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Design of tunnel linings in a creeping rock*

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Summary

The effect of long term rock deformation on lining pressure is considered using different concepts of rock behaviour. These include the conventional characteristic-line or convergence-confinement method, modified to allow for rock ageing, and lining–rock interaction methods using models of linear-viscoelastic, linear-elastic linear viscous, and linear elastic non-linear viscous rock behaviour. Calculations of lining pressures show that the former tends to underestimate, compared with the latter.

Keywords: Tunnel linings; rock creep; rock strength; rock deformation; convergence.

Introduction

The ground pressure on the lining of a tunnel of a given shape is known to depend not only on the rock mass properties and ground stress conditions, but also on the type and the rigidity of the lining and the time of its installation. After the contact with the rock mass has been established, the pressure on the lining will usually vary with time, first due to the advance of the driving face of the tunnel, and then due to the gradual changes in the rock mass properties, caused by a combined action of processes such as fracturing, consolidation, swelling, creep and weathering.

The design of tunnel linings has been extensively discussed in the recent literature, and many different design methods have been proposed for that purpose. Although the present numerical methods allow the tunnel lining–rock mass interaction problem of any complexity to be resolved, the simple characteristic-line, or convergence-confinement method has, nevertheless retained attention of the designers through the last two decades, mainly because of its clarity and versatility.

Since its first mention by Lauffer and Seeber (1961), the method has seen many variations and developments, as seen in a review by Brown *et al.* (1983). Within this general area, the authors of this paper have, in the last 15 years, concentrated their attention to the problem of time-dependence of lining pressure, particularly as affected by the rock mass creep and deterioration.

Although a number of rigorous theoretical solutions for time-dependent ground–lining interaction, covering different kinds of rheological models of rock behaviour, have been published (e.g., Gnirk and Johnson, 1964; Gill *et al.*, 1970; Sakurai, 1977; Gill and Dubé,

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1974; Panet, 1979; Lo and Yuen, 1981; Fritz, 1981), the characteristic-line approach has seen relatively few developments in this direction.

One of the first attempts to introduce the time element into the determination of ground pressure on tunnel linings by the characteristic-line method, was made by Ladanyi (1974). However, in that approach, only two limiting characteristic lines were considered: one for short-term response ($t \to 0+$), and another for long-term response ($t \to \infty$). For intermediate times, it was assumed that the surrounding rock mass undergoes a continuous deterioration, involving a decrease in its modulus of deformation and a gradual loss of strength.

In a subsequent paper (Ladanyi, 1980), it was attempted to fill the gap between the short term and long term rock mass response by assuming that the rock mass around the tunnel creeps according to a non-linear Maxwell (power law) model. This made it possible to calculate any intermediate isochronous characteristic line and to find the time-dependent tunnel wall convergence and ground pressure increase.

However, since neither of these two solutions considered the effect of the loading history on the rock response, but only the rock deterioration with time, unaffected by the lining pressure, they fall clearly in the class of solutions based on the 'ageing' theory of creep, as do, in fact, nearly all 'conventional' characteristic-line methods presently in use around the world (e.g., Deere *et al.*, 1969; Daemen and Fairhurst, 1971; Duddeck, 1979; Einstein *et al.*, 1980; Sulem *et al.*, 1987).

In the last three papers (Gill and Ladanyi, 1983, 1987; Ladanyi and Gill, 1984), the authors have analysed the implications of this simplifying assumption and found that, in general, the shape of characteristic lines is considerably affected by the presence of the lining. As a result, it was shown that the conventional convergence – confinement method generally underestimates the lining pressure at any intermediate time.

This paper gives a brief presentation and a comparative review of the authors' work on tunnel lining design, emphasizing only the most important aspects of each particular solution.

General assumptions

Although the following solutions consider different types of rock mass behaviour, they all make the same assumptions about the tunnel shape, the type of lining, and the ground stress conditions. These are as follows:

1. The underground opening is a tunnel having a shape of a straight circular cylinder.
2. The far-field pre-driving ground stress is hydrostatic.
3. The state of stress in the vicinity of the tunnel is unaffected by the gravity and the presence of any free surface.
4. The considered section is sufficiently far from the tunnel face, so that plane strain conditions apply.
5. The rate of tunnel face advance is large, relative to the creep rate of the ground.
6. The strains remain infinitely small.
7. The ground support is provided by a cylindrical lining, behaving in a linear-elastic manner.

Figure 1 shows general notation used in all solutions shown in the following.

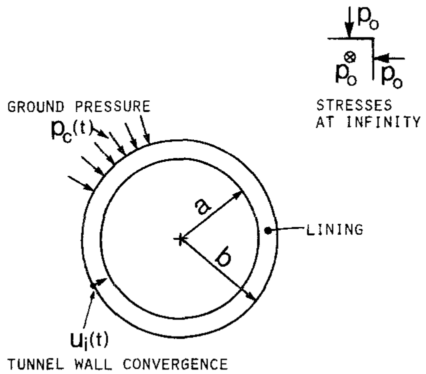

Fig. 1. Tunnel cross-section showing the notation used in the paper.

Solutions based on the long-term strength concept

This in based on the work of Ladanyi (1974), Hoek and Brown (1980) and Brown *et al.* (1983).

Assumptions on rock mass behaviour

(a) Before failure: linear elastic, characterized by the Young's modulus, E and the Poisson's ratio, v.

(b) At failure: the peak strength is defined by the Mohr failure criterion. The failure envelope is either Coulomb straight line (Ladanyi, 1974), or a second degree parabola (Ladanyi, 1974; Hoek and Brown, 1980).

(c) After failure: the strength falls to its residual value. The behaviour of broken rock is perfectly plastic, responding either to the Mohr–Coulomb failure theory (Ladanyi, 1974), or to a second degree parabola (Hoek and Brown, 1980).

(d) Volume strain: in the elastic domain, the strain is governed by the value of the Poisson's ratio, v. At failure, and up to some strain beyond failure, it is determined by the associated flow rule of the theory of plasticity.

(e) All the parameters are assumed to be time-dependent, but only their limiting values, i.e. short-term (peak) and long-term (residual) values are used in the analysis.

Considering only the original (Ladanyi, 1974) version of the theory, the results can be represented by the following equations.

Equation of the characteristic line

Short-term. If p_{icr}, given by

$$p_{icr} = p_o - MC_o \tag{1}$$

denotes the critical pressure at which the broken zone around the tunnel starts developing, then the short-term characteristic line is defined by the following two equations:

For $p_{icr} \leq p_i \leq p_o$,

$$u_i/b = (p_o - p_i)/2G \tag{2}$$

For $0 \leq p_i \leq p_{icr}$,

$$u_i/b = 1 - [(1 - e_{av})/(1 + A)]^{1/2} \tag{3}$$

in which

$$A = (MC_o/G - e_{av}) \left[\frac{p_o + H - MC_o}{p_i + H} \right]^{2/(f-1)} \tag{4}$$

In these equations:

b = initial radius of the tunnel wall.

u_i = radial tunnel closure.

$G = E/2(1+\nu)$ = shear modulus of intact rock mass.

e_{av} = average volume strain undergone by the rock mass during breakage (positive for compression). Its value can be determined from the associated flow rule, as shown by Ladanyi (1974), or Hoek and Brown (1980). A more general way for considering volume strain in the calculation, was shown by Brown *et al.* (1983).

C_o = uniaxial compression strength of intact rock mass.

M = a dimensionless parameter, depending on the shape of the failure envelope of intact rock mass and on the value of the far-field stress, p_0. For example, for Coulomb's law,

$$M = [1 + (f_s - 1)p_o/C_o]/(f_s + 1) \tag{5}$$

where $f_s = \tan^2(45 + \phi_s/2)$ for intact rock, while, for the Hoek and Brown (1980) second degree parabola

$$M = \tfrac{1}{2}[(m/4)^2 + mp_o/C_o + s]^{1/2} - m/8 \tag{6}$$

where m and s are Hoek and Brown strength parameters for rock mass, based on rock mass classification.

$H = c \cot \phi$ is valid for crushed rock in the broken zone.

Long term. The long-term characteristic line is assumed to be given by the same Equations 2 to 4, in which all rock deformation and strength parameters are replaced by their long-term values, i.e., for $t \to \infty$, $G \to G'$, $C_o \to C_o'$, $H \to H'$, etc.

Figure 2 shows schematically both characteristic lines, together with the lining resistance line, whose slope is given by

$$\frac{dp_c}{d(u_{ec}/b)} = K_s \tag{7}$$

where K_s denotes the rigidity modulus of the thick cylindrical lining, given by

$$K_s = \frac{2G_c(1 - a^2/b^2)}{1 - 2\nu_c + a^2/b^2} \tag{8}$$

G_c and ν_c being valid for concrete.

Long-term lining pressure

Reversing Equation 4, and taking into account that at $p_i = p_c$, the sum of all radial displacements will be (Fig. 2):

$$\Sigma(u_i/b) = (u_i/b)_o + \Delta/b + p_c/K_s \tag{9}$$

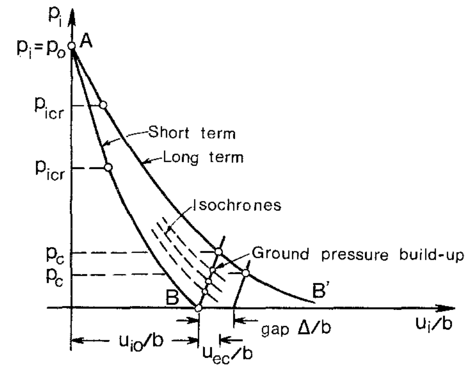

Fig. 2. Determination of ground pressure according to the long term strength concept (Ladanyi, 1974).

one gets the lining pressure

$$p_c = (p_o + H' - M'C_o')\left[(M'C_o'/G' - e_{av}')\Big/\left\{\frac{1-e_{av}'}{[1-\Sigma(u_i/b)]^2} - 1\right\}\right]^{(f'-1)/2} - H' \quad (10)$$

This theory implies that rock mass properties deteriorate with time in a continuous manner, tending to their ultimate long-term values when time tends to infinity. If one wants to find the lining pressure variation for intermediate times, $0 < t < \infty$, using this type of theory, the best way is to express each of the parameters as functions of time. For example, Gilbert and Farmer (1981) replaced the term $M'C_o'$ in Equation 10 with a 'rock support factor', which depends on the logarithm of time. On the other hand, in a similar solution, Fairhurst and Daemen (1980) expressed the strength and deformation parameters of rock mass by exponential functions of the thickness of the broken zone, which itself is a function of time.

Solution assuming a linear-viscoelastic behaviour of the rock mass without failure

An exact solution for this particular case was obtained by Gill (1970), and was published by Gill *et al.* (1970). In a recent paper by Gill and Ladanyi (1987), it was shown how this particular solution could be used for filling the time gap between $0 < t < \infty$ in a solution of the previous type, provided the support system is sufficiently strong and tight, so as to be able to prevent rock mass failure.

In this solution, the rock mass is modelled by a three-parameter solid, called also Kelvin/Voigt or Zener rheological model (Fig. 3). The model is composed of a Hooke element in series with a Kelvin element, the former being characterized by a spring constant k_1, and the latter by a spring constant k_2 and the viscosity η. Using subscript 'o' for initial, and 'f' for final values of the parameters, and noting that, for a deviatoric creep case:

$\eta_2 = 2\eta$, and

$$k_1 = 2G_o \quad (11)$$

$$k_2 = \frac{2}{1/G_f - 1/G_o} \quad (12)$$

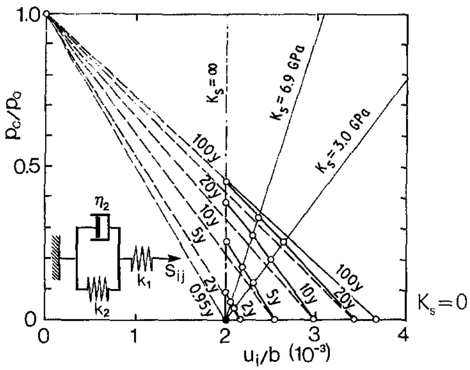

Fig. 3. Exact and conventional (ageing) characteristic lines for linear visco-elastic (Zener) behaviour of rock mass (Gill and Ladanyi, 1987). (———) Exact, (- - -) ageing.

where G_o and G_f are initial and final shear moduli, respectively, while

$$\eta_2/k_2 = t_R \tag{13}$$

is the relaxation time, one finds the following relations:

(a) Creep closure of unlined tunnel is given by:

$$\frac{u_i}{b} = \frac{p_o}{2G_o}[1 + (G_o/G_f - 1)f(t)] \tag{14}$$

where

$$f(t) = 1 - \exp(-t/t_R) \tag{15}$$

The value of u_i/b is seen to vary from $p_o/2G_o$ at $t=0$, to $p_o/2G_f$ when $t \to \infty$.

(b) 'Conventional' isochronous characteristic lines, unaffected by the lining (ageing assumption) can be obtained by replacing p_o in Equation 14 by $(p_o - p_i)$, and by varying p_i between $0 \le p_i \le p_o$ for each t = constant line. The resulting lines are straight lines passing through the point (0, 1) in the u_i/b versus p_i/p_o plot (Fig. 3).

(c) Effect of lining. If a lining of rigidity K_s is installed and becomes active at the time $t = t_s$, the conventional 'ageing' theory gives the following results:

Lining pressure increase:

$$\frac{p_c}{p_o} = \left[\frac{1 - G_f/G_o}{1 + 2G_f/K_s}\right][f(t) - f(t_s)] \tag{16}$$

and the corresponding closure is then

$$\frac{u_i}{b} = \left(\frac{u_i}{b}\right)_{t=t_s} + \frac{p_c}{K_s} \tag{17}$$

where the first term is given by Equation 14. Note that Equations 14 and 16 are equivalent to those shown by Panet (1979).

On the other hand, the exact solution by Gill (1970), taking into account the effect of lining pressure on the rock creep, gives for the lining pressure:

$$\frac{p_c}{p_o} = \left[\frac{1-G_f/G_o}{1+2G_f/K_s}\right]\exp(-t_s/t_R)\left[1-\exp C\left(-\frac{t-t_s}{t_R}\right)\right] \tag{18}$$

with

$$C = \frac{2+K_s/G_f}{2+K_s/G_o} \tag{19}$$

The corresponding closure is also given by Equation 17.

Since the characteristic lines represent a (p_c/p_o) versus (u_i/b) relationship for t = constant, it will be seen that they will be different according to the two approaches described, the exact ones lying always a little higher than the conventional ones (Fig. 3), giving higher lining pressures at any time $0 < t < \infty$. However, because of the asymptotic character of the Zener model, the two solutions (Equations 14 and 16) both tend to the same value when $t \rightarrow \infty$ (Fig. 4):

$$\left(\frac{p_c}{p_o}\right)_{t=\infty} = \left[\frac{1-G_f/G_o}{1+2G_f/K_s}\right]\exp(-t_s/t_R) \tag{20}$$

In other words, if the rock mass shows an attenuating creep with finite values of creep parameters, when $t \rightarrow \infty$, the ageing solution of the type shown previously (e.g., Ladanyi, 1974) will give a correct long-term value of lining pressure.

Note that the results shown in Figs 3 and 4, have been obtained for the following data: $G_o = 2.069$ GPa, $G_f = G_o/2$, $\eta_2 = 41.38$ GPa years, $t_R = \eta_2/k_2 = 10$ years, $t_s = 3 \times 10^7$ s $=$ 0.9513 years, $p_o = 7.586$ MPa.

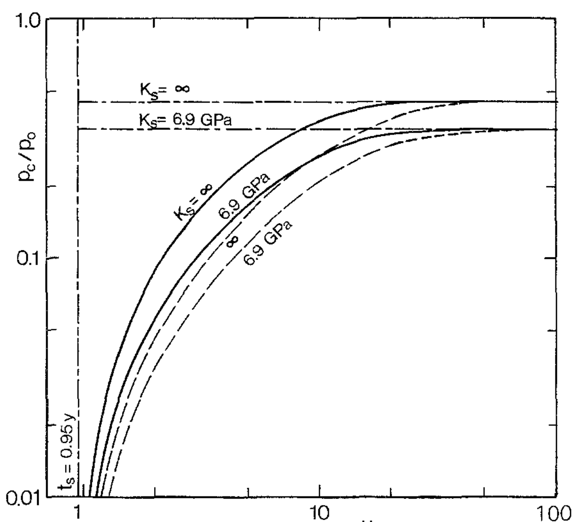

Fig. 4. Ground pressure increase curves for the case in Fig. 3. (———) Exact, (- - -) ageing. $G_o = 2.07$ GPa, $G_f = 1.035$ GPa, $\eta_2 = 41.38$ GPa years, $p_o = 7.6$ MPa.

Solution assuming a linear-elastic linear-viscous behaviour of the rock mass without failure

In this solution (Gill and Ladanyi, 1983), the rock mass is modelled by a two-parameter, or Maxwell liquid model. The model (Fig. 5) is composed of a Hooke element in series with a linear dashpot, the former being characterized by a spring constant k_1, and the latter by a viscosity η_1. Such a model represents a material which creeps at any deviatoric stress, i.e., which has neither a creep threshold nor long-term strength.

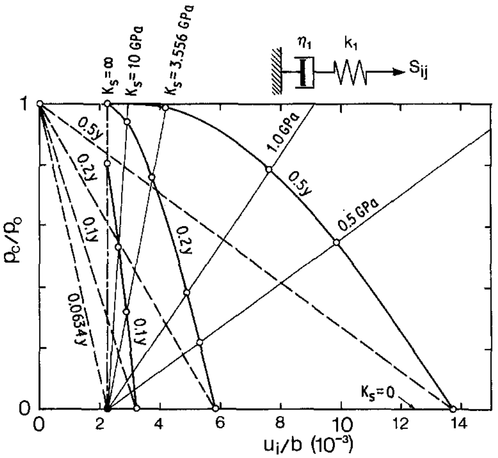

Fig. 5. Exact and conventional (ageing) characteristic lines for linear-elastic, linear-viscous (Linear Maxwell) behaviour of rock mass (Gill and Ladanyi, 1983). (———) Exact, (– – –) ageing.

Since, in the deviatoric creep case, $k_1 = 2G$, $\eta_1 = 2\eta$ and $k_1/\eta_1 = G/\eta = 1/t_R$, the following results are obtained (Gill and Ladanyi, 1983):

(a) Creep closure of unlined tunnel

$$\frac{u_i}{b} = \frac{p_o}{2G}(1 + t/t_R) \tag{21}$$

(b) 'Conventional' isochronous characteristic lines are obtained, as before, by replacing in Equation 21 p_o with $(p_o - p_i)$, and by varying p_i with t = constant for each particular line. They are represented by a set of straight lines passing through the point (0, 1) in Fig. 5.

(c) Effect of lining. For a lining of rigidity K_s, coming into contact with rock at $t = t_s$, the conventional 'ageing' theory yields:

Lining pressure increase:

$$\frac{p_c}{p_o} = \frac{t - t_s}{t + t_R(1 + 2G/K_s)} \tag{22}$$

and the corresponding closure is as before:

$$\frac{u_i}{b} = \left(\frac{u_i}{b}\right)_{t=t_s} + \frac{p_c}{K_s} \tag{23}$$

However, the exact solution (Gill and Ladanyi, 1983) gives for the lining pressure:

$$\frac{p_c}{p_o} = 1 - \exp\left[-\frac{t - t_s}{t_R(1 + 2G/K_s)}\right] \tag{24}$$

with u_i/b still given by Equation 23.

It will be seen that, for $t > t_s$, Equations 22 and 23 describe a non-linear, but different, lining pressure increase with time, both of them tending to $p_c = p_o$, when $t \to \infty$. The characteristic lines resulting from the two solutions are, however, quite different, as shown in Fig. 5. For an infinitely stiff support ($K_s = \infty$), they give, respectively:

Conventional theory:

$$\left(\frac{p_c}{p_o}\right)_{K_s=\infty} = \frac{t - t_s}{t + t_R} \tag{25}$$

Exact theory:

$$\left(\frac{p_c}{p_o}\right)_{K_s=\infty} = 1 - \exp\left(-\frac{t - t_s}{t_R}\right) \tag{26}$$

Equations 21 and 26 were also given by Panet (1979).

It will be seen in Fig. 6 that, for any given lining stiffness, the true lining pressure build-up is much faster and higher than that obtained by using the conventional (ageing) convergence-confinement theory. Note that the results shown in Figs 5 and 6 have been obtained for the following data: $E_1 = 2G = 11\ 724$ MPa, $\eta_1 = 2\eta = 8.276 \times 10^9$ MPa s, $t_R = \eta/G = 7.06 \times 10^5$ s $= 0.02238$ years, $t_s = 2 \times 10^6$ s $= 342.47$ h $= 0.06342$ years, $p_o = 6.897$ MPa.

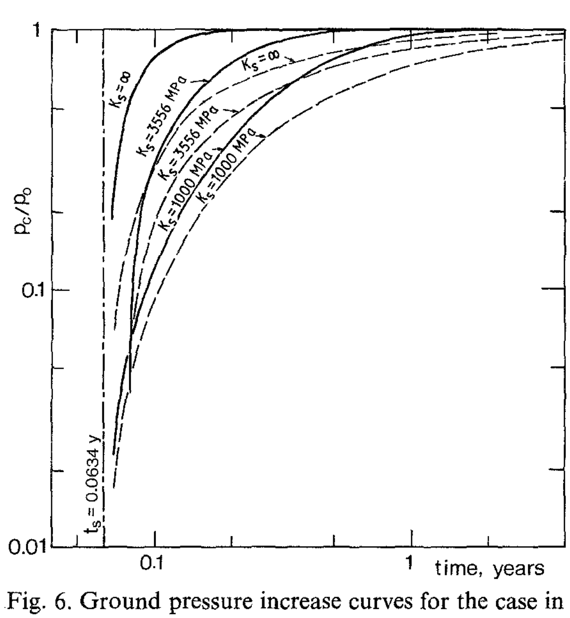

Fig. 6. Ground pressure increase curves for the case in Fig. 5. (———) Exact, (– – –) ageing.

Solutions assuming a linear-elastic non-linear-viscous behaviour of the rock mass, without failure

In this solution (Ladanyi, 1980; Ladanyi and Gill, 1984), the rock mass is represented by a two-parameter Maxwell liquid model, composed of a linear spring in series with a non-linear dashpot (Fig. 7). The spring is characterized by the constant $k_1 = 2G$, as before, but the viscosity of the dashpot, η_1, is a function of the Von Mises equivalent stress, σ_e, and the time, t.

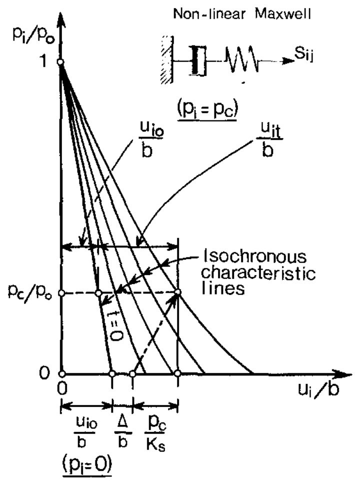

Fig. 7. Determination of ground pressure for a linear-elastic, non-linear-viscous (non-linear Maxwell) behaviour of rock mass, using the ageing concept (Ladanyi, 1980).

If power-law formulations are adopted for both the stress- and the time-dependence of the equivalent creep strain, $\varepsilon_e^{(c)}$, the latter can be expressed by

$$\varepsilon_e^{(c)} = A(\sigma_e/\sigma_c)^n \tau \tag{27}$$

where τ is the time function

$$\tau = t^c, \tag{28}$$

and

$$A = (\dot{\varepsilon}_c/c)^c \tag{29}$$

Here, $n > 1$ and $c \leq 1$ are creep exponents, and σ_c is the reference stress corresponding to an arbitrary reference strain rate $\dot{\varepsilon}_c$. The power-law equation 27 is considered to be able to represent adequately the behaviour of weak rocks, such as rock salt and potash.

Using this formulation, it can be shown that, under plane strain conditions, and with time-hardening assumption, the behaviour of a circular tunnel can be described by the following equations.

(a) Closure of an unlined tunnel (Ladanyi, 1980)

$$\frac{u_i}{b} = \frac{p_o}{2G} + B\, p_o^n\, \tau \tag{30}$$

where

$$B=(\sqrt{3}/2)(\sqrt{3}/n)^n A/\sigma_c^n \tag{31}$$

and τ and A are given by Equations 28 and 29.

(b) Conventional isochronous characteristic lines (ageing assumption), (Ladanyi, 1980) are obtained by replacing p_o in Equation 30 by $(p_o - p_i)$, and by varying p_i with t = constant for any particular line. They are represented by a set of non-linear lines passing through the point (0, 1) in Fig. 7.

(c) Effect of lining. With a lining of rigidity K_s, becoming active at $\tau=\tau_s=t_s^c$, the ageing theory gives the time $\tau=t^c$ at which a given value of lining pressure p_c will be attained (Ladanyi, 1980):

$$\tau=\frac{(p_c/2GB)(1+2G/K_s)+p_o^n\tau_s}{(p_o-p_c)^n} \tag{32}$$

and the corresponding closure is

$$\frac{u_i}{b}=\left(\frac{u_i}{b}\right)_{\tau=\tau_s}+\frac{p_c}{K_s} \tag{33}$$

with the first term calculated from Equation 30.

It will be seen that both for $p_c \to p_o$ and $K_s \to 0$, one gets $\tau \to \infty$, as expected. However, when $K_s \to \infty$, τ tends to a finite value, which is the intersection point of a characteristic line with the ordinate at $\tau=\tau_s$.

On the other hand, a more correct solution, taking into account the effect of lining on the rock mass creep rate (Ladanyi and Gill, 1984), shows the following results:

Pressure on the lining:

$$\frac{p_c}{p_o}=1-(1+t^*)^{-1/(n-1)} \tag{34}$$

where

$$t^*=K_s B(n-1)(\tau-\tau_s)p_o^{n-1} \tag{35}$$

with the corresponding closure given by Equation 33.

As required, Equation 24 implies that $p_c=0$ for both $K_s=0$ and $\tau=\tau_s$, and that $p_c \to p_o$ when $\tau \to \infty$. However, it gives also $p_c=p_o$ when $K_s \to \infty$, which means that all characteristic lines pass through the point $[(u_i/b)_s, p_o]$, as shown in Fig. 8. Clearly, for $K_s=\infty$ and $\tau=\tau_s$, Equation 35 becomes indeterminate.

Since in this model, p_c tends to p_o, the lining will fail in compression as soon as p_c attains the value:

$$p_{cf}=[f'_c(1-a^2/b^2)]/2 \tag{36}$$

where f'_c denotes the compressive strength of massive concrete.

Figure 8 shows a set of characteristic lines calculated for various values of creep exponents n and c. For the given case, the dashed line, obtained by the ageing theory, is seen to give a much smaller lining pressure than the more exact theory (1.1 instead of 2.7 MPa). Finally,

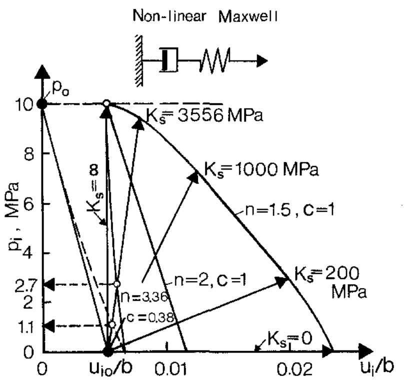

Fig. 8. Effect of ground-lining interaction on characteristic lines for a non-linear Maxwell behaviour of rock mass (Ladanyi and Gill, 1984).

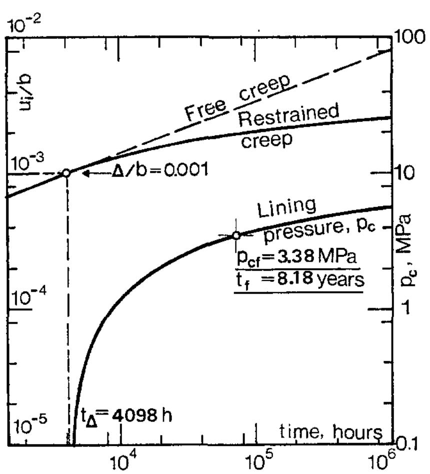

Fig. 9. Free and restrained creep and lining pressure increase for a non-linear Maxwell rock mass around a tunnel with a gap of 0.001 × radius behind the lining (Ladanyi and Gill, 1984).

Fig. 9 shows the results of calculation of tunnel wall closure and lining pressure increase for a tunnel in salt rock, with the following data and assumptions:

For tunnel: $a = 2.20$ m, $b = 2.50$ m.
For lining: $E_c = 26$ GPa, $v_c = 0.20, f'_c = 30$ MPa, giving $K_s = 3556$ MPa.
For salt rock: $E = 2700$ MPa, $v = 0.45$, $n = 3.36$, $c = 0.38$, $\sigma_c = 30$ MPa at $\dot{\varepsilon}_c = 10^{-5}\ h^{-1}$.
For ground stress: $p_o = 10$ MPa.
Lining activation time: $t_s = 4098\ h \simeq 5.6$ months.

Conclusion

The foregoing comparative analysis shows that, contrary to the conventional thinking, there is no general rule either for the position or the shape of characteristic lines for tunnel lining design, both of which are affected by the rock properties, the lining stiffness, and the support activation time. Comparing the conventional (ageing) with the exact (lining–rock interaction) methods, two clear trends can be observed:

1. For rocks with a very low or no creep threshold, like rock salt, the conventional convergence-confinement method considerably underestimates the true lining pressure at all times, up to $t \to \infty$.
2. For rock masses with a relatively high and well-defined creep threshold, the underestimate by the conventional method is much smaller and decreases with time, giving a justification to the long-term lining pressure prediction, based on long-term or residual rock mass properties.

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