PARAMETRIC INSTABILITY OF GLASS-PLASTIC CYLINDRICAL SHELLS

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UDC 539.3:534.1

Up to the present time there have been a large number of theoretical investigations of parametric vibrations of round cylindrical shells [1, 4] and others. Experimental studies have been made of the parametric vibrations of metallic and glass-plastic shells [2, 6, 7].

The present article considers the parametric instability and the effect of damping on the "threshold" of the parametric perturbation of thin glass-plastic cylindrical shells having a high density of the spectrum of the natural frequency of the bending vibrations.

This investigation was undertaken with the aim of clarifying the specific special characteristics of the parametric instability of shells made of glass-plastic, a material which is characterized by the inhomogeneity of its mechanical properties and its greater ability to absorb vibrations in comparison with metals [5].

Investigations were made of shells (L = 0.6 m, d_m = 0.32 m, h = 0.510^{-3} m), one of whose edges was rigidly fastened and played the role of a base, while the other (upper) edge was free. Studies were made both of the principal region of instability (PRI) and of secondary region (side) of instability: first (SRI 1) and second (SRI 2).

Figure 1 shows the experimental unit for the investigation of the parametric instability of cylindrical shells. The attachment of the shell corresponds to the following edge conditions:

$$\text{for } x = \alpha = 0, \ u = v = w = \frac{\partial w}{\partial \alpha} = 0;$$

$$\text{for } x = l, \text{ and } \alpha = \alpha_0, \ N_x = M_x = M_{x\beta} = Q_x = 0.$$

Here x is the distance of a point of the middle surface of the shell from the base; u, v, and w are the displacements of points of the middle surface of the shell, respectively, in longitudinal, peripheral, and radial directions; M_x and $M_{x\beta}$ are the bending and torsional moments (torque), referred to a unit of length; N_x is the normal stress at the middle surface of the shell; β is the angular coordinate in a system of cylindrical coordinates; Q_x is the reduced (in the Kirchhoff sense) transverse intersecting load; $\alpha = x/a$; $\alpha_0 = l/a$; and a is the mean radius of the shell.

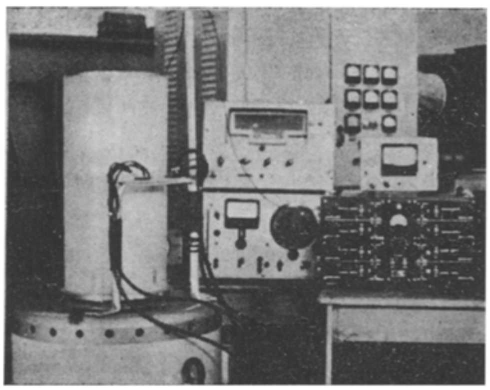

Fig. 1

Institute of Mechanics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Prikladnaya Mekhanika, Vol. 13, No. 4, pp. 124-128, April, 1977. Original article submitted March 10, 1976.

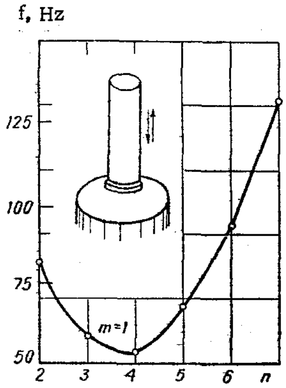

Fig. 2

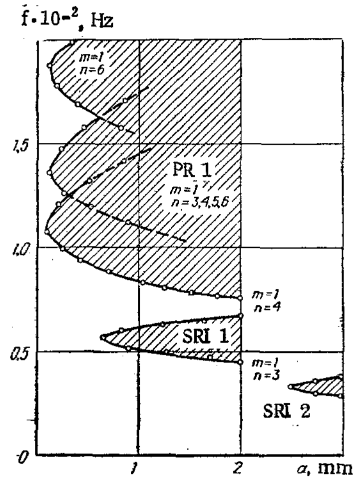

Fig. 3

The investigations were made in a glass-plastic shells with longitudinal – transverse windings. Specifically, a study was made of a shell, obtained by the two-layer winding of nonalkaline "Steklonite" No. 9, 6 on a special mandrel. The direction of the filaments in the inner and outer layers coincides with the guide and the generatrix of the shell. The composition of the binder used in preparing of the shells included ÉD-6 epoxide resin, Bakelite lacquer A, and glue BF-4. The shells were hardened at a temperature of 150°C for a period of 30 h.

To assure rigid attachment of the lower edge, the shell was inserted into a round groove with a depth of 10 mm and a width of 3 mm in an aluminum disk. Before this, the groove was filled with epoxide resin, after whose hardening the above edge conditions were assured.

The disk was fastened by bolts to the table of a D-10 electrodynamic vibrator, fed from a UPV-15 amplifier. The amplitude and frequency of the vibrations were varied using a GZ-34 sonic generator. The vibrational accelerations were measured on a PIU-1M piezoelectric acceleration measuring instrument, which had a PDU-1 acceleration pickup; the instrument was mounted on the table of the vibrator. The frequency of the longitudinal and transverse vibrations with the onset of instability was measured using strain gauges, an 8AN4 amplifying device, and an F-519 frequency meter. Two strain-gauge pickups for each of the investigated modes of the vibrations were glued one above the other in such a way that their sensing elements were perpendicular to one another. Simultaneously, the signal from the strain gauges entered the loops of an MPO-2 oscillograph. From the appearance of vibrations on the screen of the oscillograph, the limits of the principal and secondary regions of instability were observed. The strain gauges were installed over the whole circumference of the shell at a distance of 0.34 m from the base.

Before going on to investigation of the parametric instability of glass-plastic cylindrical shells, for each shell a frequency curve was determined experimentally; one such curve is shown in Fig. 2, where along the axis of ordinates there are plotted the natural frequencies of the vibrations of the shell and along the axis of abscissas, the number of surrounding waves of vibrations n. The frequency curve was recorded for one longitudinal half-wave m = 1. The experimental investigations of the parametric instability were made in a range of frequencies 80-200 Hz for wave parameters m = 1 and n = 2-6, with determination of the principal region of instability, and below, with the investigation of the side regions of instability SRI 1 (secondary region of instability 1) and SRI 2 (secondary region of instability 2).

Figure 3 shows the regions of instability of the shell under investigation; along the axis of ordinates there are plotted the frequencies of the perturbation and, along the axis of abscissas, the amplitudes of the perturbation. The limits of the principal region of instability are observed with vibrations taking place with a frequency close to the doubled frequency of the natural frequencies; therefore, they can be established with a high degree of accuracy. Outside the region of instability, the vibrations of the shell take place with the frequency of the external load. The approach to the limits of the region of instability is effected from the side of the stable region; by varying the frequency and the amplitude, they can be displaced along the boundaries in any given direction.

Thus, as a result of the investigations carried out, it has been established that the principal region of instability is bounded from below by a curve corresponding to a mode of vibration with the wave parameters m = 1 and n = 4, which corresponds to the lowest natural frequency of the vibrations of the shell. From above, the

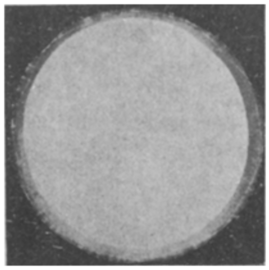

Fig. 4

principal region of instability in the present case is bounded by a curve corresponding to a mode of vibrations with the wave parameters $m = 1$ and $n = 6$.

As the experimental investigations showed, the presence of damping in the shell decreases the regions of instability by shifting them to the right of the axis of ordinates. A "threshold" of the parametric perturbation appears, whose presence excludes the onset of a loss of stability with small amplitudes of the perturbation (in the present case, down to 0.1 mm).

Figure 4 shows a photograph of the free edge of a shell, in which there can be seen characteristic transverse vibrations of the shell for loss of stability with five surrounding waves. The amplitude of the transverse vibrations of the upper end of the shell attained approximately 1-1.5 cm, depending on the mode of the vibrations. Under these circumstances, the base of the shell was displaced with an amplitude of tenths of a millimeter.

An investigation of the first (SRI 1) and second (SRI 2) side regions of instability was carried out in a GRW 3.05.02 – DK vibration test stand with a frequency range of 20-1000 Hz, which permitted attaining large amplitudes (1-4 mm). With respect to the side regions of instability, the effect of damping becomes more considerable; the "threshold" of the perturbation is found to be higher. Thus, in the given case, a loss of stability for the side resonances (SRI 1 and SRI 2) did not set in with amplitudes of the perturbation of 0.6 and 2.8 mm. During the course of the investigations, it was observed that the transition of a vibrating shell from the region of stability to the region of instability is accompanied by a "bang," which can lead to failure of the shell. Transitions from one mode of vibrations to another inside of the principal region of instability can also bring about failure of the shell, as has been observed during the course of an experiment. Changes in the phase of exactly the same mode of vibrations within the principal region of instability are also dangerous.

As a result of the investigations carried out, the following conclusions can be drawn.

1. The experimental unit described permits investigating the principal and side regions of dynamic stability of cylindrical shells in a wide range of parameters of the kinematic perturbation.

2. As a result of the large density of the spectrum of the natural frequencies of the bending vibrations, the principal regions of dynamic instability of thin cylindrical shells occupy a large part of the plane of the parameters of the kinematic perturbation.

3. The effect of internal friction in the material of the shell has a considerable effect only on the side regions of dynamic instability. For a glass-plastic shell, characterized by high damping properties [3], the "threshold" level of the kinematic perturbation for the first side region exceeds by 5-6 times, and for the second side region by 30 times, the corresponding value of the level for the principal region of parametric instability.

4. Effects have been observed, taking place in regions of parametric instability and accompanied by a "bang," which can lead to failure of the shell.

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CALCULATION OF TEMPERATURE FIELDS IN PROBLEMS OF THERMOELASTICITY OF A TWO-TEMPERATURE ISOTROPIC MEDIUM

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UDC 536.241

The investigation of the stress–strain state in multicomponent materials and structural elements made from them under conditions of nonuniform heating is bound up with a need to determine the temperature fields. Under these circumstances, the multicomponent (multiphase) nature of the medium leads to the concept of a multitemperature continuum [1, 4]. The construction of models of multiphase media has been carried out by a number of authors [1-4]. The description of the thermal conductivity of a multiphase heterogeneous medium on the basis of a multitemperature homogeneous continuum, in the case of different temperatures of the phases, should lead to a thermal-conductivity equation with effective characteristics of the medium. In [4] this condition is not satisfied. In [6] thermal-conductivity equations are derived for a two-phase solid body, satisfying the above requirement.

The present article, on the basis of the equations of the thermal conductivity of a two-phase isotropic solid body [6], considers the problem of the distribution of the mean temperature and the difference in the temperatures of the phases in an unbounded plate.

For solution of this problem, we use the equations of the thermal conductivity of a two-phase isotropic solid body [6]

$$\lambda^* \langle t \rangle_{,kk} + \langle F \rangle = c^* \rho^* \frac{\partial \langle t \rangle}{\partial \tau} + p_1 p_2 (c_1 \rho_1 - c_2 \rho_2) \frac{\partial (\langle t_1 \rangle - \langle t_2 \rangle)}{\partial \tau};$$
$$p_1 p_2 (c_1 \rho_1 \mu + c_2 \rho_2 \lambda) \frac{\partial (\langle t_1 \rangle - \langle t_2 \rangle)}{\partial \tau} + \lambda^* \alpha (\langle t_1 \rangle - \langle t_2 \rangle) = (p_2 c_2 \rho_2 \lambda - p_1 c_1 \rho_1 \mu) \frac{\partial \langle t \rangle}{\partial \tau} + p_1 \mu \langle F_1 \rangle - p_2 \lambda \langle F_2 \rangle. \quad (1)$$

Here p_i is the volumetric concentration of the i-th phase; $\langle F_i \rangle$ is the mean density of the heat source over the i-th phase; $\langle t_i \rangle$ is the mean temperature of the i-th phase; $\langle t \rangle$ is the mean temperature; λ_i, c_i, ρ_i are, respectively, the coefficient of thermal conductivity, the heat capacity, and the density of the i-th phase; λ^*, c^*, ρ^* are, respectively, the macroscopic coefficient of the thermal conductivity, the macroscopic heat capacity, and the density of the two-phase body; α is the heat-transfer coefficient between the phases, which depends on the thermophysical properties and the microstructure of the two-phase medium.

We note that λ, μ, and α can be taken in the form [6]

$$\lambda = \frac{\lambda_1 (\lambda^* - \lambda_2)}{\lambda_1 - \lambda_2}; \quad \mu = \frac{\lambda_2 (\lambda^* - \lambda_1)}{\lambda_2 - \lambda_1}; \quad \alpha = 2\sqrt{15} \frac{p_1 p_2 \lambda_1 \lambda_2}{(p_1 \lambda_1 + p_2 \lambda_2) l^2}, \quad (2)$$

where l is the scale of the correlation.

Institute of Mechanics, Academy of Sciences of the Ukrainian SSR, Kiev. Translated from Prikladnaya Mekhanika, Vol. 13, No. 4, pp. 128-131, April, 1977. Original article submitted May 5, 1976.