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Optimal mine production scheduling: evaluation of large scale mathematical programming approaches

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Summary

Computational difficulties arising in the solution of linear programming models have mitigated against their widespread use in mine scheduling. These difficulties are identified and discussed and key avenues for further research are isolated.

Key words: Optimization; mine production scheduling; linear programming

Introduction

The advantages of using linear programming for solving mine planning and scheduling problems have been recognized since the 1960s. Manula (1965), Kim (1967), Johnson (1969), Meyer (1969) and Ramani (1970) all addressed these problems using linear programming formulations. While a number of applications were performed over the next decade (Gangwar, 1973; Wilke and Reimer, 1977; Smith, 1978), linear programming has not become the predominant method of mine scheduling. One reason this has not happened is that, although the theory may be accepted, computational difficulties arise in the solution of the linear programming models.

This paper identifies and addresses these computational difficulties by presenting the various formulations for mine scheduling, pointing out the advantages and disadvantages of each, and making recommendations for their use. In addition, key avenues for further research are identified. The primary aim of the paper is to stimulate and direct new research toward achieving an efficient scheme for the accomplishment of mine scheduling optimization.

The next section summarizes a general mine scheduling optimization approach (Gershon, 1983). This approach addresses a number of problems, the most important and difficult being the mine production scheduling problem. This problem is addressed in the remaining

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sections of the paper. Finally, conclusions are summarized and suggestions outlined for future research.

Mine scheduling optimization

The mine scheduling optimization (MSO) approach (Gershon, 1983) is an application of linear programming to the problem of mine production scheduling. The philosophy leading to its development can be summarized as follows.

Production scheduling should include the mining locations, not just the tonnages

Most linear programming applications for mine production scheduling define the production levels in terms of tonnage mined from one or more mines (Albach, 1967; Barbaro and Ramani, 1983). The advantage of taking this approach is that the computational difficulties discussed later in this paper are avoided. The disadvantage is that the detailed sequence of mining is ignored. This mining sequence is the most important aspect of mine production scheduling and, despite any difficulties caused, must be the major result of any production scheduling algorithm. The necessary input required to achieve this goal is a detailed set of block precedence relationships for the production blocks.

Production scheduling cannot be done independently of hauling, blending, processing, sales contracts, etc.; simultaneous optimization is required

The problems shown in Fig. 1 can be solved by various optimization methods. However, if each is solved separately, the resulting solutions may not be consistent. For example, the

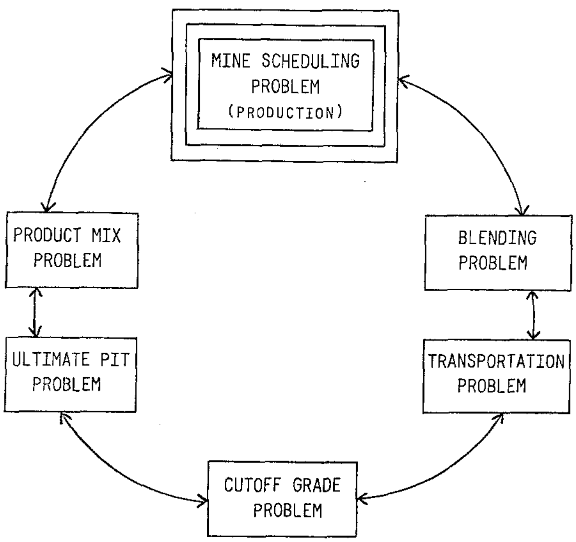

Fig. 1. Linear programming subproblems.

'optimal' mining sequence may yield a different mill feed grade than the derived 'optimal' grade for the mill. To ensure that a true optimum is found for the overall level of mine operations, these problems must be solved simultaneously, each being solved subject to the restrictions imposed by the others.

Long term considerations should guide short term production scheduling

Past attempts to increase short term profits without considering long term consequences have resulted in increased costs for many operating mines today. Long term strategies must be developed prior to considering short term schedules and these schedules must be consistent with those strategies. In addition to requiring that long term strategies guide short term schedules, the time periods should be arranged to achieve a greater level of detail in the short term than in the more distant future. This can be accomplished by using variable length scheduling periods, the early ones being of shorter duration.

Ultimate pit limits and cut-off grades should result from these long term considerations: not specified in advance

The idea of dynamic cut-off grades has been implemented and described elsewhere (Lane, 1964). Essentially, it asserts that the economic cut-off grade at any given time is a function of the ore availability and the needs of the mill at that time. One cut-off grade, dictated by management, fails to allow the flexibility needed to maximize profits in different situations.

Ultimate pit limits cannot take into account the time value of revenues and costs unless the sequence of production within those limits is known. Therefore, the ultimate pit limits, as they are currently developed, are based on seriously skewed economic data. While they have value for other reasons, they should not be given a great deal of validity for use in production scheduling. Actually, the long term production schedule should yield the ultimate pit if the scheduling horizon is equal to the life of the mine.

Analysis of long term plans requires consideration of a large number of interdependent variables

Long term schedules are often designed to achieve a balance over time with respect to strip ratios or some characteristic of the ore being mined. Trade-offs in decisions between one period and another or between content of two or more different minerals becomes very complex.

No group of people can consider more than a few of these interdependencies at a time

The human mind can only handle five to seven variables at one time when evaluating decisions (Shepard, 1964). Usually, decisions are based on just one or two primary considerations, while some attempt is made to take into account an additional one or two secondary considerations.

The interdependencies can be identified

The decision to produce at a given location at a given time can be evaluated with respect to its impact on mill feed, hauling capacity, ore availability for the next period, wall slope and other factors. These impacts can be specified individually, yet they cannot all be considered at once unless a computer is used.

Once identified, an optimization algorithm can analyse these variables through the use of a computer

Thousands of variables need to be considered for long term mine scheduling. A computer can be programmed to evaluate all of these variables simultaneously. In addition, a procedure can be programmed to satisfy all mining requirements while choosing the set of decisions that optimize some objective such as profits or life of mine.

Any algorithm applied must be applicable to different mines and different ores

A core model should be developed describing, for example, an open pit mining situation. This model can then be used, with only minor changes, for any open pit mine. Then, only ore-specific or property-specific additions need to be made.

Linear programming is the applicable optimization algorithm to optimize production scheduling

Linear programming has five major advantages for use in mine production scheduling. First, it can optimize in terms of thousands of decisions and thousands of restrictions. Second, it does this more efficiently than a number of other optimization procedures, when certain assumptions hold (Hillier and Lieberman, 1981). These assumptions do hold for mine production scheduling. Third, valuable information is provided concerning the value of obtaining additional resources and the value of relaxing operating restrictions. This is helpful in identifying areas of management concern for additional study. Fourth, the means for accomplishing this additional study, sensitivity analysis on the inputs, is provided. Linear programming performs sensitivity analysis efficiently. There is no need to solve the entire problem a second time. Finally, since linear programming (LP) models are generated as systems of equations, general sets can be used for multiple projects without having to redesign the complete model for each mine encountered.

This philosophy led to the development of the MSO approach as shown in Fig. 2. The critical step in this approach is the building of the mathematical model. After this, the remaining steps can be viewed as a 'black box'. Once this mathematical model is developed, only the model is optimized, not the mine. Therefore, careful attention must be given to assure that the model accurately reflects the mining realities.

Having achieved a number of successes in applying this approach, it can now be considered to be a proven tool for the mining engineer. It has yielded workable mine plans as well as improving upon long term strategies. Many short term and mine specific savings have also been realized. Most important, it is consistent with all of the required philosophical design motivations.

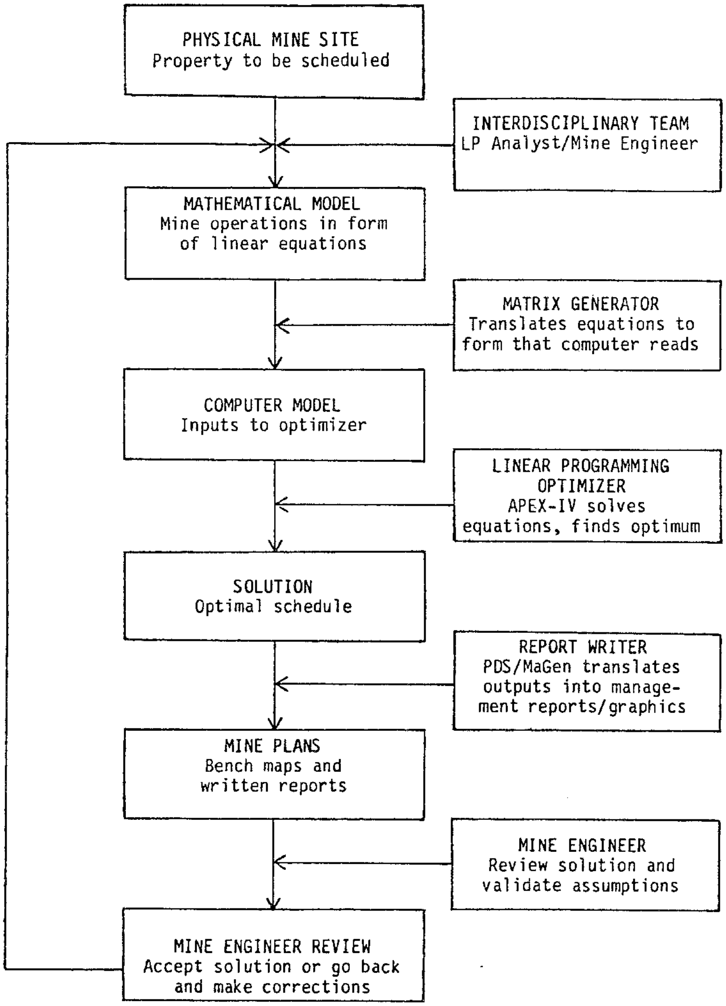

Fig. 2. MSO project organization.

The production scheduling problem

The production scheduling problem, while only one part of MSO, is the most difficult part to solve. It involves the sequencing of the scheduling units (production volumes). The two assumptions for each production volume are:

1. that its content can be estimated;
2. that all other volumes that must be mined prior to it can be identified.

While no assumption need be placed on the shape of the production volumes, the remainder of this discussion will make use of rectangular blocks.

These assumptions cause no problems in practice. They are the same pieces of information that any mining engineer must take into account in the development of mine schedules (the sequencing of the blocks). In many cases, the estimates of the content of each block can be taken directly from computerized geologic ore reserve models. The precedence relationships between the blocks are a function of slope stability considerations.

Four formulations are outlined in this section. The first three follow a block scheduling approach where the scheduling units are three-dimensional blocks of specified length, width and height having known (estimated) ore content. The fourth, a column or directional approach, is applicable to bedded deposits. In this case, the scheduling units are columns or strips of waste and ore material rather than blocks. The four formulations are:

1. Linear programming
2. Pure integer programming
3. Mixed integer programming

} Block formulations

4. Linear programming — Directional formulation

For simplicity in presentation, these will be presented in terms of a single time period. Although this reduces the problem to an ultimate pit problem, there are other, more efficient approaches for solving that problem and these formulations should not be used for that purpose.

The problem described in terms of the block sequencing is limited to a discussion of the constraints. The objectives are not discussed in detail until later. Fig. 3 shows the block precedence relationship described. A nine-above relationship is used, again for simplicity in presentation. Alternatives to the nine-above relationship, and reasons for modifying it, are also discussed later in this paper.

Linear programming

The easiest way to model the situation shown in Fig. 3 is to start with a decision variable for each block as follows:

$$X_{ijk} = \begin{array}{l} 0 - \text{If block } ijk \text{ is not mined} \\ 1 - \text{If block } ijk \text{ is mined} \end{array}$$

Then, in order to mine a given block, nine separate linear constraints can be developed

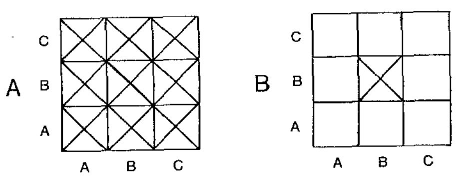

Fig. 3. Simple precedence relationship.

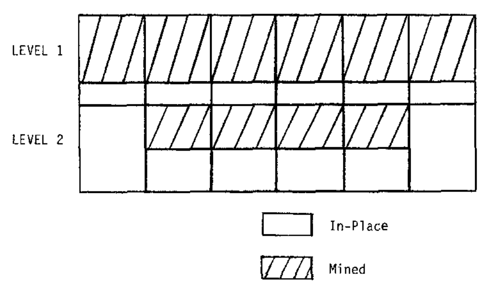

Fig. 4. Cross section – problem of partial blocks.

saying, in effect, that nine other blocks must be mined first. For Fig. 3, these equations are:

$$\begin{aligned} X_{AAA} - X_{BBB} &\geq 0 \\ X_{ABA} - X_{BBB} &\geq 0 \\ X_{ACA} - X_{BBB} &\geq 0 \\ X_{BAA} - X_{BBB} &\geq 0 \\ X_{BBA} - X_{BBB} &\geq 0 \\ X_{BCA} - X_{BBB} &\geq 0 \\ X_{CAA} - X_{BBB} &\geq 0 \\ X_{CBA} - X_{BBB} &\geq 0 \\ X_{CCA} - X_{BBB} &\geq 0 \end{aligned} \tag{1}$$

In more general terms, Equations 1 can be written:

$$X_{l,m,k-1} - X_{ijk} \geq 0 \qquad l = i-1, i, i+1; \quad m = j-1, j, j+1 \tag{2}$$

$$X_{ijk} = 0 \quad \text{or} \quad 1$$

and this equation holds for all i, j, k, in the geologic model. At nine constraints per block, this set of equations can be too large to handle (this is why linear programming is not used to find ultimate pit limits). Using modern commercial LP software, this formulation would limit the user to about 1000 blocks and one time period.

An advantageous aspect of this formulation results from the special structure exhibited by the LP matrix. The resulting matrix has a property known as unimodularity, which ensures that all decision variables will take on integer values in the optimal solution. Because of this property, the integer requirement, $X_{ijk} = 0$ or 1, can be dropped from the formulation and the desired results are obtained without having to resort to the more difficult integer programming procedures.

Unfortunately, in MSO, where other groups of constraint equations are added to these, this property is lost. When this occurs, another problem arises. Partial blocks can be mined as long as less is taken from a given block than from each of its precedent blocks. This may lead to the situation shown in Fig. 4 where half of each block on the second level is mined, while only three-quarters of those on the first level are mined. The remaining one-quarter of each precedent block is left suspended in air.

Pure integer programming

A standard integer programming reformulation of Equation 2 is to use just one equation to describe the same situation (Williams, 1978). As before,

$$X_{ijk} = \begin{cases} 0 - \text{If block } ijk \text{ is not mined} \\ 1 - \text{If block } ijk \text{ is mined} \end{cases}$$

Then, the constraint is

$$\sum_{l=i-1}^{i+1} \sum_{m=j-1}^{j+1} X_{l,m,k-1} - 9X_{ijk} \geq 0 \tag{3}$$

This equation forces all precedent blocks to be completely mined if block ijk is to be mined. It also allows for any, all or none of the precedent blocks to be mined if X_{ijk} is not mined. The major advantage of this formulation is that the precedences are incorporated using one constraint per block instead of nine, allowing more blocks (finer detail) to be included. A more practical advantage is that this approach overcomes the problem shown in Fig. 4.

The major disadvantage is that it over compensates for the situation shown in Fig. 4 by being too restrictive on the mining process. It is possible that there may be no combination of complete blocks that, for example, meet some blending requirement. This is especially true if the blocks are large or if the time periods are short.

If this occurs, it may be necessary to mine some partial blocks. Furthermore, the actual mining situation does allow for the mining of partial blocks. Fig. 5 shows, for the same cross section, a practical mining situation that would not be allowed by this formulation. Neither the partial block on the right nor the three partial blocks on the second level could result from this model. Fig. 6 shows an allowable situation for the same cross section. Any block that is mined is mined in its entirety.

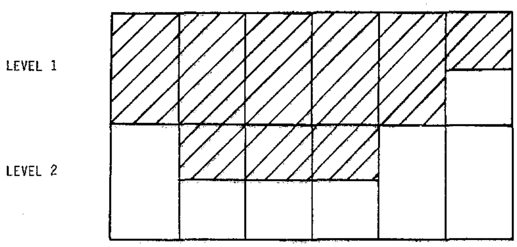

Fig. 5. Partial block mined – precedences satisfied.

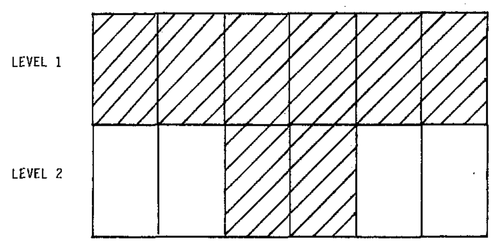

Fig. 6. Pure integer results.

Mixed integer programming

It is apparent at this point that Fig. 5 represents the most desirable situation. It incorporates the best points of Figs. 4 and 6. In Fig. 4, the LP formulation, partial blocks may be mined, but proper precedence relationships are not maintained. In Fig. 6, precedences are maintained properly but partial blocks cannot be mined.

The mixed integer programming approach allows for partial blocks to be mined only if all precedent blocks have been completely removed. This situation is the closest to the actual mining requirements. The decision variables are defined as follows:

$X_{ijk}(t) =$ Per cent of block ijk mined in period t

$$B_{ijk}(t) = \begin{cases} 0 & \text{If all precedent blocks have not been completed in period } t \\ 1 & \text{If all precedent blocks have been completed in period } t \end{cases}$$

$C_{ijk}(t) =$ Per cent of block remaining at start of period t

$D_{ijk}(t) =$ Per cent of the precedent blocks mined at start of period t

where $X_{ijk} \in [0,1]$, $C_{ijk} \in [0,1]$, $D_{ijk} \in [0,9]$.

The constraints are of the form:

$$B_{ijk}(t) + C_{ijk}(t) - C_{ijk}(t+1) - X_{ijk}(t) \geq 0$$

$$D_{ijk}(t) + \sum_{l=i-1}^{i+1} \sum_{m=j-1}^{j+1} X_{l,m,k-1}(t) - D_{ijk}(t+1) - 9B_{ijk}(t) \geq 0$$

This approach has a number of advantages over the previous two. The most important is discussed in the previous paragraphs: it provides for the most flexibility and practicality in the mine sequencing. The other advantages concern the mathematical formulation in terms of reducing the computer time required to derive the optimal sequence of production. As in the pure integer case, only one constraint per block is required. Also, the inclusion of a mechanism for keeping track of the status of a block at any given time (C_{ijk}), reduces the need for extra 'book keeping' constraints.

There is still a major drawback to this approach. It does not overcome the difficulties involved in solving a large integer programming problem. This is a problem with all of the formulations presented so far. It is the major problem that needs to be overcome prior to achieving the development of an efficient mine scheduling optimization system. Despite the problem, this formulation is currently the best available for block models and is the one that should be used.

Linear programming–directional model

For bedded ore deposits, the sequencing problem becomes simplified because it is no longer necessary to work with blocks (Meyer, 1968). Instead, the deposit can be divided into columns (or pillars), each column extending down to the bottom of the ore bed. Fig. 7 shows an example of the column representation of the ore deposit. In plan view, it is the same as the block model. While only the case of a single seam is discussed here, the same formulation can be extended to the case of multiple seams.

The decision variables for this case are continuous rather than integer. Although some integer variables are still required in other supporting constraints (for example, differentiating ore from waste), far less are required than in the block model formulations. The decision variables are:

X_{ij} = Mining elevation at column ij

$X_{ij} \in$ [min elevation, max elevation]

These variables represent distances (really, depth) mined in a given area. The constraints, reflecting slope stability requirements, state that the difference in the depths at two adjacent columns must be within some permissable limit d_{ij}. This limit corresponds to the required slope angle in area ij. Using this notation, the constraints are:

$$X_{lm} - X_{ij} \leq d_{ij} \qquad l = i-1, i, i+1 \qquad m = j-1, j, j+1 \qquad lm \neq ij \tag{5}$$

The major advantage of this approach is the reduction in the use of integer variables. This is compounded by the fact that the number of production volumes (columns) is reduced

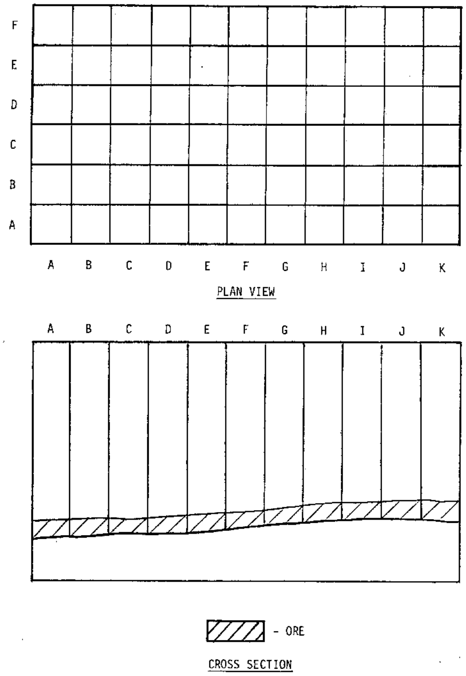

Fig. 7. Column representation of ore deposit.

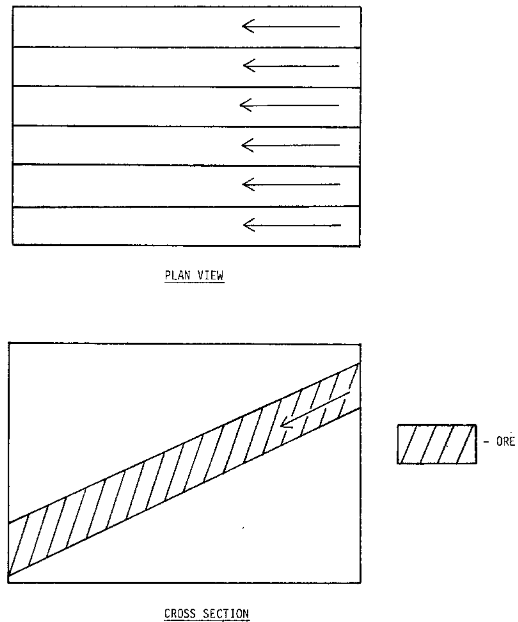

Fig. 8. Directional representation of ore deposit.

from the case of the block model. For example, a small block model consisting of 20 blocks north by 20 blocks east by 10 levels deep contains 4000 production volumes. The column model of the same dimension would have only 400 production volumes (800 if ore and waste are considered separately) and still provide the same level of detail for the resulting production schedule.

The drawback to this approach is that it has very limited applicability. However, it is the most efficient approach of those presented here. For every production scheduling project, this approach should be considered to determine if it can be successfully applied.

A slight modification of the column model leads to an even simpler model, the directional model. As shown in Fig. 8, this formulation defines strips of ore along a given direction as the production volumes. Stripping required to advance a given distance along a strip can either be specified in advance or calculated as a function of the angle at which the ore lies. Changes in this angle (see Fig. 9) can be accommodated and this approach is especially useful for the case of multiple seams (see Fig. 10).

The direction of advance in Fig. 8 need not be the direction of mining. The criteria on which this direction should be selected is the direction of homogeneity of the ore. If a direction of mining has been predefined, then the model must be constrained to follow it.

Again, this model is more efficient than the column model. For the example given above where the column model had 400 production volumes, the directional model has only 20. Of course, a more detailed production schedule can then be obtained by decreasing the

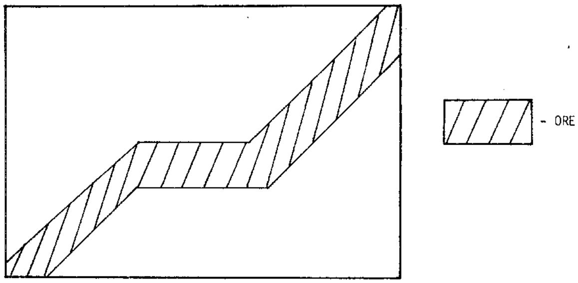

Fig. 9. Seam cross section.

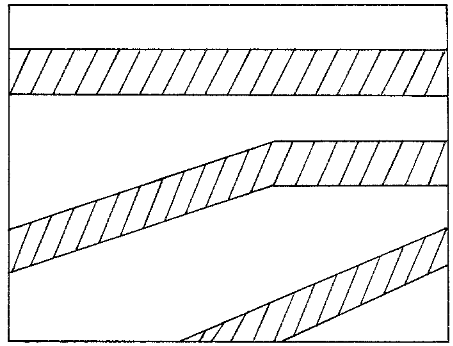

Fig. 10. Multiple seams.

width of the strips. However, this consideration is usually more a function of equipment capabilities than anything else.

Summary of alternative formulations

Clearly the directional approach is the most efficient and practical for frequent use. The only other formulation that can yield good results for practical use is the linear programming model, since no integer variables are needed. However, the addition of other constraints requires that the variables be of the integer type. In this case, this approach is definitely the least practical of those presented here.

The difficulty that all of these formulations have in common is that each results in a large integer programming problem. For the block models, the number of integer values can be as many as a few thousand. No technique has been proven to be effective in guaranteeing predictable and practical run times on these problems. This is the major hurdle to overcome and recommendations for specific approaches to overcoming it are discussed in a later section.

Of the block models, the mixed integer formulation is the best. It is no worse than the others in terms of the number of integer variables and it provides better results, as has been described earlier. While results from this model have shown outstanding savings for operating mines, the run times have been unpredictable. Wide variations have been seen from one property to another. This model can be considered to be adequate for use on a project or consulting basis or for the development of long term development strategies, but may not be practical for frequent usage due to the unpredictability of solving the large integer programming problems.

The decision to implement these kinds of approaches will ultimately be based on their cost effectiveness. Through use of the mixed integer approach, costs have already been reduced to the point where they are comparable to the cost of a similar project done on a consulting basis by a large consulting firm. Costs must be reduced more before this type of approach can overcome the 'proven' techniques now in use. However, the optimization approach has the following advantages:

1. The model can be run repeatedly under different assumptions at a small fraction of the original cost. Traditional methods would have to repeat almost the entire project.
2. The resulting production schedule is in some sense the best possible. The traditional approach seeks to create a feasible schedule, not necessarily the best one.
3. A basis for production scheduling decisions is available for analysis, review and improvement. Usually, this only exists in the mind of the responsible person and replacing that person can be difficult.
4. Information gained during the model-building process allows management to be more aware of the interdependencies of their many operations. This leads to an increased decision making capability in terms of the quality of their future decisions, possibly even unrelated to scheduling.

Related considerations

The choice of an objective is central to the use of any optimization algorithm. This choice serves to quantify the definition of 'best'. The best production schedule may be the one that can meet fixed production specifications at a minimum cost. The associated objective is to minimize cost. If production levels are not fixed, or if multiple products are being produced, a better objective may be to maximize net present value of the property over some timespan. This timespan should be compatible with the life of the mine, which it is desirable to extend. A third objective, then, is to maximize the life of the mine, given that some minimum acceptable rate of return must be maintained. A fourth objective might be to maximize production (cash flow), given that some minimum life of mine or rate of return is maintained.

Each of the objectives mentioned above can take on a variety of forms when the actual quantification is performed. During one recent discussion on the objective of balancing strip ratios, five different interpretations of the word 'balance' were given, each leading to a different 'balance strip ratio' objective.

The choice of the objective will depend on the commodity being mined, the philosophy of the company, the mining methods and equipment employed, and even the experience and personality of the mine engineer. Many managers today do not attempt to accomplish their tasks in the best possible manner as much as they try to minimize their risk of missing future targets. This crisis avoidance objective, to minimize vulnerability, is as legitimate for planning as any of the others mentioned above.

A second consideration is the manner by which the appropriate slope stability angles are achieved for block models. The nine above block precedence relationship used in a previous section is often overly conservative. One method of achieving the desired slope angles is to develop a set of precedence rules, each rule corresponding to some desired angle. Then, for any given angle, the appropriate precedence rule is used. This is a fairly standard approach for ultimate pit limit analysis. Variable angles can be handled by specifying an angle for every area or every block in the model. This is also done in ultimate pit limit analysis.

An alternative method is to fix one precedence rule and vary the dimensions of the blocks to achieve the desired angle. While this will work, there are a few drawbacks. First, block sizes should be derived from the density of drilling so that there can be some confidence in the accuracy of the estimation of the block parameters. Second, it is difficult to have many different block sizes in one model. This difficulty then creates a situation where variable slopes cannot be utilized. For these reasons, the first approach should be used.

A third consideration concerns the accuracy of hauling costs used in a long term optimization model. One mechanism for deciding to mine in one area over another area is the difference in haulage costs, based on location. However, the haulage routes are not known until after the long range development strategy for mining is known. Therefore, one of the required inputs to the optimization (or any other mine planning approach) is not known. The only case where this may be known is in an underground operation.

The mining engineers at a mine should have a general 'feel' for alternative development strategies as well as alternative haul road locations associated with each. The approach for the optimization should be to select a conservative haulage route for each location so as to try not to be too high or too low in the resulting estimates of haulage distance. This haulage distance applies not only to cost of hauling, but to haulage fleet size requirements as well.

Futures – avenues of research

As mentioned in the introduction, one major purpose of this paper is, by outlining the problem and various approaches to it, to motivate the required research needed to improve the techniques so that it will be implemented on a regular basis in practice. Future work in the optimization of mine production schedules should be directed towards overcoming the need to solve an integer programming problem that is really too large to solve using conventional software packages. An approach to circumvent this problem is outlined in this section. Also, approaches are suggested for solving the large integer problem more efficiently and foı reducing the size of the problem.

The dual of the first problem formulation (Equation 2) can be solved using a network algorithm and network problems can be solved many times faster than linear programming. Therefore, it is desirable to solve this problem as a network problem. However, the remaining constraints needed for a complete scheduling system cannot be fit into the network framework.

The approach that must be researched is a decomposition algorithm for this problem. In this way, the large sequencing problem is solved efficiently as a network subproblem and the remaining constraints serve as a master problem, modifying the block values toward the achievement of a consistent solution. This network approach has been suggested for ultimate pit limit analysis and the subproblem discussed here is virtually the same except that time periods are included for scheduling.

A great deal of research is currently being conducted on techniques to solve very large integer problems. Two promising techniques, in particular, have been used successfully on problems that are as large or larger than those discussed here (Garfinkel and Nemhauser, 1972). These are the methods of Lagrangian relaxation and surrogate constraints.

Researching the effects of these techniques applied to the mine production scheduling problem would provide valuable alternatives to the network decomposition approach. That approach is really, in a sense, also a relaxation approach because a problem (the network problem) that is easy to solve is formed by relaxing the complicating constraints.

Another possibility involves the use of automatic cut generating techniques, available and currently under development. While cut generation techniques have never realized their initial expectations, specific techniques for 0–1 problems have recently been tested successfully (Crowder *et al.*, 1981). This development could impact the problem encountered here.

Both the automatic cut generation and the capability of implementing a 'presolve' problem reformulation routine at each node of a branch and bound process are currently being considered as enhancements in commercial packages (Ashford, 1983). This raises the hope of being able to use this readily available software for the large integer problems.

Finally, some research needs to be done in the model formulation (problem definition) stage. If small blocks are to be grouped into larger blocks to reduce problem size, there must be a sound basis for accomplishing the aggregation. While geologic boundaries provide natural groupings, a geostatistical basis needs to be developed within a given rock type. Another approach for reducing the number of blocks is to use the smaller blocks for exposed areas (near term production) and increase the size of more distant blocks just as the length of time periods can be increased for more distant years. This provides the greatest level of detail in the shorter term where it is most needed while still considering the needs of the longer term.

Recommendations and conclusions

1. A set of philosophical considerations has been put forth for the development of a system for the optimization of mine production scheduling.
2. An approach consistent with these considerations and requirements has been developed and implemented.
3. A variety of mathematical formulations are available within the general approach.
4. Advantages and disadvantages of each formulation have been discussed.
5. When applicable, the column or directional approach should be used. For block models, the mixed integer formulation is best.
6. These formulations lead to the use of up to a few thousand integer variables, creating severe computational difficulties (for block models).
7. Attacking these difficulties should be the primary focus of future mine planning related research.
8. Suggested approaches are provided for concentration of this research.
9. A great deal of research is being done towards providing better techniques for the solution of problems of this type.

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