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Trend Surface Analysis as a Special Case of IRF-*k* Kriging[1]

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This short note establishes the equivalence between trend surface analysis with polynomials of order k *and IRF-*k *(intrinsic random function of order* k*) kriging with a nugget effect covariance model.*

KEY WORDS: IRF-*k* kriging, trend surface analysis.

INTRODUCTION

Trend surface analysis has been a popular method of mapping (Davis, 1973) which is superseded now by more sophisticated methods like splines and kriging. Formal equivalence between splines and kriging has been demonstrated (Matheron, 1981) and illustrated (Dubrule, 1981; Galli et al., 1984). Coefficients of a polynomial drift (or trend) can be obtained from the IRF-*k* kriging equations (Dubrule, 1981, Dowd, 1985). Equivalence between trend surface analysis and IRF-*k* kriging when a pure nugget effect covariance model is used, will be demonstrated hereafter.

The proof will be presented without loss of generality in a 2D context.

Trend Surface Analysis (Order *k*)

Let Z be the measured variable at *n* different locations having coordinates (u, v).

The model is simply

$$Z = \sum_{\substack{i+j=0 \\ i,j>0}}^{k} \beta_{ij} u^i v^j + \epsilon \tag{1}$$

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that is, the value is expressed as a polynomial of the coordinates (deterministic component) plus a random error (white noise). Equation (1) may be rewritten in matrix notation as

$$Z = X\beta \qquad \text{where} \tag{2}$$

- X is the $n \times p$ matrix of p monomials of Eq. (1) measured at n locations
- Z is the $n \times 1$ vector of random variables measured at n locations
- β is the $p \times 1$ vector of coefficients to estimate

Estimation of the βs takes place under the minimum square error criterion. This leads to the usual normal equations

$$b = (X'X)^{-1}X'Z \tag{3}$$

where b is the $p \times 1$ vector of estimates of β

Estimates at the n locations are obtained from

$$Z^* = Xb \tag{4}$$

Errors or residuals at n locations are thus

$$e = Z - Z^* \tag{5}$$

where e is the $n \times 1$ vector of residuals

Any estimate at a new location (u_0, v_0) is obtained from

$$Z_0^* = x_0'b \tag{6}$$

where x_0' is the $1 \times p$ vector with the monomials placed in the same order as in X.

IRF-*k* Kriging

In the IRF-*k* theory (Matheron, 1973), a linear estimate at any point is to be constructed from n data points, that is

$$Z_0^* = \sum_{i=1}^{n} \lambda_i Z_i = \lambda' Z \tag{7}$$

where Z is, as before, the $n \times 1$ vector of random variables for n locations, and λ is the $n \times 1$ vector of kriging weights.

In order for Z^* to be a minimum variance unbiased estimator, λs must be derived from the following system of linear equations

$$\begin{bmatrix} K & X \\ X' & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} k_0 \\ x_0 \end{bmatrix} \tag{8}$$

where K is the $n \times n$ covariance matrix between random variables at n locations, k_0 is the $n \times 1$ vector of covariances between data points and point to estimate, X is the $n \times p$ matrix with monomials (same as X in trend surface), λ is the $n \times 1$ vector of kriging weights, μ is the $p \times 1$ vector of Lagrange multipliers, and x_0 is the $p \times 1$ vector of monomials corresponding to the estimated point.

System (8) can be rewritten as

$$\begin{bmatrix} \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} K & X \\ X' & 0 \end{bmatrix}^{-1} \begin{bmatrix} k_0 \\ x_0 \end{bmatrix}$$

$$= \begin{bmatrix} K^{-1}(I - X(X'K^{-1}X)^{-1}X'K^{-1}) & K^{-1}X(X'K^{-1}X)^{-1} \\ (X'K^{-1}X)^{-1}X'K^{-1} & -(X'K^{-1}X)^{-1} \end{bmatrix} \begin{bmatrix} k_0 \\ x_0 \end{bmatrix}$$

$$= \begin{bmatrix} K^{-1}(I - X(X'K^{-1}X)^{-1}X'K^{-1}) & k_0 + K^{-1}X(X'K^{-1}X)^{-1}x_0 \\ (X'K^{-1}X)^{-1}X'K^{-1}k_0 & -(X'K^{-1}X)^{-1}x_0 \end{bmatrix} \tag{9}$$

Then, using Eq. (7)

$$Z_0^* = k_0'[I - X(X'K^{-1}X)^{-1}X'K^{-1}]'K^{-1}Z + x_0'(X'K^{-1}X)^{-1}X'K^{-1}Z \tag{10}$$

When $K = C_0 I$, Eq. (10) simplifies to

$$Z_0^* = 1/C_0 k_0'[I - X(X'X)^{-1}X']Z + x_0'(X'X)^{-1}X'Z \tag{11}$$

Substituting Eqs. (3), (4), and (5) into Eq. (11):

$$Z_0^* = 1/C_0 k_0' e + x_0' b \tag{12}$$

Comparing Eq. (12), with Eqs. (5) and (6), trend surface analysis and IRF-k kriging with a nugget effect covariance model yield the same estimates at every point except at data points where IRF-k acts as an exact interpolator.

From Eq. (12), k_0 will be a vector of zero's, when a nugget effect covariance is adopted, unless Z_0 coincides with Z_i. In the latter case, only the ith value of k_0 is nonzero and equal to C_0.

Hence, universal kriging and trend surface analysis give the same estimates except at data points where universal kriging restitutes the observed value (exact interpolator). (However, the estimated value from trend surface analysis can be retrieved with Eq. (12).

CONCLUSION

Trend surface analysis is a special case of IRF-k kriging where the covariance model is a nugget effect. IRF-k kriging is a more powerful model because it allows the use of any admissible covariance function of the random component.

In trend surface analyses, the order of the polynomial can be increased to improve the fit. However, this is hardly a solution because large-order polynomials lead to numerical precision problems in calculating the vector of estimates.

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