SOMMERFELD EFFECT IN SYSTEMS WITHOUT INTERNAL DAMPING

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The Sommerfeld effect is a manifestation of the law of conservation of energy and is a universal phenomenon. It is due to direct and feedback coupling between the excitation mechanisms and the vibrational loads. It must always exist, to a certain degree. A rather complete study of the effect has been given in the works of Kononenko and his group [1, 3, 5, 6] for different vibrating systems and for limited power electric motors. It follows from this study that in the steady-state case the various interaction effects collectively known as the Sommerfeld effect are caused by consumption of energy in internal damping of the vibrational system. The vibrational load "creates" a torque on the shaft of a motor proportional to the damping coefficient and this torque brakes the rotation of the shaft.

The present paper is concerned with the interaction effects in the case when a different energy dissipation channel is significant for the excitation mechanism: the case when radiation of elastic waves and sound in the surrounding objects is significant in the dynamics of the shaft. Machines are sources of noise and the radiation of acoustic energy is an undesirable factor which must be controlled. Hence it is necessary to estimate the effect of this factor on the operation of the machine. We consider the characteristic features of the limited excitation of elastic systems when a significant fraction of the consumed energy is transported by means of waves. We assume that the internal damping of the system is negligibly small. As an example, we consider the vibrations of an infinite plate in contact with an acoustic medium (a fluid) when the plate is subjected to a point excitation by an electric motor of limited power. For a system of this type, it is natural to represent the solution in the form of waves, since such dynamical characteristics as normal frequencies and normal modes do not occur because of the infinite extent of the system. Resonance methods are therefore inapplicable.

1. Derivation of the Basic Equations of the Interaction. We consider the bending vibrations of a thin elastic plate subjected to a point-force excitation generated by an electric motor of limited power. One of the surfaces of the plate is in contact with a fluid. We note that the vibrations of elastic plates in contact with a fluid and subject to applied loads is a key problem in understanding the dynamics of structures in naval engineering. This problem, in its various aspects, has attracted the attention of researchers in the last decade. The fundamental paper is [12], in which the significant effect of the fluid on the vibrations of the plate was established. The radiated power and far acoustic field was studied in [2, 4, 9, 13]. In the more recent papers [7, 14] the velocity distribution in the near field was studied in detail and in [10, 15] a detailed analysis was made of the energy flux in the near field for the two-dimensional case.

This problem graphically demonstrates the difficulties in hydroelasticity. These difficulties are compounded when, in addition to considering the dynamics of the elastic body and the acoustic fluid, we take into account their reaction on the driving force [6]. The present paper is concerned with an analysis of the combined system made up of the infinite elastic plate, the acoustic half-space, and the driving force.

We consider an elastic plate of thickness h, density ρ_0, and mean surface coinciding with the plane $z = 0$. We assume that the half-space $z < 0$ is occupied by a fluid of density ρ and speed of sound c (Fig. 1). We further assume that an electric motor is placed at the origin O of a cylindrical coordinate system r, φ, z. The motor has an unbalanced mass m at a distance a from the axis of the shaft. When the shaft rotates the vertical component of the inertial force of the mass m is $ma(d^2/dt^2(1 - \cos\Theta))$, where Θ is the deflection angle of the shaft, measured with respect to upward vertical. We will consider the bending vibration w of the plate, therefore only this vertical component is taken into account, assuming that it has

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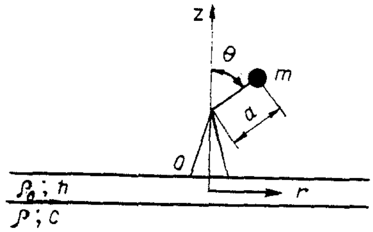

Fig. 1

circular symmetry with respect to the point O. Hence the problem is azimuthally symmetric with respect to the z axis. The equation describing the vibrations of the plate can then be written as [11]:

$$D\Delta^2 w(r, t) + \rho_0 h \frac{\partial^2 w(r, t)}{\partial t^2} = \frac{ma}{2\pi r}\delta(r)\frac{d^2}{dt^2}(1-\cos\Theta) + p(r, 0, t), \tag{1.1}$$

where D is the bending rigidity of the plate; $\Delta = \partial^2/\partial r^2 + [(1/r)(\partial/\partial r)]$; $\delta(r)$ is the Dirac function; $p(r, z, t)$ is the acoustic pressure of the fluid, which satisfies a wave equation of the form

$$\Delta p(r, z, t) = \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}. \tag{1.2}$$

The boundary condition is that the normal component of the velocity must be continuous in passing from the plate (where it is $\partial w(r, t)/\partial t$) to the fluid (where it is v_z). Therefore we have $\partial w/\partial t = v_z|_{z=0}$. Using the relation $\partial p(r, z, t)/\partial z = -\rho(\partial v_z/\partial t)$, which is satisfied in the acoustic field, the boundary condition transforms to

$$-\rho\frac{\partial^2 w(r, t)}{\partial t^2} = \frac{\partial p(r, z, t)}{\partial z}\bigg|_{z=0}. \tag{1.3}$$

The rotation of the shaft is caused by an electric motor of limited power [3], such that its output power is comparable to the power consumed by the vibrating plate. We let the moment of inertia of the rotor shaft be I, and the driving torque be $M(\dot\Theta)$. Then the equation of motion of the shaft, including the vibration $w(0, t)$, which the motor performs along with the plate, can be written in the form:

$$I\ddot\Theta = M(\dot\Theta) + ma\sin\Theta\left[g + \frac{\partial^2 w(0, t)}{\partial t^2}\right], \tag{1.4}$$

where g is the acceleration of gravity.

Equations (1.1) through (1.4) describe the complicated process of energy redistribution from the electric motor into the acoustic field of the vibrating plate. Using the Laplace transform with respect to time, and the Hankel transform with respect to the radial coordinate, general expressions for the bending deflection and pressure can be represented in the form

$$w(r, t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\int_0^\infty u(\lambda, s) J_0(\lambda r)\lambda e^{st} d\lambda ds;$$

$$p(r, z, t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\int_0^\infty \zeta(\lambda, s) J_0(\lambda r) e^{z\sqrt{\lambda^2+s^2c^{-2}}}\lambda e^{st} d\lambda ds. \tag{1.5}$$

Substituting these expressions into the boundary condition (1.3) and into (1.1) leads to the relation $u(\lambda, s) = \Phi(s)/\tau(\lambda, s)$, where $\tau(\lambda, s) = D\lambda^4 + \rho_0 hs^2 + \rho s^2/\sqrt{\lambda^2 + s^2c^{-2}}$; $\Phi(s)$ is the Laplace transform of the function $ma/2\pi[d^2/dt^2(1 - \cos\Theta)]$; hence $\Phi(s) = \frac{ma}{2\pi}\int_0^\infty \frac{d^2}{dt^2}(1 - \cos\Theta)e^{-st}dt$. The relations (1.5) can then be written as

$$w(r, t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\int_0^\infty \frac{\Phi(s)}{\tau(\lambda, s)} J_0(\lambda r)\lambda e^{st} d\lambda ds;$$

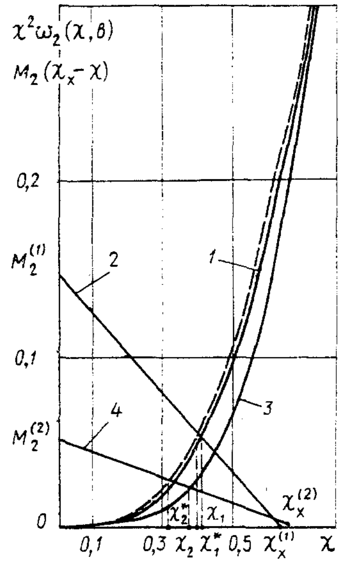

Fig. 2

$$p(r, z, t) = -\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\int_{0}^{\infty}\frac{s^2\rho\Phi(s)}{\tau(\lambda, s)\sqrt{\lambda^2+\frac{s^2}{c^2}}}e^{z\sqrt{\lambda^2+s^2c^{-2}}}J_0(\lambda r)\lambda e^{st}d\lambda ds. \qquad (1.6)$$

In order to obtain the explicit form of the solution for the bending vibrations of the plate w(r, t) and the pressure of the fluid p(r, z, t), the Laplace transform of the driving force of the hydroelastic system Φ(s) must be known. We therefore return to the equation for the rotation of the motor shaft (1.1). With the help of (1.6) it can be written in the form

$$I\ddot{\Theta} = M(\dot{\Theta}) + ma\sin\Theta\left\{g + \frac{ma}{(2\pi)^2 i}\int_{\sigma-i\infty}^{\sigma+i\infty}\int_{0}^{\infty}\frac{s^2e^{st}\lambda}{\tau(\lambda, s)}\left[\int_{0}^{\infty}\frac{d^2}{dt^2}(1-\cos\Theta)\,e^{-st}\,dt\right]d\lambda ds\right\}.$$

This is a nonexistence integrodifferential equation for the variable Θ. An exact solution of this equation for arbitrary initial conditions would be difficult to find. Therefore we study the special, but practically important case, when the interaction has reached the steady state.

2. Steady-State Interaction. The parameters describing the vibration of the plate and the angular velocity of the shaft can be obtained using the approximate Poincaré method. We introduce the small positive parameter $\varepsilon = mag/I\Omega_x^2$. Here Ω_x is the angular velocity of the shaft when the electric motor is idling. In this case

$$M(\Omega_x) = 0. \qquad (2.1)$$

For a steady-state interaction the driving torque corresponds to the static characteristics of the energy source. We put $M(\dot{\Theta})/I = \varepsilon M_1(\dot{\Theta})$. Then (1.7) can be written as

$$\ddot{\Theta} = \varepsilon M_1(\dot{\Theta}) + \varepsilon\Omega_x^2\sin\Theta + \frac{\varepsilon q\sin\Theta}{(2\pi)^2 i}\int_{\sigma-i\infty}^{\sigma+i\infty}\int_{0}^{\infty}\frac{s^2e^{st}\lambda}{\tau(\lambda, s)}\left[\int_{0}^{\infty}\frac{d^2}{dt^2}(1-\cos\Theta)\,e^{-st}dt\right]d\lambda ds, \qquad (2.2)$$

where

$$q = \frac{ma\Omega_x^2}{g}.$$

We introduce the traditional change of variable $\dot{\Theta} = \Omega$. The solution in the cases of interest to us can be written as an expansion

$$\Omega = \Omega_0 + \varepsilon\alpha_1\cos\Omega_0 t + \varepsilon\alpha_2\cos 2\Omega_0 t + \varepsilon\alpha_3\sin 2\Omega_0 t + \varepsilon^2\ldots$$

$$\Theta = \Omega_0 t + \frac{\varepsilon}{\Omega_0}\left(\alpha_1\sin\Omega_0 t + \frac{\alpha_2}{2}\sin 2\Omega_0 t - \frac{\alpha_3}{2}\cos 2\Omega_0 t + \frac{\alpha_3}{2}\right) + \varepsilon^2\ldots$$

$$w(r, t) = \frac{\Omega_0^2 ma}{2\pi} \operatorname{Re}\left\{e^{i\Omega_0 t} \int_0^\infty \frac{J_0(\lambda r)\lambda}{\tau(\lambda, i\Omega_0)} d\lambda\right\} + \varepsilon \ldots$$

$$p(r, z, t) = \frac{\Omega_0^4 ma}{2\pi} \rho \operatorname{Re}\left\{e^{i\Omega_0 t} \int_0^\infty \frac{J_0(\lambda r)\lambda e^{z\sqrt{\lambda^2-\Omega_0^2 c^{-2}}}}{\tau(\lambda, i\Omega_0)\sqrt{\lambda^2-\Omega_0^2 c^{-2}}} d\lambda\right\} + \varepsilon \ldots \tag{2.3}$$

Here Re denotes the real part; Ω_0, $\varepsilon\alpha_j$ ($j = 1, 2, 3$) are unknown constants which are to be determined.

In order to determine the constants we use an approximate representation of the integral $I_1 = \frac{c^2}{\Omega_0^2} \int_0^\infty \frac{\lambda d\lambda}{\tau(\lambda, i\Omega_0)}$, which appears in the expression for the bending deflection $w(0, t)$ of the plate. We write the integral as an expansion in the small parameter $b = \rho/c\,(D^{1/2}(\rho_0 h)^{3/2})$ and the dimensionless frequency $\chi = \Omega_0/\omega_c$ (where the frequency ω_c is given by $\omega_c = c^2(\rho_0 h/D)^{1/2}$). Then from [4] $I_1 = w_1(X, b) + iw_2(X, b)$. From (2.2) and (2.3) we obtain the following torque balance equation which can be used to find the frequency of the steady-state interaction:

$$\varepsilon M_1(\chi\omega_c) - \frac{\varepsilon q c^2 \omega_c^2}{4\pi D} \chi^2 w_2(\chi, b) = 0. \tag{2.4}$$

Approximating the static characteristic of the motor by a linear dependence of the form $\varepsilon M_1(\Omega_0) = N_0/I - (N_1/I)\Omega_0$ (where N_0 and N_1 are constants), we can determine χ from the equation

$$M_2(\chi_x - \chi) = \chi^2 w_2(\chi, b). \tag{2.5}$$

Here

$$M_2 = \frac{4\pi N_1 D}{(mac)^2 \omega_c}; \quad \chi_x = \frac{\Omega_x}{\omega_c} = \frac{N_0}{N_1 \omega_c}.$$

From (4) and (10) we also obtain $\alpha_1 = -\Omega_X^2/\Omega_0$, $\alpha_2 = q/4(\Omega_0 W_1)$, $\alpha_3 = -q/4\,(\Omega_0 W_2)$.

In addition, knowing $\chi = \Omega/\omega_C$, the solution of the hydroelastic problem (2.3) (by an appropriate choice of the contour of integration with respect to λ) can be represented as a sum of waves [8] in the plate and fluid: uniform waves propagating to infinity in the radial and vertical directions, and nonuniform waves localized near the origin and the surface of the plate.

Graphs of the functions $M_2(\chi_X - \chi)$ and $\chi^2 w_2(\chi, b)$ are shown in Fig. 2 corresponding to dimensionless driving and vibrational torques for $\chi < 0.7$ in the case of steel (curves 1, 2; $b = 0.13$; $M_2^{(1)} = 0.143$, $\chi_X^{(1)} = 0.646$) and aluminum (curves 3 and 4; $b = 0.39$; $M_2^{(2)} = 0.05$, $\chi_X^{(2)} = 0.65$) plates of equal thickness h in contact with water and excited by electric motors with identical characteristics.

The dashed curve corresponds to the case of no fluid ($b = 0$). It is evident from Fig. 2 that equation (2.5) has a single root. Unlike the case of a resonant interaction of a vibrating system with an electric motor, when even for a linear oscillator there are several roots to the analog of equation (2.5), in the system considered here there is a single interaction regime, when the motor has the angular velocity $\Omega_0 = \chi w_C$. This is because the admittance of a hydroelastic subsystem of infinite extent is approximately a power law in the excitation frequency [8]. We also point out that the decrease of the angular velocity of rotation of the motor shaft ($\chi_{1,2} < \chi_X^{(1,2)}$) is due in this case to excitation and radiation of waves, rather than to internal damping in the plate and the viscosity of the fluid [6]. Our results show that in this case the steel plate (both with and without a fluid in contact with the plate) consumes less energy than the aluminum plate, since the wave amplitude in the steel plate is smaller; this is manifested in an inequality of the form $\chi_1 > \chi_2$ for the steady-state case (the larger the consumption of energy, the smaller the angular velocity).

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THERMOELASTIC DEFORMATION OF A TRANSVERSALLY ISOTROPIC PROLATE SPHEROID

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In this article we present the exact solution of the static thermoelastic problem for a transversally isotropic prolate spheroid when an arbitrary temperature distribution is specified on its surface. It is assumed that the surface of the spheroid is free of external forces. Both interior and exterior problems are solved for the spheroid.

1. The static thermoelastic problem in absence of heat sources and bulk forces is described by the equations [3]

$$c_{11}\frac{\partial^2 u}{\partial x^2}+\frac{1}{2}(c_{11}-c_{12})\frac{\partial^2 u}{\partial y^2}+c_{44}\frac{\partial^2 u}{\partial z^2}+\frac{\partial}{\partial x}\left[\frac{1}{2}(c_{11}+c_{12})\frac{\partial v}{\partial y}+(c_{13}+c_{44})\frac{\partial w}{\partial z}\right]=\beta\frac{\partial T}{\partial x};$$

$$\frac{1}{2}(c_{11}-c_{12})\frac{\partial^2 v}{\partial x^2}+c_{11}\frac{\partial^2 v}{\partial y^2}+c_{44}\frac{\partial^2 v}{\partial z^2}+\frac{\partial}{\partial y}\left[\frac{1}{2}(c_{11}+c_{12})\frac{\partial u}{\partial x}+(c_{13}+c_{44})\frac{\partial w}{\partial z}\right]=\beta\frac{\partial T}{\partial y};\quad(1.1)$$

$$c_{44}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial x^2}\right)+c_{33}\frac{\partial^2 w}{\partial z^2}+(c_{13}+c_{44})\frac{\partial}{\partial z}\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)=\beta'\frac{\partial T}{\partial z};\quad(1.2)$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\beta''\frac{\partial^2 T}{\partial z^2}=0,$$

where c_{ij}, β, β', β'' are constants depending on mechanical and thermal properties of the body.

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