{2}\right)^{1+\frac{r_0}{\sqrt{\Delta}}} = r^{2};\ M = N + rN',\ \varphi' = \frac{A}{r^{2}}\,N^{-1}.$$

An analysis of this solution shows that a Schwarzchild surface (where $g_{00} = 0$) does not exist for however a small γ. It may also be shown that the singularity of this solution is exposed and not hidden behind the horizon and that it is reached in a finite time. If the existence of a Schwarzchild surface were fundamental it would be strange to find it affected by an arbitrarily small field. A topological analysis of the exposed singularity at r = 0 shows that it is of dimension 2 (not counting the time dimension) as in the scalar-tensor theory.

A scalar field with a nonzero mass also removes the degeneracy, thus as it were "opposing" the formation of a Schwarzchild surface. The solution of the field equations here is more complicated but the following important general property can be stated: the expression for the mass of a central body obtained from the equations

$$2\varkappa m = \lim_{r\to\infty} r(1 - e^{-\lambda}),$$

$$2\varkappa m = \xi + \int_{\xi}^{\infty} \left[1 - (1 - \mu^2 R^2 \varphi^2) \exp\left\{-\int_{R}^{\infty} r\varphi^2\, dr\right\}\right] dR,$$

shows that for a sufficiently small mass $m < (r_{0\min}/2\varkappa) \neq 0$, there does not exist any ξ for which $e^{-\lambda(\xi)} = e^{\nu} = 0$; in the usual Schwarzchild solution, on the other hand, we have $1/g_{11} = e^{-\lambda(2\varkappa m)} = 0$ at the horizon for any value of the central mass. Solving the equations in the weak-field approximation we get an addition to the Schwarzchild field which clearly shows the effect of a scalar field in general relativity. Remembering that a scalar field in flat space (or, in other words, at a point sufficiently far from the center) has the form $\varphi = c\cdot\exp(-\mu r)/r$ we get the first-order correction to the Schwarzchild field as

$$\exp(-\lambda) \approx 1 - 1/r\,(r_0 - c^2 \exp(-\mu r)).$$

Thus as we approach the center, this field tends to "reduce" r_0 and it can be assumed that for a sufficiently large "charge" c the Schwarzchild surface is liquidated, i.e., we get the result quoted above. As in the Nordstrom—Reissner case that the Schwarzchild surface disappears for a sufficiently large charge, we now see that a scalar field is also capable of liquidating it.

A vector field in general relativity can also remove the degeneracy characteristic of the Schwarzchild solution. We take an action of the type

$$S = \int (\beta A_{\mu;\nu} A^{\mu;\nu} + \alpha A_{\mu;\nu} A^{\nu;\mu}) \sqrt{-g}\, d^4 x.$$

In the particular case $\alpha = -\beta$ we have the usual electrodynamics and the Nordstrom—Reissner solution. When $\alpha = \beta > 0$, an arbitrarily small field eliminates the Schwarzchild-type surface in the solution but there is a singularity at r = 0 of the same type as in the Nordstrom—Reissner solution (when the neutral test particle is repulsed from the central charged body). For $\alpha = \beta$ there is a special solution which does not have a Newtonian asymptote. With $\alpha = \beta$ (< 0) we get a singular Schwarzchild surface, where $R = \infty$.

These examples of scalar and vector fields with a central mass in general relativity, and also the example of a scalar-tensor field, and the analysis of the cosmological term, show the necessity for a more detailed analysis of singularities and collapse; this analysis must not be based only on the degenerate Schwarzchild solution. Despite the Thorne—Penrose idea of a "cosmic censureship" which would forbid exposed singularities, these singularities might in fact correspond to the actual situation. The question of the eternal existence of objects forming exposed singularities or of their formation by various processes requires further study. It is also necessary to clarify the role of the inevitable fluctuations, which up to now have been ignored in the Hocking thermodynamic analogy of the surface of a black hole with entropy. Of course, the allowance for quantum effects in the form of the Planck length, quantum fluctuations, the existence of maximum density and so on will make illusory the idea of purely classical singularities, of the absence of "hairs" and so on.

We now turn to a discussion of the analog of gravitational singularities with elementary particle models. The interesting possibility was recently noted of constructing geon of any size (possibly atomic) in the Kerr metric [11]; in the presence of charge, moreover, the Kerr—Newman field leads to an anomalous gyromagnetic ratio as with the Dirac electron (Carter). A point Schwarzchild singularity "swells" in the presence of a Kerr rotation into a ring filament (with a radius of the order of the Compton length for the electron case). In the more general class of solutions with additional parameters describing the mass dipole and magnetic monopole (Newman—Unti—Tamburino—Demyanskii) another singularity arises in the form of a filament which passes through the symmetry axis.

Burinskii and the present author suggest that these filaments can be compared to the relativistic 'string' or filament which in a number of models (Nambu)unitesas gluon the quarks which together with (or in place of) the charges carry the magnetic monopoles inside the proton or neutron.

The filaments of superconductor electrodynamics (Abrikosov) will correspond to singular solutions of nonlinear spinor theory of the same symmetry. The general analogy between this theory and superconductivity has previously been pointed out by Nambu. In the development of such interesting analogies, we must remember the theory of superluminal particles (tachyons) since the transition $v \gtrless c$ in generalized Lorentzian transformations (Terletskii, Sudarshan, Yudin, and others) can properly be compared with a transition through a Schwarzchild surface separating the so-called r- and t-regions (Novikov). Tachyonization is in turn related to monopoles. Is it not possible that tachyons play a role both inside protons and super-dense stars or collapsers? The possibility is not excluded that the analogies between gravitation and elementary particles will not only prove to be helpful models but will actually reflect a part of physical reality, especially in the use of Kerr strings inside particles and, according to our other hypothesis, in the idea of quarks and partons inside stars.

In any case, in entering the epoch of subparticles, super-dense stars, and cosmological formations, and the construction of a unified picture of the world, physics cannot avoid using the means of modern relativity theory and relativistic quantum mechanics. On the difficult path to the establishment of deeper theories it must make use of the undoubted riches of the compensation approach to fields and the nonlinear spinor theory.

LITERATURE CITED

1. D. D. Ivanenko, Catalogs of Gravitational Theories [in Russian]; Theory of Gravitation and the Principle of Equivalence by H. Treder [Russian translation], Atomizdat, Moscow (1973).
2. A. Salam, Preprint, Trieste, IC (73) 28 (1973).
3. B. N. Frolov and G. A. Sardanashvili, Izv. Vyssh. Uchebn. Zaved., Fiz., No.9 (1974).
4. B. N. Frolov, Vestnik MGU, Fiz., No.6, 48 (1963).
5. N. P. Konopleva and V. N. Popov, Calibration Fields [in Russian], Atomizdat, Moscow (1972).
6. D. Ivanenko, Tensor (Japan), 25, 161 (1972); Acta Phys. Hung., 32, 341 (1972); Convegno Relativita Generale, Firenze (1964).
7. F. Hehl, Phys. Letters, 36A, 225 (1971); J. Math. Phys., 12, 7 (1971).
8. V. N. Ponomarev, Bull. Acad. Pol. Sci., 19, 585 (1971).
9. G. E. Gorelik, Vestnik MGU, Fiz., No.6, 13 (1973).
10. G. E. Gorelik, Izv. Vyssh. Uchebn. Zaved., Fiz., No.1, 56 (1973).
11. A. Ya. Burinskii, Zh. Éksper. Teor. Fiz., 66, 406 (1974).