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The brain a geometry engine

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Summary. According to Kant, spacetime is a form of the mind. If so, the brain must be a geometry engine. This idea is taken seriously, and consequently the implementation of space and time in terms of machines is considered. This enables one to conceive of spacetime as really "embodied."

According to Kant (1787) *spacetime* is a form of the mind:

> Space is a necassary a priori intuition, that lies at the root of every awareness. One cannot imagine its negation, although one can imagine it to be empty. It is the necessary condition for the possibility of perception, and not a consequence of perception, it is an intuition a priori, the necessary basis for external perceptions. (p. 38)

Whereas it is hard to refute the fundamental truth of this insight, it appears to be even harder to accept it in practice. This is abundantly clear from the scientific literature. Mind and brain are studied in terms of space and time, rather than the converse. In this paper I take Kant's notion very seriously, and attempt to indicate how the brain can be understood as an *embodiment* of spacetime in a non-spatiotemporal manner. The ultimate objective of the enterprise is to cleanse brain theory from the last vestiges of the flock of *homunculi.*

Stratification of scientific description

A recurrent problem in brain theory is the broad spectrum of logical levels of description required in order to catch the intuitively interesting features of the phenomena. Confusions of levels are the *rule* rather than the exception in the scientific literature. Of necessity a few remarks will have to suffice here. Most of the discussion will be focussed on visual perception and the visual system, although the concepts are of much broader scope.

A property that almost all vital functions hold in common is their *cyclical nature* (cycles juxtaposed and nested in tremendous numbers and to great depth). The cycles may encompass very disparate levels of description (*e.g.*, swimming in dolphins has to be understood in terms of the hydrodynamics of the medium, the biomechanics of skeleton and muscle groups, the molecular structure of muscle fibers, the global climate over the ages, the structure of the DNA molecules, brain activity, foraging, prey-catching and mating goals, etc.). Some levels are merely physically disparate or complementary (*e.g.*, molecular vs. global), others are logically disparate or complementary (*e.g.*, mating goals, brain activity, hydrodynamics). Thus not only must one accept a pluricausality, but even more important, in cyclical systems "causality" (any variant) applies only to limited parts of a cycle: any "input-output analysis" is necessarily restricted to a limited – and essentially whimsically defined – subsystem.

In this paper I (arbitrarily) let the visual process start at the eye and end in either overt behavior or in *potential behavior*. The former I shall denote *sensorimotor behavior*, the latter *visual perception*. Both subserve visually guided behavior. Behavior is defined in purely physicochemical terms: a body movement such as "a left-arm pronation of 10 degrees" is a prime example. *Potential behavior* is the invariant core of behavior under variation of goals and structure of the environment to the extent that it is conditional to prior optical exposure. Here I assume that the *goals* of the agent under study are under partial control of the scientist (by dehydration or food deprivation in monkeys or rats, payment or fines for human subjects, etc.). Thus *perception* is different from *reflexive behavior*, but still an objective fact (Koenderink, 1984a). Perception is defined in much the same manner as the "acquired knowledge" due to teaching. The pupil is tested over a variety of problems and the knowledge gained is judged in terms of the ability to act efficaciously over that range. The knowledge gained is the potential action conditional to the teaching.

A *picture* is a spatiotemporally structured field in the description of the scientist. The retinal irradiance in a psychophysical experiment is a picture. Operationally it is a set of observations, each observation being characterized by a spatiotemporal aperture (a photocell with diaphraghm and shutter) and a recorded flux. A page from the physicist's diary with entries recording spatiotemporal coordinates, structure of apparatus, recorded fluxes, etc., is a paradigmatic picture.

A *record* is a set of items obtained via some recording device, without further order. If the physicist has forgotten to note the coordinates, the set of fluxes is a record. The electrophysiologist sticking an electrode into the brain more or less at random – a fair approximation of daily practice – collects data that are at least spatially a record of brain activity. These examples are paradigmatic.

An *image* is a perception. In the diary of the psychophysicist it is the common core of recorded behavior over varied trials following a well-defined optical exposure. Thus an introspective report is *not* considered to be an image. A perception is a scientific – thus public – fact.

An image is not to be confused with a picture, nor a picture with an image. A record is to be treated as different from both. These terms indicate *generic classes of scientific descriptions* of certain vital functions, not parts of the body or mind. If you look seriously for pictures in the brain or for images in the mind, you are using the terms in a different sense than I do in this paper.

Lotze's concept of "local sign"

Soon after Kepler (1604) correctly explained the geometrical optics of the eye and Descartes (1637) contemplated the inverted world on the back of an excised ox's eye, vision was considered to be basically *understood* by the hardcore scientist. A picture could be recorded in part of the visual cycle and was considered to "explain" the images. (That optical exposure influences the potential behavior of many organisms was of course clear to all.)

It took centuries before Hermann Lotze (1884) for the first time in history, clearly distinguished what I have called pictures, records, and images. Lotze is very explicit and extremely pellucid. He understands clearly that optic-nerve activity is merely a record, and that a record differs from a picture by the absence of what he called a "local sign" (*Localzeichen:* in the physicist's diary the coordinates are the local signs). Lotze saw that local signs cannot be *spatial positions in the brain*, as this would let in the homunculus by the back door. Brain signals are of an *intensive*, rather than an *extensive* nature. Lotze even foresees the discovery of somatotopic maps and warns against the trap of considering these to be the embodiments of local sign. Regrettably, his warning has not been heeded. In modern neuroscience somatotopic maps are generally considered to be the embodiments of the visual field.

Although this is rarely acknowledged, Lotze's concept of a local sign leads to *experimentally verifiable* predictions. It is clear by now that spatial discrimination is learned from experience, and that this process sometimes fails. There exist visual systems that appear to be normal in every respect, except for the absence of any local sign: certain varieties of amblyopia. A striking case is described by Hess (1982). Hess's patient had one normal and one amblyopic eye. The normal eye showed normal acuity on the classical letter-chart test, whereas the amblyopic eye was almost useless. Yet on other tests (e.g., spatial-frequency transfer) the eyes came out equally well, as a clear indication that the optical and neural resolving power of the amblyopic eye were fully normal. Introspective reports revealed that the amblyopic visual field was scrambled: the patient had access to all the pieces, but could not put the puzzle together. The only thing awry with people like the patient is apparently the absence of any local sign in the amblyopic eye.

The problem then becomes: how does the brain manage to generate efficacious action on the basis of the optic-nerve record of $10^6 - 10^7$ signals? Or, to put it in more operational terms, can we build a machine that *unscrambles* an arbitrary permutation of such an input? If so, then at least we have shown that the problem of a local sign admits of other than *deus ex machina* (or homunculus) solutions, and we have something to look for (namely: specific types of circuitry, instead of little men) in the brain.

The physiological substrate of the local sign

An entertaining thought experiment may help to find the direction in which to look for an answer. Scramble a jigsaw puzzle. Can you then reorder the pieces? Yes, that's the whole idea! Now scramble the stones of a mosaic put together from cubical homogeneously colored glass or ceramic pieces of similar size. Can you reorder the mosaic? No! You can make another version, or order the stones according to color or something, but you can't *restore* the mosaic to its original order. Why this difference? Obviously because neighboring jigsaw pieces are *related* in a way that can be put to a test, whereas the mosaic stones are unrelated, except in the cases in which you number them (adding an artificial local sign) before you scramble the mosaic. When London Bridge was transferred to some desert in the United States, the stones had to be numbered for this very reason. That such a numbering is not needed for the jigsaw puzzle derives from the fact that the pieces are related by their very nature. Either the shape of the boundary and/or the color and texture let you decide unambiguously whether two pieces match or not. If two pieces actually possessed the same relation to all other pieces, you could not help confusing the pair.

This latter observation has an interesting analogy in psychophysics, originally described by Helmholtz (1896). In case of acute toothache the patient is often uncertain whether the trouble is in the upper or in the lower jaw. The dentist finds out by judiciously applying force to both spots. Why? Helmholtz explains this – I think rightly – as a *fused local sign* due to the fact that the brain always receives similar signals from both places. In the act of chewing food, locations immediately above each other in the upper and the lower jaw are stimulated in synchrony. Thus the brain has no option but to confuse these locations. This indicates the fundamental role of the *correlation of nerve activity*. The relation between the pieces is exactly this *correlation*. It can be measured in the brain by simple circuitry, and need not be "given" *a priori*.

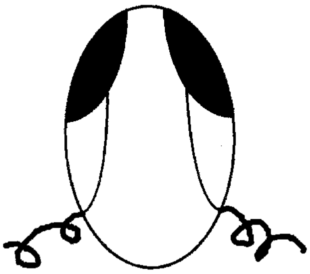

Fig. 1. A simple photophobic animal. The excitation of the two eyespots activates the kinocilia on the ipsilateral side. As a consequence, the animal tends to turn away from the light source, at least in an environment dominated by low Reynolds numbers.

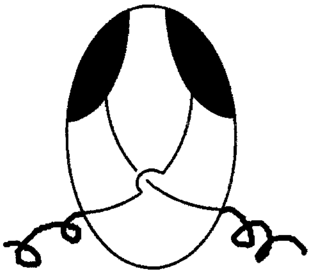

Fig. 2. A simple phototropic animal. The excitation of the two eyespots activates the kinocilia on the contralateral side. As a consequence, the animal tends to turn towards the light source, at least in an environment dominated by low Reynolds numbers.

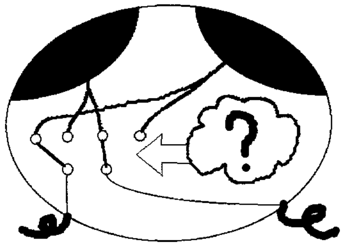

Fig. 3. A simple animal with a choice between photophobic and phototropic behavior. The switch box is governed by a "goal" system. The current goal decides whether the animal seeks or flees from the light. Of course the goal unit could be wired up to other sensors.

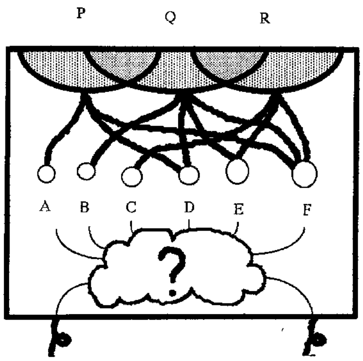

Fig. 4. A rather complicated animal featuring three eyespots, P, Q, and R. The somatotopic projections to the brain are also quite sophisticated: there is a high-resolution projection A, B, and C, a medium-resolution projection D and E, and a low-resolution projection on a single cell F. A network with its state governed by the current goals maps these somatotopic representations upon the motor system, which simply consists of a pair of kinocilia. A problem of local sign would be: Can this animal figure out the fact that Q is *between* P and Q?

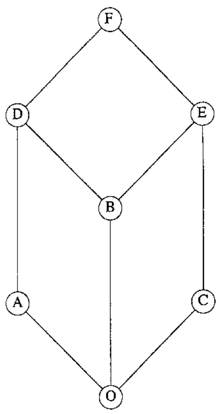

Fig. 5. The Hasse diagram of inclusions as derived from the pairwise correlations of brain-cell activities. (This structure is available to the animal itself, not merely to an outside observer. A cell I is said to cover, or include another cell J if, and only if, it is the case that for every other cell K (say) a correlation of J and K implies a correlation between I and K. A node *above* and *connected with* another node in the Hasse diagram indicates that the corresponding cell covers the other one. All cells formally cover the null cell (0).

Fig. 6. The segment (or linear chain) PQR has a covering structure isomorphic to the Hasse diagram in Fig. 5, which again characterizes the correlation structure of the brain of the animal depicted in Fig. 4. For PQR covers PQ and QR, PQ covers P and Q, P covers the null set, *etc.* Thus the correlation structure of the brain of the animal drawn in Fig. 4 can be interpreted as a linear chain, hence Q is between P and R. We have provided the animal with a rudimentary local sign.

The local sign as the scaffold of sensory space

In order to elucidate the power of the correlation concept I shall introduce some simple artificial organisms as "concrete" thought experiments (Braitenberg, 1984; Koenderink, 1984a).

First consider a simple (single-cell) aquatic organism equipped with two eyespots, each eyespot sensitive to about a halfspace. The eyespot activities activate two kinocilia that may propel the animal through its medium. Ipsilateral connections (Figure 1) make the organism "photophobic," whereas contralateral connections (Figure 2) make it "phototropic." Clearly the two varieties of animal are *identical* as abstract machines (Minsky, 1967), even though they behave in completely opposite ways. The lesson is that there is more to organisms than their structure as abstract machines: the nature of the interface to the environment is of crucial importance.

As a simple step in the evolution towards more complicated autonomous behavior you may consider a combination of the two opposites, in which the sensorimotor connections can be controlled via a switch box (Figure 3). This animal may freely decide whether to act in a phototropic or a photophobic fashion. The switch is controlled by the "current goal." (You may imagine it to be wired up to a diffuse-radiation-density sensor, or whatever you wish. Even a random generator would do, leading to a whimsical animal displaying distinct "moods.")

As a follow-up you may consider a similar organism, but one higher up in the chain of evolution, sporting three eyespots (Figure 4). The receptive fields of the spots are distinct, but overlap to an appreciable extent. The eyespots project upon three "brain areas" according to a neat somatotopic scheme. The projections are of different resolution (Koenderink & van Doorn, 1978). There are six "cortical receptive fields." It is an easy exercise to find the correlation matrix for this brain. This can also be done automatically through simple circuitry, which I add to this brain: a "superagent" is not required. The correlation structure is a relation between the brain cells, which is a subset of the set of all pairs of brain cells. Formally it is most conveniently described as a *partial order* through the Hasse diagram of inclusion (Figure 5). Curiously enough, this structure is isomorphic to the Hasse diagram of the coverings of the linear chain of three elements (Figure 6).

The point is that the correlation structure, which is the structure of the time averaged activity of the corresponding brain area, is *isomorphic* to the overlap structure of a *linear chain* of three elements (P, Q, R) (say). *Thus this simple brain has embodied the fact that its visual field is a linear chain of three elements.* This is an embodiment of local sign, the birth of space.

It is possible to follow up these thought experiments with the necessary mathematics (Koenderink, 1984b; Toet, Blom, & Koenderink, 1987 a–c, 1988) in order to reveal the following result. In many cases (which can be precisely specified) the correlation structure of the signals in some nerve bundle suffices to construct an isomorphism to a *topology* (dimension, neighborhood relations, connectivity structure) and even a simple *geometry* (convex sets, linear subspaces) of an "ideal sensory space." The system need not be bothered by arbitrary permutations of the input bundle (artificial destruction of somatotopic local sign), since the only information it uses is in the correlations. The main precondition is that the system must be in a position to establish the correlations through experience. Precise conditions can be specified in which this procedure must fail in specific ways leading to amblyopia. The failures may be of many varieties, *e.g.*, the geometry may be one-dimensional, but branched; a point on a linear segment may fail to divide that segment into two pieces; *etc.*

The differential texture of sensory space

Much of the structure of space in mathematics (Schouten, 1954) and physics (Misner, Thorne, & Wheeler, 1973) is studied in the immediate neighborhood of each point. This is the so-called "differential structure." This type of structure is exemplified in the set of all spatial directional derivatives up to a certain order for arbitrary functions (or "fields") defined on the space. Examples are *points*, which assign the value of any field (the zero-order derivative) at a specific location to that field, or *vectors*, which assign the first-order directional derivative of any field at that location to the field. Notice that *geometrical entities* such as point, or vector, acquire an *operational*, rather than an *extensive*, meaning.

Although a *vector* is usually introduced as a "bilocal entity," say the directed stretch AB defined by two points A and B, the more viable mathematical definition regards the vector as a *directional derivative*. This is actually closely connected to the bilocal notion if you remember the old-fashioned definition of directional differentiation as the limit of functional values at two points (A and B say) divided by their distance for vanishing distance. The vector then *lives at a single point*, in its own local "tangent space."

Euclid defines a point as "that which has no parts." This is usually interpreted as "having no size" or even of "being vanishingly small." This is not at all necessary (see Huntington, 1913; Koenderink & van Doorn, 1987; Koenderink, 1988), however; "having no parts" is best interpreted in the sense of "having no windows." *One declines to investigate the internal structure.* For the physicist a point is a physical operation yielding an observation that is relevant for a certain fiducial volume. He never dreams of setting any limit to zero size: such an operation has no meaning to the physicist. The earth is a point in the Kepler problem, the galaxy is a point in cosmological theories. Thus the operational definition of a point is a linear zero-order isotropic operator that assigns a number to scalar fields. The size is arbitrary. Although not often recognized, this abstract notion is closely related to that of the receptive field in the neurosciences.

These concepts can be formalized into a powerful theory if you admit two extra assumptions: That *size invariance* holds in the visual system, and that *sampling* with points of a coarser size than the present level of resolution results in a loss (never a gain) of spatial structure. You require the so-called absence of spurious resolution; technically the resulting structure is known as *scale space* (Babaud, Witkin, Baudin, & Duda, 1986; Koenderink & van Doorn, 1978; Koenderink, 1988). This formalism leads to a complete taxonomy of possible local operators (Koenderink, 1988a; Koenderink & van Doorn, 1988), points, vectors, tensors, etc. of all orders, symmetries, and sizes (Figure 7). It is an exciting fact that the receptive fields found in the primate visual system (Jones & Palmer, 1987a,b; Young, 1985; Zucker & Hummel, 1986) are almost invariably of this class. These neurons can be interpreted as embodiments of differential operators, and the activities of cortical "columns" likewise as *jetbundles* of the retinal irradiance. This interpretation immediately suggests implementations of circuits for the encoding of geometrical entities (*e.g.*, curvatures) that are already implied by certain psychophysical results (Koenderink & Richards, 1988). These operators are also in widespread use in image processing and machine vision nowadays (Coggins & Jain, 1985; Coggins, Fay, & Fogarty, 1986).

Thus you may understand a large part of the structure of the front-end visual system as an embodiment of differential geometry of the visual field. Such an interpretation is purely syntactical; "feature detectors" have no place here. Instead of the concrete "edge detectors" and "bar detectors," one speaks of the abstract first- and second-order directional derivatives. The difference in emphasis is crucial. The derivatives are meaningless as such, their meaning derives purely from their use. A bar detector *may* well be used to detect bars, but may equally well play a role in the encoding of curvature. The very same cell may well take part in various calculations. Outside their context the signals carried by these cells are completely meaningless.

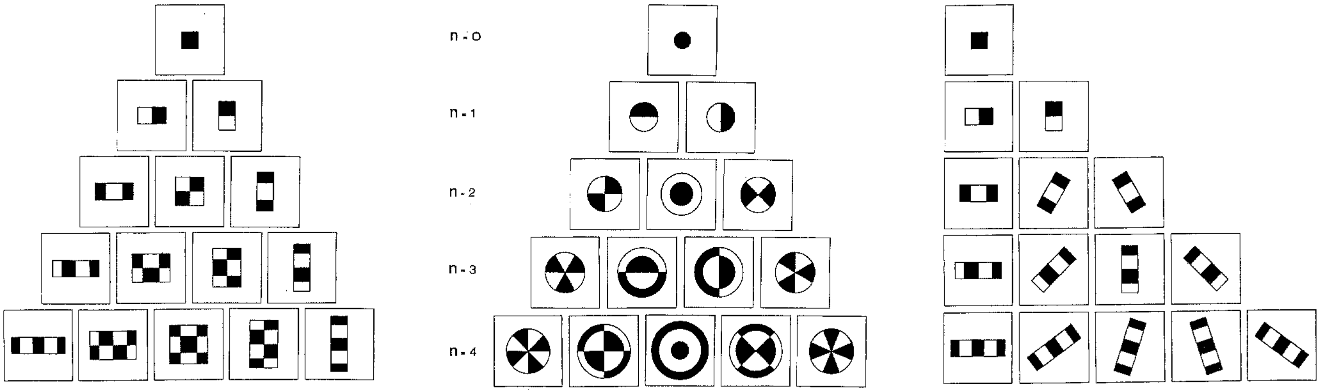

Fig. 7a – c. The taxonomy of receptive fields: **a** Receptive fields characterized by translational symmetry. These operators yield activities that can be interpreted as mixed partial derivatives of the retinal irradiance distribution in Cartesian coordinates. All distinct receptive fields up to the fourth order are shown. **b** Receptive fields characterized by rotational and radial symmetry. These operators are mixed partial derivatives in polar coordinates. All distinct receptive fields up to the fourth order are shown. **c** Receptive fields structured like Gabor wave trains which can be interpreted as (higher-order) directional derivatives. They must occur in various orientations. The figure gives a minimal set for orders less or equal to four. Such fields (with orders of differentiation up to four) occur abundantly in primate brains with a uniform orientational distribution. The second-order ones (the "bar detectors") led to a nobel prize (Hubel and Wiesel).

Helmholtz's geodesics: The global structure of sensory space

The differential structure is confined to single locations. A *multilocal* operation requires a lot of additional structure. A minimal requirement is that of an *affine connection*, or roughly the ability to judge parallelity of directions at proximate, but distinct, locations (Schouten, 1954). Many types of geometrical expertise within the capacity of the human visual system imply the existence of exactly such an affine connection. But it is not easy to base this on such fundamental properties as the simultaneous-correlation structure or local receptive-field structure.

An elegant solution was put forward by Helmholtz (1986) and very forcefully defended by Poincaré (1902, 1905). The idea is that *eye movements* can be used to superimpose optical structure on different retinal locations under control of the organism. This is much like the ability to carry a yardstick around; it makes comparison of remote structure possible. If an eye movement can carry A to P and at the same time B, and C to Q and R, then the triangle ABC must be congruent with the triangle PQR. This would even seem to induce a metrical connection. In actual fact the eye movements are highly constrained (Donders's and Listing's laws constrain the number of degrees of freedom to a mere pair). Thus only parallel transport is possible and you do not obtain a full metric.

Helmholtz was among the first to understand the kinematics of human eye movements clearly and to draw the consequences. The kinematical constraints determine the position of the eye's orb fully once the direction of gaze is fixed. What remains is the possibility of establishing geodesics with an affine parametrization on each. Geodesic congruences were calculated by Helmholtz. They have been implicated as a model for the subjective curvatures of objectively straight lines, as had already been noticed by the ancients, who used them in their architecture (Hauck, 1875).

A system of this kind is a likely candidate for the global tuning of the geometrical structure of the visual field, built upon the fundamentals of topology (correlation structure) and local differential geometry (receptive-field assemblies in columns).

Discussion

As we have shown in this paper, homunculi or God-given local signs are not required in a formal scientific understanding of the geometrical expertise of brains – that is, the apparent ability to generate efficacious potential action based on optical exposure. The brain can organize *itself* through information obtained via interactions with the physical world into an embodiment of geometry, it becomes a veritable *geometry engine*.

Whether space is in the head, or the head is in space, remains undecided here. In the final analysis the distinction is scientifically meaningless anyway in view of the inherent circular nature of vital processes, including optically guided behavior.

There may be a point in holding that many of the better-known brain processes are most easily understood in terms of differential geometrical calculations running on massively parallel processor arrays whose nodes can be understood quite directly in terms of multilinear operators (vectors, tensors, etc.). In this view brain processes in fact are space. But the spaces operated upon are quite remote from Kant's simple notion of three-dimensional Euclidean space.

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