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Fourier Transforms of Magnetic Anomalies of Two-Dimensional Bodies

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Summary – Fourier transforms of theoretical magnetic anomalies of two-dimensional bodies (vertical fault and horizontal cylinder of infinite length) are computed. In both cases it is shown that for a particular body the amplitude spectra of horizontal and vertical anomalies are the same and the phase spectra differ by $\pi/2$. Different parameters like depth and susceptibility contrast can be calculated from the amplitude spectrum.

1. Introduction

The present paper deals with the Fourier transform of theoretical anomalies of two-dimensional magnetic bodies. The magnetic anomaly of a regularly shaped body is a function of the direction of magnetization, shape and size of the body and its susceptibility contrast with the surrounding material. The Fourier transform of a magnetic anomaly, therefore, contains information regarding all these parameters.

In recent years extensive work has been done regarding the application of spectral analysis in gravity and magnetic interpretation (DEAN [3], ODEGARD and BERG [7]). ODEGARD and BERG [7] have applied Fourier integration for the interpretation of gravity anomalies due to regular bodies.

2. Method of analysis

The theoretical magnetic anomalies due to a horizontal cylinder of infinite length and a vertical fault are noted from HEILAND [5]. In both cases the plane of observation is perpendicular to the strike direction. In the case of a horizontal cylinder the axis of the cylinder is perpendicular to the plane of magnetization. The strike of the fault makes an angle α with the magnetic north through the magnetic west. Exact Fourier sine and cosine transforms of these anomalies are calculated. The nature of the phase and amplitude spectra are discussed and different parameters like depth of burial and throw of fault are calculated from the amplitude spectra.

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i) Horizontal cylinder of infinite length

The theoretical vertical and horizontal magnetic anomalies due to a horizontal cylinder of infinite length are given as,

$$Z = -\frac{2KS}{r^4}[2Z_0 xd - H_0(x^2 - d^2)]$$

and

$$H = -\frac{2KS}{r^4}[2H_0 xd + Z_0(x^2 - d^2)]$$

respectively where

Z = vertical magnetic anomaly,
H = horizontal magnetic anomaly,
K = susceptibility contrast,
S = cross-sectional area of the cylinder,
Z_0 = vertical component of Earth's magnetic field,
H_0 = horizontal component of Earth's magnetic field,
x = horizontal distance of point of observation from origin,
d = depth of the axis of the cylinder from the Earth's surface,
$r^2 = x^2 + d^2$.

The Fourier transforms of Z and H are taken as $F_Z(\omega)$ and $F_H(\omega)$ respectively. The necessary formulas for integration are given in the Appendix.

$$F_Z(\omega) = -2KS\left[4iZ_0 d\int_0^\infty \frac{x}{(x^2+d^2)^2}\sin\omega x\,dx - 2H_0\int_0^\infty \frac{x^2-d^2}{(x^2+d^2)^2}\cos\omega x\,dx\right]$$

$$= -2\pi KS\omega(H_0 + iZ_0)e^{-\omega d} \tag{1}$$

In a similar way it can be shown that

$$F_H(\omega) = -2\pi KS\omega(Z_0 - iH_0)e^{-\omega d} \tag{2}$$

where $i = \sqrt{-1}$ and ω is frequency in radians per unit distance. The amplitude spectrum for both the horizontal and vertical anomalies is given as

$$|F(\omega)| = 2\pi KS\omega(Z_0^2 + H_0^2)^{1/2}e^{-\omega d}$$

$$= 2\pi KS\omega T e^{-\omega d} \tag{3}$$

where T is the total magnetizing field.

The nature of $|F(\omega)|$ versus ω is shown in Fig. 1 by plotting $|F(\omega)|$ as $\omega e^{-\omega d}$ versus ω. $|F(\omega)|$ will be maximum when $\omega = 1/d$ as

$$\frac{d|F(\omega)|}{d\omega} = 2\pi KS\omega T(1 - \omega d)e^{-\omega d} = 0$$

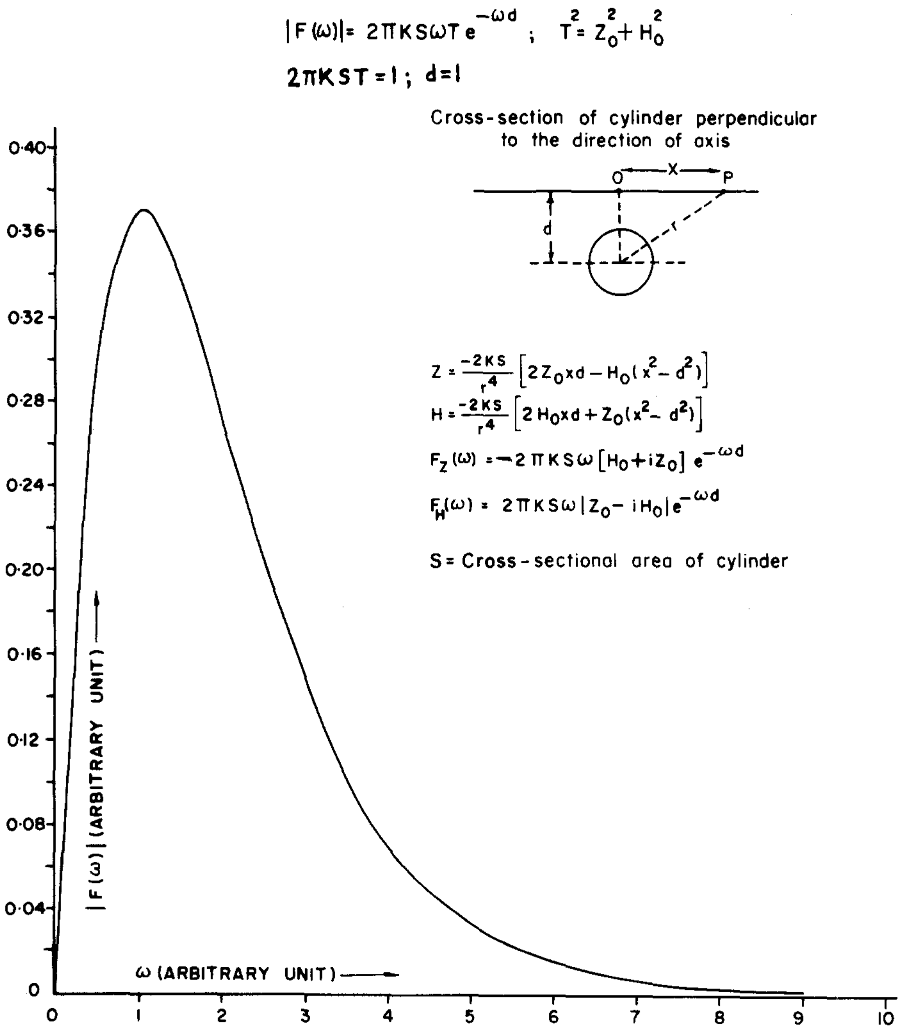

Figure 1
Fourier transform of vertical and horizontal magnetic anomalies over a cylinder of infinite length (amplitude spectrum)

when $\omega = 1/d$. Let $|f(\omega)| = |F(\omega)|/\omega$, then

$$\ln|f(\omega)| = \text{lin}\, 2\pi KST - \omega d \tag{4}$$

A plot of $|f(\omega)|$ versus ω on a semilog paper will be a straight line with slope $-d$ and intercept $2\pi KST$. The nature of $\ln|f(\omega)|$ versus ω is shown in Fig. 2, where d and $2\pi KST$ are taken as unity.

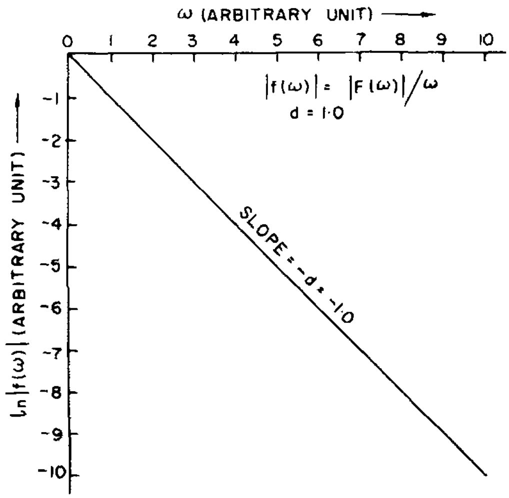

Figure 2
Modified amplitude spectrum $|f(\omega)|$ versus frequency ω for a horizontal cylinder

The phase spectrum of vertical and horizontal anomalies are related as

$$\phi_Z(\omega) = \tan^{-1}[Z_0/H_0] = \pi/2 - \tan^{-1}[H_0/Z_0] = \pi/2 + \phi_H(\omega) \tag{5}$$

This shows that the horizontal magnetic anomaly lags the vertical anomaly by $\pi/2$.

ii) Vertical fault

The vertical and horizontal magnetic anomalies due to a vertical fault are given as

$$Z = 2K\left[H_0 \sin\alpha . \ln\frac{r_2}{r_1} - Z_0(\theta_1 - \theta_2)\right]$$

and

$$H = 2K\left[H_0 \sin\alpha . (\theta_1 - \theta_2) + Z_0 \ln\frac{r_2}{r_1}\right]$$

respectively. The meaning of r_1, r_2, θ_1 and θ_2 is explanied in Fig. 3. The strike of the fault makes an angle α with the magnetic north through west. E.g. for a fault with strike in the north–south direction $\alpha = 0$ and when the strike of the fault is in an east–west direction $\alpha = \pi/2$. The Fourier transforms of Z and H are given as $F_Z(\omega)$ and $F_H(\omega)$ respectively.

$$F_Z(\omega) = 2K\left[2H_0 \sin\alpha \int_0^\infty \ln\frac{\sqrt{(d+h)^2 + x^2}}{\sqrt{d^2 + x^2}} \cos\omega x \, dx - i2Z_0 \int_0^\infty \left(\tan^{-1}\frac{d}{x} - \tan^{-1}\frac{d+h}{x}\right) \sin\omega x \, dx\right]$$

$$= \frac{2\pi K}{\omega}\left[e^{-\omega d} - e^{-\omega(d+h)}\right].\left[H_0 \sin\alpha + iZ_0\right] \tag{6}$$

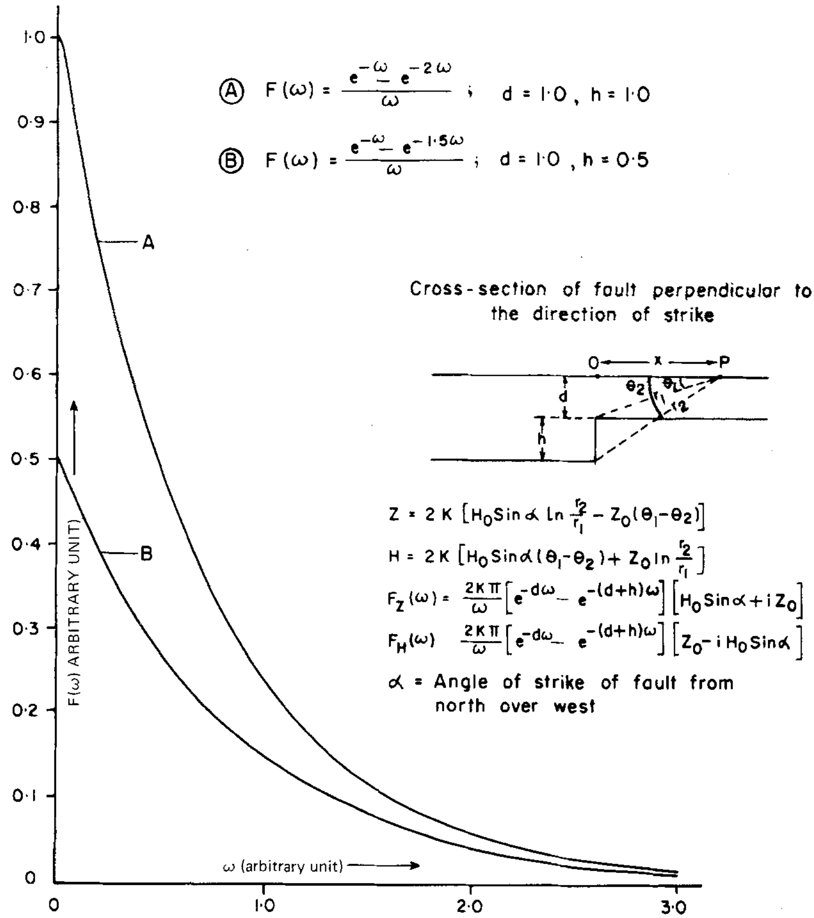

Figure 3
Fourier transform of horizontal and vertical magnetic anomalies over a vertical fault (amplitude spectrum)

In a similar way it can be shown that

$$F_H(\omega) = \frac{2\pi K}{\omega}[e^{-\omega d} - e^{-\omega(d+h)}].[Z_0 - iH_0 \sin\alpha] \tag{7}$$

The amplitude spectrum $|F(\omega)|$ is the same for both the horizontal and vertical magnetic anomalies.

$$|F(\omega)| = \frac{2\pi K}{\omega}[e^{-\omega d} - e^{-\omega(d+h)}].[Z_0^2 + H_0^2 \sin^2\alpha]^{1/2}$$

$$= \frac{2\pi KT'}{\omega}[e^{-\omega d} - e^{-\omega(d+h)}] \tag{8}$$

where T' is the total intensity of the magnetizing field in the plane perpendicular to the strike of the fault.

The phase spectra $\phi_Z(\omega)$ and $\phi_H(\omega)$ are given as

$$\phi_Z(\omega) = \tan^{-1}\frac{Z_0}{H_0 \sin\alpha} \quad \text{and} \quad \phi_H(\omega) = -\tan^{-1}\frac{H_0 \sin\alpha}{Z_0} \tag{9}$$

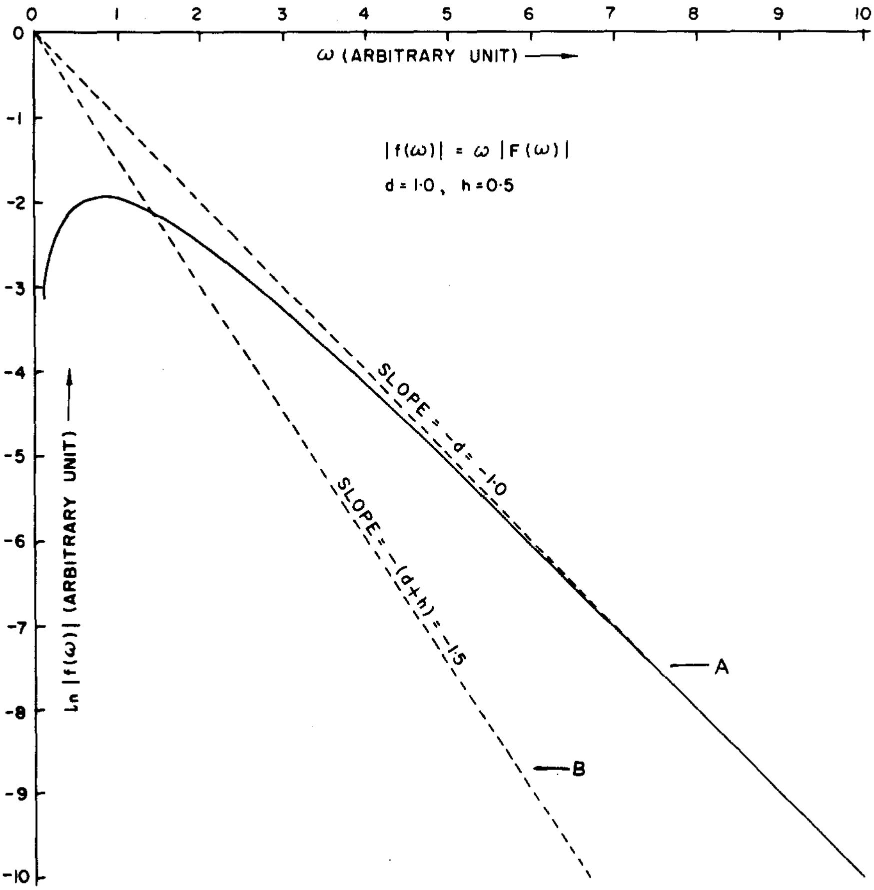

Figure 4
Modified amplitude spectrum $|f(\omega)|$ versus frequency ω for a vertical fault ($d = h$)

It can be shown from the above equation that

$$\phi_Z(\omega) - \phi_H(\omega) = \pi/2. \tag{10}$$

Equation (10) is the same as equation (5). The phase of the vertical anomaly thus differs from the phase of the horizontal anomaly by an amount of $\pi/2$. Figure 3 shows a plot of $|F(\omega)|$ versus ω in arbitrary units. For simplicity, and as it does not change the nature of the amplitude spectrum, $2\pi KT'$ is taken as unity for computing $|F(\omega)|$. Two cases of depth and throw of fault are taken. In the first case d and h both are taken as unity and in the second case d is unity and h is 0.5. From equation (8) it can also be shown that

$$\lim_{\omega\to 0}|F(\omega)| = \lim_{\omega\to 0}\frac{2\pi KT'[e^{-\omega d} - e^{-\omega(d+h)}]}{\omega} = \lim_{\omega\to 0}\frac{2\pi KT'[-de^{-\omega d} + (d+h)e^{-\omega(d+h)}]}{1}$$
$$= 2\pi KT'h \tag{11}$$

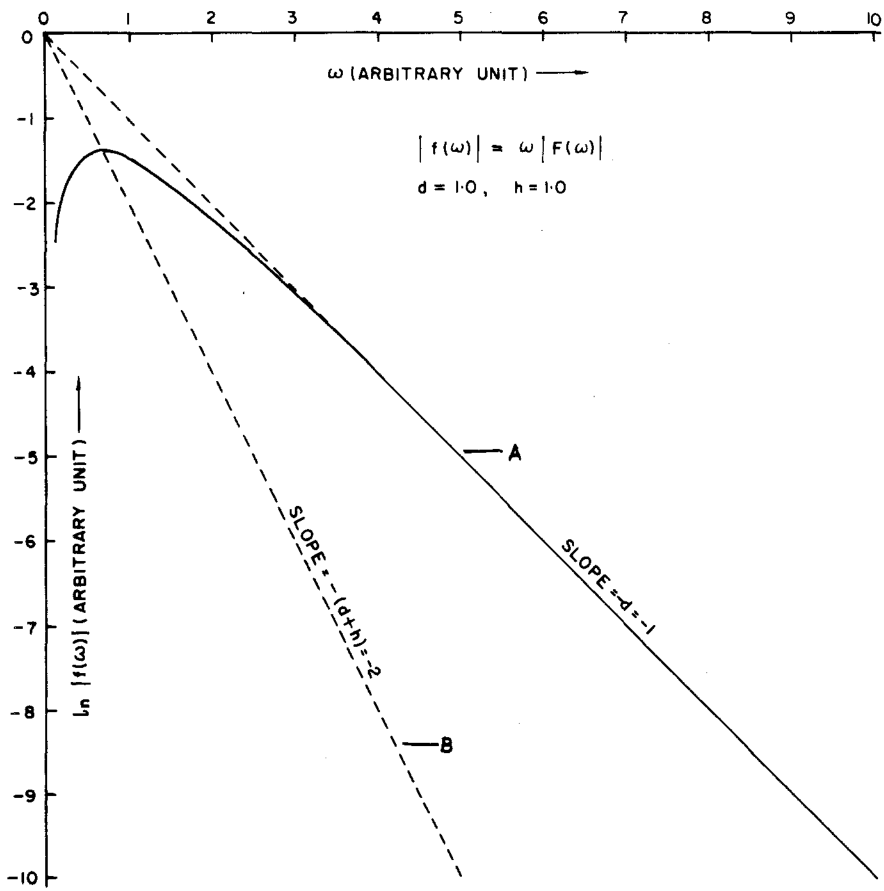

Figure 5
Modified amplitude spectrum $|f(\omega)|$ versus frequency ω for a vertical fault ($d = 2h$)

The peak value of the amplitude spectrum is, therefore, equal to $2\pi KT'h$. Since in the present case $2\pi KT'$ is taken as unity so the peak value of $|F(\omega)|$ is 1.0 and 0.5 in the first and second case respectively (cf. Fig. 3). If we define $|f(\omega)| = \omega|F(\omega)|$ as modified amplitude spectrum then

$$|f(\omega)| = 2\pi KT'[e^{-d\omega} - e^{-\omega(d+h)}] \tag{12}$$

To study the nature of $|f(\omega)|$, variation of $\ln[e^{-\omega d} - e^{-\omega(d+h)}]$ versus ω is shown in Figs. 4 and 5. Two combinations of d and h, namely $d = 1$, $h = 1$ and $d = 1$, $h = 0.5$ are taken into consideration. In both cases the latter part of the graph (part A) is linear. This part actually shows the linear variation of $\ln e^{-\omega d}$ or $-\omega d$ with respect to ω. The slope of part A gives $-d$, which is the depth to the top surface of the fault plane. This linear part is extended to the origin and the exponential values of $e^{-\omega d}$ are subtracted from the values of $|f(\omega)|$. The residue (part B), when plotted on the same semilog graph, gives a straight line with slope as $-(d+h)$. As d is known earlier from the slope of part A, h now be calculated from the slope of part B. Details of this technique are discussed in KAPLAN [6]. As h is known so from equation (11), $2\pi KT'$ and hence K can be determined if T' is known.

3. *Discussion*

The present investigation is carried out to study the nature of theoretical horizontal and vertical magnetic anomalies due to simple two-dimensional geological bodies in frequency domain. Similar derivations for other types of two-dimensional bodies and three-dimensional bodies are also found. A set of master curves for different values of parameters is under preparation. It is interesting to note that in the two-dimensional cases the amplitude spectrum remains the same for both the horizontal and vertical magnetic anomalies. The variation in phase spectra is $\pi/2$. In such a case a set of numerical filters (dispersive filters) can be designed to convert horizontal magnetic anomalies into vertical magnetic anomalies and vice versa.

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Appendix

I. Fourier transform of $f(x) = \tan^{-1}\frac{d}{x}$ (Ref. [2], p. 87).

As $f(x)$ is an odd function of x its Fourier transform $F(\omega)$ is

$$F(\omega) = 2i\int_0^\infty \tan^{-1}\frac{d}{x}\sin\omega x\,dx = \frac{2\pi i}{\omega}e^{-\omega d/2}\sinh\left(\frac{\omega d}{2}\right)$$

$$= \frac{2\pi i}{\omega}e^{-\omega d/2}.\tfrac{1}{2}[e^{\omega d/2} - e^{-\omega d/2}]$$

$$= \frac{i\pi}{\omega}[1 - e^{-\omega d}]$$

when $d > 0$, $\omega \geqslant 0$.

II. Fourier transform of $f(x) = \ln\frac{\sqrt{\alpha^2 + x^2}}{\sqrt{\beta^2 + x^2}}$ (Ref. [2], p. 18).

As $f(x)$ is an even function of x its Fourier transform $F(\omega)$ is

$$F(\omega) = 2\int_0^\infty \ln\frac{\sqrt{\alpha^2 + x^2}}{\sqrt{\beta^2 + x^2}}\cos\omega x\,dx = \frac{\pi}{\omega}(e^{-\beta\omega} - e^{-\alpha\omega})$$

when $\alpha > 0$, $\beta > 0$ and $\omega \geqslant 0$.

III. $$2\int_0^\infty \frac{d^2\cos\omega x}{(d^2 + x^2)^2}\,dx = \frac{\pi}{2d}(1 + \omega d)e^{-\omega d}$$ (Ref. [4], p. 225).

IV. $$2\int_0^\infty \frac{x^2\cos\omega x}{(d^2 + x^2)^2}\,dx = \frac{\pi}{2d}(1 - \omega d)e^{-\omega d}$$ (Ref. [4], p. 225).

V. $$2\int_0^\infty \frac{x\sin\omega x}{(d^2 + x^2)^2}\,dx = \frac{\pi\omega}{2d}e^{-\omega d}$$ (Ref. [4], p. 225).

when $d > 0$, $\omega \geqslant 0$.