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Incentives to Adopt New Technologies Under Different Pollution-Control Policies

TILL REQUATE
Institute of Mathematical Economics, P.O. Box 100131, University of Bielefeld, 33501 Bielefeld, Germany, e-mail: requate@nw42.wiwi.uni-bielefeld.de

Abstract

The paper investigates the incentives for polluting firms to adopt new technologies under pollution-control policies such as effluent taxes and auctioned permits. We pay explicit attention to the output market. Firms can choose among two types of technologies, a conventional one with high marginal abatement costs and a new one with low margainl abatement costs but higher fixed costs. We find that taxes almost always induce complete adoption or no adoption at all. Permits, in contrast, allow for partial adoption. Moreover, ex post, permits can always induce first best, whereas taxes cannot if partial adoption is socially optimal.

Key words: emission taxes, auctioned permits, adoption of new technology

Environmental policies based on prices and decentralized decision making, such as emission taxes or tradeable permits, have been proved to be powerful and efficient instruments of pollution control. It is well known that these policy tools are equivalent if the regulator is well informed and markets are perfectly competitive. This equivalence result has been derived, for the most part, under static conditions—that is, the firms' technologies have been assumed to be exogenously given. Spulber (1985) has demonstrated that if firms are identical, emission taxes as well as auctioned permits are optimal also in the long run. This means that they induce the optimal number of firms to enter the market. But Kneese and C. Schultze (1975, p. 38) have stressed that "over the long haul, perhaps the most important single criterion on which to judge environmental policies is the extent to which they spur new technology toward the efficient conservation of environmental quality" (see also Orr, 1978).

In this paper we investigate the two most prominent policies of pollution control—namely, taxes and auctioned permits—with respect to the incentives for firms to adopt new technologies. Similar to Spulber (1985) but in contrast to the bulk of literature in environmental economics, we pay explicit attention to an output market where firms engage in perfect competition. We assume that there are two possible technologies, a conventional one that causes high marginal abatement costs and an innovative one that leads to low marginal abatement costs but that incurs an additional fixed cost due to installation and maintenance of abatement equipment. We do not endogenize technology itself. We rather endogenize the choice of technology and study how many firms will keep the conventional technology and how many firms will invest in the new technology under either of the two policies.

In order to examine the efficiency of the policy tools, we start to examine socially optimal allocations when two types of technologies are available. As usual in partial analysis, we assume that pollution is valued by a social damage function, and we find that the optimal level of adoption depends heavily on the slope of that function. In particular, the new technology should not be adopted if the social damage function is relatively flat. *Complete adoption* is optimal if the damage function is relatively steep, whereas for intermediately steep damage functions *partial adoption* is optimal—that is, *both* types of technologies should be employed.

The question arises, whether Spulber's optimality result does still hold if firms can choose among different technologies. Or to put it another way, whether permits and taxes can be used to induce the socially optimal degree of adoption. The answer is yes and no. For we will discover a remarkable asymmetry between permits and taxes here. It turns out that for almost all tax levels only conventional firms *or* only innovators can stay in the market at the same time. On the other hand, there is *a* tax level that causes a whole set of competitive equilibria such that the number of conventional firms and the number of innovators are not uniquely determined. Permits, in contrast, always lead to a *unique* competitive equilibrium, and both types of firms can stay together in the market for different quotas of permits. This means that, ex post, permits can *always* implement the socially optimal allocation, whereas with taxes one can achieve this only if complete adoption of the new technology is socially optimal. If partial adoption is optimal, taxes fail to implement first best in general. Among the multiplicity of free-entry market equilibria only one equilibrium is efficient, and decentralization of first best could happen only by chance.

If a polluting industry has been regulated optimally before innovation, the original tax or permit policy is clearly not optimal any longer as soon as a new technology is available. In practice, however, adjustment of environmental policy happens with large time delays, if at all. Hence it is important to know how efficient those policy tools are if the regulator cannot anticipate a new technology and if the institutional framework does not allow for fast and efficient policy adjustment. It turns out that, especially if the conventional industry is optimally regulated before the innovative technology has been available, both policy instruments can cause too much but also too little adoption of new technology if the original policy levels are still valid. In particular, we find that for a considerable range of parameters where partial adoption is optimal, taxes lead to overinvestment, whereas permits cause underinvestment in new technology. Even more striking, welfare may decrease through innovation under taxes, whereas this can never happen under permits. These results apparently provide strong arguments in favor of auctioned permits. If, on the other hand, the social damage function is sufficiently steep such that complete adoption of the new technology is optimal, the pattern can be reversed: there may be overinvestment in the new technology under permits but underinvestment under taxes!

Recently, other authors, such as Downing and White (1986), Milliman and Prince (1989), and Malueg (1989), have alluded to this topic. In particular, Milliman and Prince provide a detailed comparison of the incentives to adopt new technologies under different policies such as command and control, effluent taxes, auctioned permits, free permits, and subsidies on abatement. Under different scenarios they rank those policies with respect to the firms' incentives to adopt less polluting technologies. In most of the cases, auctioned permits turn out to be the winner. However, as fashionable in the environmental economics literature, those authors pursue partial-partial analysis—that is, they focus on the pollution sector only and do not pay attention to an output market. In contrast to their findings, our more general model does *not* allow for the conclusion that one of the two policies is superior in general.

Laffont and Tirole (1994a, 1994b) analyze permit markets where firms can bypass the cost of buying permits by investing into emission-free technology (1994a) or by engaging in R&D in order to develop a technology that does not cause pollution (1994b). In the first model permits lead to overinvestment,[1] whereas in the second model permits lead to underinvestment.[2] Those models differ from ours in several points. First, these authors also do not pay explicit attention to the output market. Second, the number of firms is not endogenous. And finally it is assumed that innovation always leads to emission-free production.

The paper is organized as follows. The following section contains the basic assumptions. Section 2 investigates the socially optimal allocation if both types of technologies are available. In Section 3 we characterize free-entry equilibria under any tax or permit policy, and we draw conclusions with respect to welfare. The final section concludes. Technical proofs are relegated to the appendix.

1. The model

Throughout this paper we consider a partial model where an endogenous number of n firms causes pollution while producing a homogeneous consumption good. Let q_i and e_i, $i = 1, \ldots, n$, denote firm i's output and emission level, respectively. Industry output is written as $Q := \Sigma_{i=1}^n q_i$, total emissions as $E := \Sigma_{i=1}^n e_i$. Welfare, as typical for partial models, is the sum of consumer welfare derived from consumption of the homogeneous good, minus the damage from pollution, minus production costs—that is,

$$W(q_1, \ldots, q_n, e_1, \ldots, e_n; s) := \int_0^Q P(z)dz - S(E, s) - \sum_{i=1}^n C^i(q_i, e_i), \qquad (1)$$

where $P(\cdot)$ is a downward sloping inverse demand function for the consumption good with a finite choke-off price $\bar{p}$. Moreover, P satisfies[3]

$$P''(Q) < -2P'(Q)/Q \qquad (2)$$

for all $Q > 0$. $S(\cdot, \cdot)$ is the social damage function depending on a damage parameter s. Damage is increasing and convex in total pollution E—that is, $S_1 > 0$, $S_{11} > 0$,[4] and marginal damage increases in s—that is, $S_{12} > 0$. The damage function is supposed to represent the disutility that consumers suffer from pollution plus the economic damage incurred by other industries. The damage parameter s is an exogenous parameter of the model and can be interpreted as an indicator of how hazardous the pollutant is. It also determines the slope of the social-marginal rate of substitution between consumption and pollution (or abatement). Parameterizing S via s allows us to completely characterize the social optimum and also regulatory policies as a function of the damage function's steepness.[5]

A firm's cost function slits up into a fixed cost $F > 0$ and the variable cost v, which depends on output q and emissions e:

$$C(q, e) = \begin{cases} 0 & \text{if } (q, e) = (0, 0), \\ F + v(q, e) & \text{else.} \end{cases} \tag{3}$$

Since the derivatives of v and C coincide, we write all assumptions about v in terms of the total cost function C. Assuming sufficient smoothness of C we define

$$Y_{MAC}(C) \equiv \left\{ (q, e) > 0 \;\middle|\; \frac{C(q, e)}{\langle \nabla C(q, e), (q, e) \rangle} = 1 \right\} \tag{4}$$

as the set of all quantities and emission levels (q, e) for which average costs are minimized. Note that if C depended on output only, the set defined in (4) would be a singleton satisfying $C(q)/q = C'(q)$—that is, average cost equals marginal cost, which determines a firm's optimal scale. In a multiproduct case we get a whole set of production plans for which we have (generalized) *minimized average costs* (hence the index MAC).[6] In order to compare *different* technologies, it is necessary to normalize those production plans by their total costs. Hence we define

$$\tilde{Y}_{MAC}(C) \equiv \left\{ \left(\frac{q}{C(q, e)}, \frac{e}{C(q, e)} \right) \text{with } (q, e) \in Y_{MAC}(C) \right\}. \tag{5}$$

If the cost function is sufficiently regular, to be defined in Assumption 1, (5) defines an implicit function h such that $\forall (y_1, y_2) \in \tilde{Y}_{MAC}(C)$ we have $y_1 = h(y_2)$. See Figure 1 for the shape of two different sets $\tilde{Y}_{MAC}$ and corresponding functions h. The next assumption guarantees that the variable cost is sufficiently convex:

Assumption 1. *For all $i = 1, \ldots, n$ the firms' cost functions $C^i : \mathbb{R}^2_+ \to \mathbb{R}$ are twice continuously differentiable and satisfy (we omit the superscript i) the following:*

1. $C_1 > 0$, $C_{11} > 0$, $C_{22} > 0$, $C_{12} < 0$.
2. *For all q there is $e(q)$ such that $C_2(q, e(q)) = 0$, and $C_2(q, e) < 0$ if $e < e(q)$, and $C_2(q, e) \geq 0$ if $e > e(q)$.*
3. $C_{11}C_{22} - [C_{12}]^2 > 0.$ (6)
4. *For all (q, e) in an open stripe containing $Y_{MAC}(C)$ we have*

$$\langle \nabla C_1, (q, e) \rangle \equiv C_{11}q + C_{12}e > 0, \tag{7}$$

$$\langle \nabla C_2, (q, e) \rangle \equiv C_{12}q + C_{22}e > 0. \tag{8}$$

5. *Moreover h is* concave.[7]

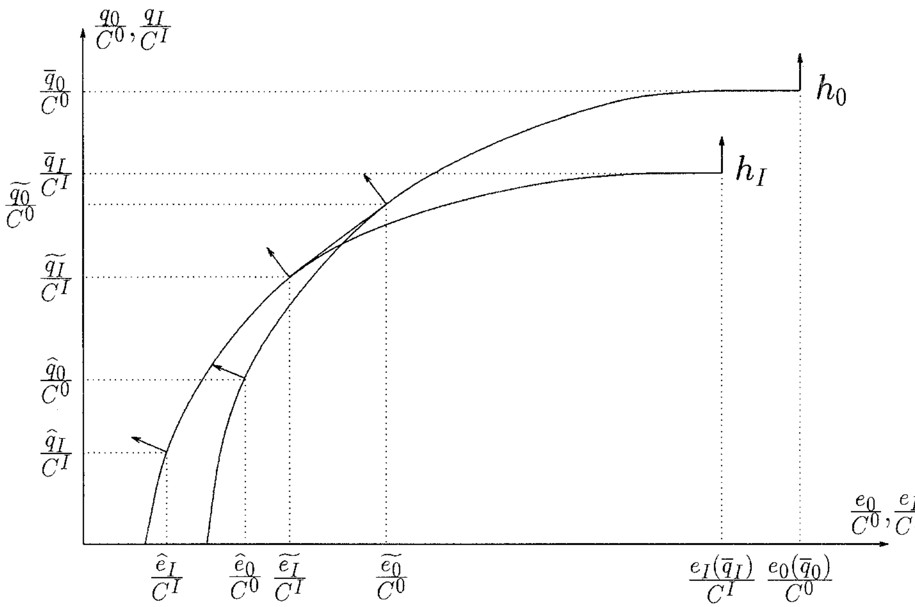

Figure 1. The sets $\tilde{Y}_{MAC}(C^0)$ and $\tilde{Y}_{MAC}(C^I)$ represented by the functions h_0 and h_I. In order to not burst the labels of the points on the axes we wrote for short q_i/C^i instead of $q_i/C^i(q_i, e_i)$. The arrows give the directions of the gradients of the cost function along the h_i curves. Note that they are perpendicular to those curves.

Items 1 through 3 imply that the variable cost function is convex. In particular we have increasing marginal costs for fixed emission levels, abatement costs are convex for each fixed output, and output and emissions are cost complements ($C_{12} < 0$). Item 2 says that each output level has a cost-minimizing emission level that the firms would choose in the absence of regulation. Item 4 guarantees that $\tilde{Y}_{MAC}(C)$ is a one-to-one relation—that is, that the function h is well defined.[8]

To make the analysis interesting we assume that in the absence of regulation there exists a market for the homogeneous commodity. For that case we can define a reduced cost function by $\tilde{C}(q) := C(q, e(q))$, and define $\bar{q}$ by

$$\frac{\tilde{C}(\bar{q})}{\bar{q}} = \tilde{C}'(\bar{q}) \tag{9}$$

as the output level that minimizes the average cost in the absence of regulation. By Assumption 1 such an $\bar{q}$ exists. A market exists if $\tilde{C}^i(\bar{q})/\bar{q} < \bar{p}$. By continuity this implies that there is also a market under moderate regulation—that is, if the government sets a lax emission standard $\bar{e}$ slightly smaller than $e(\bar{q})$ (or charges a sufficiently low emission tax). Finally, to exclude corner solutions, we assume that production is not possible without any pollution, which is certainly realistic. Formally this means that for any output $q > 0$ there is a sufficiently low emission level e^* such that for all $e < e^*$ the average cost exceeds the choke-off price—that is, $C(q, e)/q > \bar{p}$.[9]

1.1. Two different types of firms

Assume now that there are two types of technologies, a conventional one of type 0 with high pollution—that is, $e_0(q)$ is large—and an innovative technology with relatively low pollution—that is, $e_I(q) < e_0(q)$. The technologies are represented by their cost functions C^0 and C^I, respectively. The corresponding quantities and emission levels are denoted by q_0, q_I, e_0, and e_I. The innovator's fixed cost F^I include fixed costs for buying and installing the new technology, such that $F^I > F^0$. Since we allow for free entry (up to the fixed costs), we assume for simplicity that new entry and switching of technology cause the same size of fixed costs. Our crucial assumption on the relationship among the two technologies says that the conventional technology provides a cost advantage in the absence of regulation—that is, $\tilde{C}^0(\bar{q}_0)/\bar{q}_0 < \tilde{C}^I(\bar{q}_I)/\bar{q}_I$. For a sufficiently small emission level $\bar{e}$, however, the new technology has the lower minimal average costs—that is, $C^0(q_0, \bar{e})/q_0 > C^I(q_I, \bar{e})/q_I$ for $(q_0, \bar{e}) \in Y_{MAC}(C^0)$ and $(q_I, \bar{e}) \in Y_{MAC}(C^I)$. For simplicity we assume that there is no back switching of the cost advantages. These properties are depicted in Figure 1 and are formalized more technically and more generally in Assumption 2 in the appendix.

2. The social optimum

Since we are interested in the efficiency of policy tools like taxes and permits, it is useful to first study socially optimal allocations if both types of technologies—the conventional and the innovative one—are available. The social planner would maximize welfare with respect to quantities, emissions levels, and the numbers of firms. Let n_0 and n_I denote the numbers of firms with the conventional cost function C^0 and with the new cost function C^I, respectively. Since the variable-cost functions are convex, the conventional firms must have the same production plan (q_0, e_0), and the innovative firms must have the same plan (q_I, e_I). Hence the social planner's program is

$$\max_{q_0,e_0,q_I,e_I,n_0,n_I} W(q_0, q_I, e_0, e_I, n_0, n_I; s) \equiv$$

$$\max_{q_0,e_0,q_i,e_I,n_0,n_I} \left\{ \int_0^{n_0q_0+n_Iq_I} P(z)dz - S(n_0e_0 + n_Ie_I, s) - \right.$$

$$\left. - n_0C^0(q_0, e_0) - n_IC^I(q_I, e_I) \right\} \tag{10}$$

Denote by $q_0^*(s)$, $q_I^*(s)$, $e_0^*(s)$, $e_I^*(s)$, $n_0^*(s)$, $n_I^*(s)$ the solution, and by $Q_I^*(s) = n_0^*(s)q_0^*(s) + n_I^*(s)q_I^*(s)$, and $E_I^*(s) = n_0^*(s)e_0^*(s) + n_I^*(s)e_I^*(s)$ the aggregate output and emission levels, respectively. Then the solution of (10) is characterized by the following properties:

Theorem 1. *There is an interval of damage parameters* $[\underline{s}, \bar{s}]$ *such that*

1. *For all* $s \in [\underline{s}, \bar{s}]$ *we have that*

$$q_0^*(s) \equiv \tilde{q}_0, \qquad e_0^*(s) \equiv \tilde{e}_0, \qquad Q_I^*(s) \equiv \tilde{Q},$$
$$q_I^*(s) \equiv \tilde{q}_I, \qquad e_I^*(s) \equiv \tilde{e}_I, \qquad S_1(E_I^*(s), s) \equiv \tilde{S}$$

are constant in s. $E_I^*(s)$ is decreasing, $n_0^*(s)$ is decreasing, $n_I^*(s)$ is increasing in s, and these variables satisfy

$$C_1^0(\tilde{q}_0, \tilde{e}_0) = C_1^I(\tilde{q}_1, \tilde{e}_I) = P(\tilde{Q}) \tag{11}$$

$$-C_2^0(\tilde{q}_0, \tilde{e}_0) = -C_2^I(\tilde{q}_I, \tilde{e}_I) = S_1(E_I^*(s), s) = \tilde{S} \tag{12}$$

$$P(\tilde{Q}) \cdot \tilde{q}_0 - C^0(\tilde{q}_0, \tilde{e}_0) - \tilde{S} \cdot \tilde{e}_0 = 0 \tag{13}$$

$$P(\tilde{Q}) \cdot \tilde{q}_I - C^I(\tilde{q}_I, \tilde{e}_I) - \tilde{S} \cdot \tilde{e}_I = 0 \tag{14}$$

2. *For all $s < \underline{s}$ the new technology should not be employed—that is, $n_I^*(s) = 0$. For all $s > \bar{s}$ the conventional technology should not be employed—that is, $n_0^*(s) = 0$. In either case, aggregate emissions $E_I^*(s)$ and output $Q_I^*(s)$ are decreasing in s, thus $P(Q_I^*(s))$ is increasing in s. Moreover, marginal damage $S_1(E_I^*(s), s)$, as well as the social marginal rate of substitution between pollution and consumption denoted by*

$$MRS(s) \equiv \frac{S_1(E_I^*(s), s)}{P(Q_I^*(s))} \tag{15}$$

are increasing in s.

$n^*(s)$ and $e^*(s)$ are decreasing whereas $q^*(s)$ is increasing in s (without Assumption 1.4 this is not necessarily so).

3. *There is $\bar{\bar{s}} > \bar{s}$ such that for all $s \in (\underline{s}, \bar{\bar{s}})$ we have more aggregate output—that is, $Q_I^*(s) > Q_0^*(s)$—and less aggregate pollution—that is, $E_I^*(s) < E_0^*(s)$—where $Q_0^*(s)$ and $E_0^*(s)$ denote the optimal output and pollution levels before the new technology was available.*

Proof. See the appendix.

Theorem 1 says the following: if the damage parameter is low (the first part of part 2), only the conventional type of firms should produce. This is quite intuitive since by assumption those firms have the lower average cost if they do not reduce pollution. If the damage parameter is zero, or close to zero, there is no or only little need for regulation. Hence it is efficient to employ the conventional firms only. If the damage parameter is very high (the second part of part 2), only innovative firms should be active since those have the lower average cost for low emission levels. For intermediate values of damage parameters (part 1) it is optimal to employ *both* types of technologies. Production is shifted continuously from the conventional to the innovative firms, as s increases. In that case only the number of firms varies. Each type of firms keeps its efficient production plan $(\tilde{q}_i, \tilde{e}_i)$ for $i = 0, I$. Total output as well as marginal damage remain constant on the whole interval $[\underline{s}, \bar{s}]$ whereas emissions go down.

Outside that interval, $Q_I^*(s)$ and $E_I^*(s)$ are decreasing. Note, however, that for s sufficiently high, emissions may go up through innovation—that is, $E_I^*(s) > E_0^*(s)$ (part 3). The intuition is that if the damage function is sufficiently steep, output must be close to zero—that is, marginal utility of consumption is close to its maximum. If the new technology is much cleaner—that is, $e_I(q) \ll e_0(q)$—one can produce much more output with one unit of pollution. If q is close to zero before innovation, a raise of output after innovation may more than outweigh a small increase of pollution.[10]

The shape of the aggregate quantity of output is depicted in Figure 2, the shape of aggregate emissions in Figure 3. For marginal social damage and the optimal numbers of firms see Figures 5 and 6.

Note that in the one-dimensional models of Weitzman (1974), Adar and Griffin (1976), and Fishelson (1976), the relative slopes of the damage functions and the marginal abatement curve turned out to be crucial. In our more general model with two commodities (the consumption good and pollution), the relative slopes of the social marginal rate of substitution between consumption and pollution versus the slope of the marginal rate of transformation between output and abatement are crucial. This can be seen at best from Figure 4, which combines Figure 1 with different social indifference curves, which become steeper with higher s.[11]

3. Free-entry market equilibrium under taxes and permits with both types of technologies

The main goal of this paper is to investigate how environmental policy tools like taxes and tradeable permits spur the adoption of new technologies. Hence in this core section we

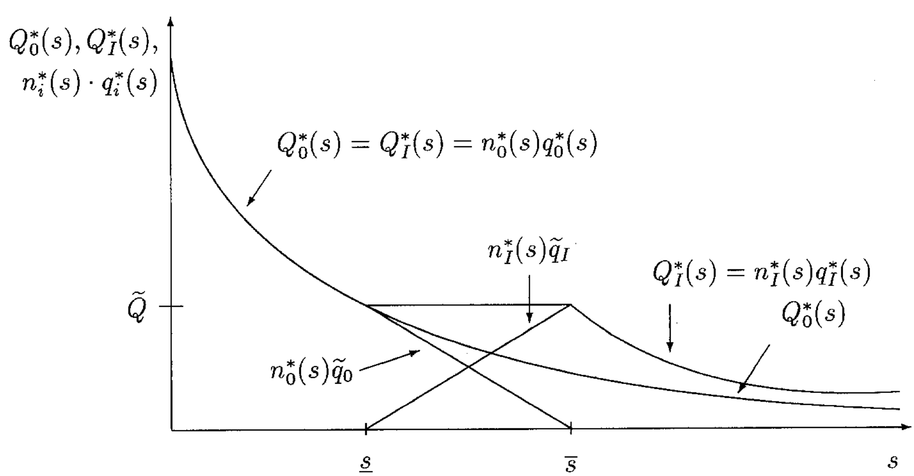

Figure 2. Optimal aggregate output as a function of the damage parameter s before and after the new technology is available. $Q_0^*(s)$ and $Q_I^*(s)$ denote optimal aggregate outputs before and after innovation, respectively. $n_0^*(s)q_0^*(s)$ and $n_I^*(s)q_I^*(s)$ are the optimal aggregate outputs after innovation produced by the conventional industry and the innovators, respectively. Note that $Q_0^*(s) = Q_I^*(s)$ for $s \le \underline{s}$.

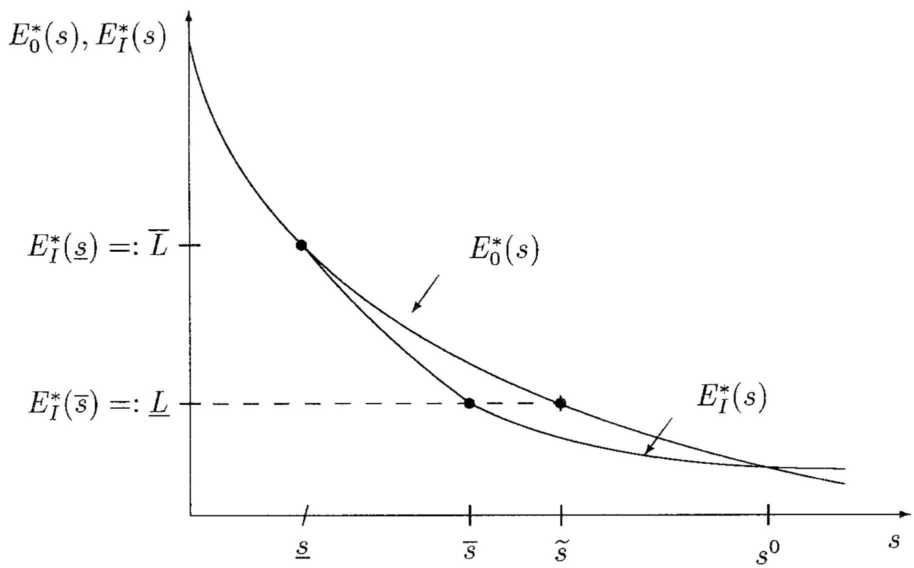

Figure 3. The optimal aggregate emissions (= optimal permit policy) before and after innovation.

examine competitive market equilibria under any tax or under any number of permits if both types of firms potentially may enter the market. We also look at the welfare implications. In particular we are interested in the question of whether the appearing of a new technology always raises welfare under those policy tools, as one should expect, and whether those policy tools induce too little or possibly too much adoption of the new technology. Too much and too little is, of course, to be taken relative to the social optimum.

From textbook partial analysis we know that different types of firms cannot survive under free entry, unless the minimum average costs happen to be equal. In *our* model, the firms' minimum average costs depend heavily on the level of regulation—that is, on the size of the emission tax or the number of permits issued by the government, respectively. Thus we investigate how many and which types of firms will operate in a competitive free-entry equilibrium if an arbitrary tax is charged on emissions or an arbitrary number of permits is issued. We treat taxes first.

3.1. *Free-entry market equilibrium under taxes*

Suppose that any tax τ is given. For a given output price p, the competitive firms' first-order conditions are $p = C_1^i(q_i, e_i)$ and $-C_2^i(q_i, e_i) = \tau$. Free entry leads to zero profits. Thus in equilibrium we have

$$P(Q) = p = C_1^i(q_i, e_i) \qquad i = 0, I, \tag{16}$$

$$\tau = -C_2^i(q_i, e_i) \qquad i = 0, I, \tag{17}$$

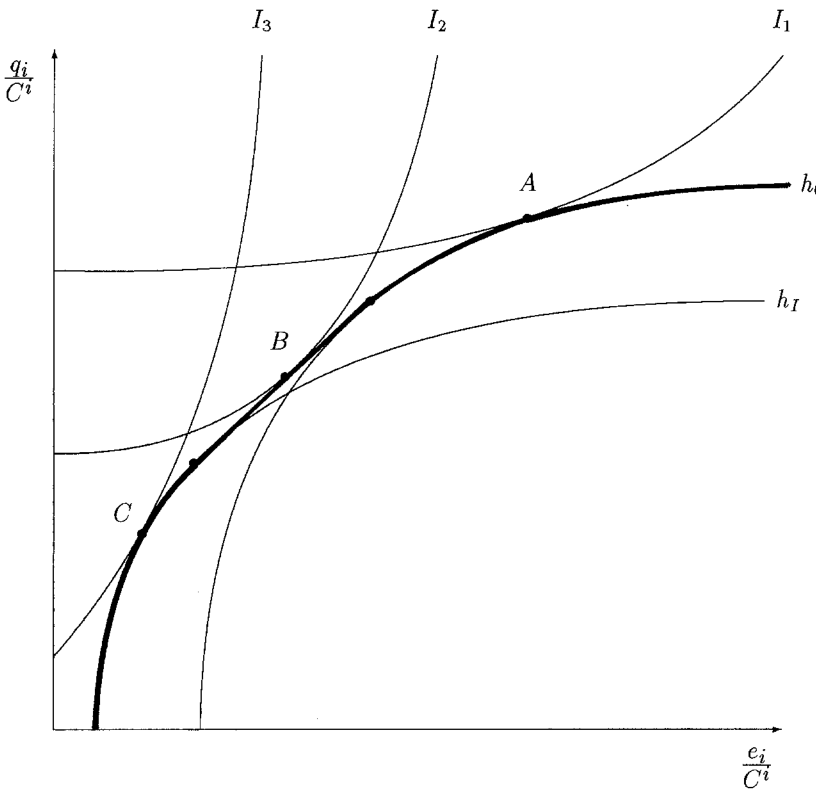

Figure 4. The bold line can be considered as the social marginal rate of transformation. The social indifference curves I_i have the slope $S_1(E, s)/P(Q)$. For I_1 only conventional firms produce (point A), for I_2 both types of firms produce (point B), and for I_3 only innovators produce (point C). The social indifference curves get steeper with s.

$$0 = P(Q(\tau)) - \tau e_i(\tau) - C^i(q_i(\tau), e_i(\tau)) \qquad i = 0, I, \tag{18}$$

if both types of firms are active, otherwise only for $i = 0$ or $i = I$. Moreover,

$$Q = n_0 q_0 + n_I q_I. \tag{19}$$

The following result characterizes the different types of equilibria contingent on the tax level. Recall from the last section that $\tilde{S}$ denotes the marginal social damage, in social optimum if both types of firms produce—that is, for $s \in (\underline{s}, \bar{s})$. It is important to note that this is also the marginal abatement cost if both types of firms have the same marginal costs, the same marginal abatement costs, and the same minimum average costs. This fact will drive the following theorem. Recall also that $\tilde{Q}$ is the corresponding aggregate output (which is independent of the number of firms of each type).

Theorem 2. *Let τ be any emission tax.*

1. *If $\tau < \tilde{S}$, there is a unique free-entry competitive equilibrium with only the conventional firms being active.*
2. *If $\tau > \tilde{S}$, there is a unique free-entry competitive equilibrium with only the innovative firms being active, or there is no market at all.*
3. For $\tau = \tilde{S}$, *there is a whole set of equilibria with $n_0 \in [0, \tilde{Q}/\tilde{q}_0]$ and $n_I = (\tilde{Q} - n_0\tilde{q}_0)/\tilde{q}_I$, and $q_0(\tau) = \tilde{q}_0$, $q_I(\tau) = \tilde{q}_I$, $e_0(\tau) = \tilde{e}_0$, $e_I(\tau) = \tilde{e}_I$.*
   Aggregate output equals $\tilde{Q}$, independent of (n_0, n_I). Aggregate emissions are the greater, the larger n_0.

Proof. We will only sketch the proof here. A more detailed one can be found in Requate (1994). Assume first $\tau = \tilde{S}$. Theorem 1 implies that for $\tau = \tilde{S}$ the average cost of the two firms break even. Hence there exists a competitive equilibrium with $q_0 = \tilde{q}_0$, $q_I = \tilde{q}_I$, $e_0 = \tilde{e}_0$, $e_I = \tilde{e}_I$, and $Q = \tilde{Q}$. However, the numbers of firms are *not* uniquely determined. All $n_0 \neq 0$ and $n_I \neq 0$ satisfying (19) yield a competitive free-entry equilibrium. On the other hand, no further equilibrium with $n_0 > \tilde{Q}/\tilde{q}_0$ or $n_I > \tilde{Q}/\tilde{q}_I$ can exist. This means that for $\tau = \tilde{S}$ there exists a whole set of competitive free-entry equilibria with different pollution levels.

Following the lines of proof of Theorem 1 one can easily show that for $\tau < \tilde{S}$ there is a unique equilibrium with only conventional firms operating, and for $\tau > \tilde{S}$ there is a unique equilibrium with only innovative firms operating. □

Up to now we have treated both types of firms symmetrically and the notions of conventional and innovative firms seemed to be somewhat arbitrary. If we take the interpretation seriously and assume that the type 0 firms are incumbent, and that after the new technology is available, innovators consider to enter the market, or some incumbents consider to adopt the new technology, the multiplicity of equilibria vanishes for $\tau = \tilde{S}$. For if τ has been set equal to $\tilde{S}$ before the new technology has been available, the number of conventional firms must be $n_0 = \tilde{Q}/\tilde{q}_0$. But this is still an equilibrium after the new technology is available! The innovative firms do not have a comparative advantage to crowd out the conventional firms. Under this story we would have

$$n_I(\tau) = 0 \text{ if } \tau \leq \tilde{S},$$
$$n_0(\tau) = 0 \text{ if } \tau > \tilde{S}.$$

We see that in a situation where firms with a conventional technology are incumbent and a new technology becomes available, in the long run a Pigouvian tax does not allow conventional and new technologies to live together. Moreover, the market equilibrium may behave very discontinuously as the tax rises. In particular, a small tax increase from, say, $\tau_1 = \tilde{S} - \epsilon$ to $\tau_2 = \tilde{S} + \epsilon'$ leads to an industrial revolution in the long run. Under free entry, or free choice of technology, the conventional technology will be completely substituted by the new one. A small tax increase from $\tau_1 = \tilde{S} - \epsilon$ to $\tau_2 = \tilde{S} + \epsilon'$ also induces the pollution level to discontinuously jump downward, whereas aggregate output varies continuously. On the other hand, a variation of taxes within $(0, \tilde{S})$, or for $\tau > \tilde{S}$, leads to continuous changes of all endogenous variables.

*3.1.1. **Welfare implications.*** Clearly, after the new technology has been adopted, the resulting allocation is not optimal if the regulator cannot adjust the tax immediately. Two questions arise. First, does, nevertheless, welfare rise through innovation? And second, do taxes cause too much or eventually too little adoption of the new technology? The answer, of course, depends on the prevailing tax level before innovation. Although not quite realistic (see Marin's 1989 criticism), but as a useful benchmark, we follow Milliman and Prince (1989) by assuming that the conventional industry is regulated optimally and the new technology becomes available suddenly. The regulator is assumed to neither be able to forecast the new technology, nor to adjust his policy immediately. This leads us to the following result:

Corollary 1. *If the conventional industry is optimally regulated by taxes—that is,*

$$\tau_0(s) = S_1(E_0^*(s), s), \tag{20}$$

and the new technology becomes available to any firm, then

1. *if $s \le \underline{s}$, no firm will run the new technology.*
2. *If $s > \underline{s}$, all firms will run the new technology.*

Proof. Theorem 1 implies that $\tau_0(s) = S_1(E^*(s), s) < \tilde{S}$ if $s \le \underline{s}$, and $\tau_0(s) = S_1(E^*(s), s) = \tilde{S}$ if $s = \underline{s}$. Since $S_1(E^*(s), s)$ is strictly increasing in s if only conventional firms are around (this follows from Theorem 1.2), $\tau_0(\underline{s})$ is strictly increasing in s if only conventional firms are around. This implies $\tau_0(s) > \tilde{S}$ if and only if $s \ge \underline{s}$. By virtue of Theorem 2 only conventional firms produce for $s < \underline{s}$, and only innovative firms for $s > \underline{s}$. For $s = \underline{s}$ we employ the fact that the conventional firms are already incumbent, and $(n_0, n_I) = (\tilde{Q}/\tilde{q}_0, 0)$ is an equilibrium pair of numbers of firms, leaving no place for new technologies. □

Corollary 1 implies that there is no adoption of the new technology whenever this is socially optimal—that is, if $s \le \underline{s}$. This is fine. On the other hand, for $s \in (\underline{s}, \bar{s})$ the original tax always induces complete adoption although only partial adoption is socially optimal. As a consequence we obtain that decentralized adoption of new technologies under taxes may even result in a *decrease* in welfare compared to the situation before the new technology has been available:

Corollary 2. *For $s > \underline{s}$ but sufficiently close to $\underline{s}$, innovation under the original optimal tax leads to a* decrease *in welfare.*

Proof. The proof is based on a continuity argument. The social optimum requires the number of innovators to be small for s close to $\underline{s}$. But since $\tau_0(s) > \tilde{S}$ for $s > \underline{s}$, we get complete innovation by Theorem 2. Hence welfare must decrease. □

Where does the welfare loss come from? Since output changes continuously if the tax rises from $\tau_1 = \tilde{S}$ to $\tau_2 = \tilde{S} + \epsilon$ but pollution falls discontinuously, the net loss in welfare must result from a dissipation of production costs for the sake of too clean an environment. Too much money will be spent on new abatement equipment.

If s is greater than $\bar{s}$, complete adoption *is* optimal. However, since the original tax *exceeds* the socially optimal marginal damage, unless s is extremely high, the original tax induces too little a number of firms to be in the market. This can be considered as underinvestment or too little adoption of the new technology.

Finally, if the damage parameter is extremely high, the optimal emission level after innovation may exceed the optimal emission level before innovation (see Theorem 1.3 and Figure 5). Thus the original tax may *fall short* of the socially optimal marginal damage after innovation. Hence also the number of firms induced by the original tax policy *exceeds* the optimal number of firms after innovation. By continuity of $\tau_0(s)$ and the socially optimal marginal damage in s, we immediately obtain the following characterization of the regions where taxes cause too much or too little adoption of the new technology.

Corollary 3. *Suppose the originally optimal tax $\tau_0(s)$ is still valid and the new technology is available. Then there is an interval (s_a, s_b), with $s_a \in (\underline{s}, \bar{s})$ and $s_b > \bar{s}$, such that there is*

1. *excessive adoption of the new technology, or overinvestment, for all $s \in (\underline{s}, s_a)$, and possibly for $s > s_b$ if there is still a market for those s;*
2. *too little adoption, or underinvestment, for all $s \in (s_a, s_b)$.*

The result is illustrated in Figure 5. Note that for $s = s_a$, $\tau_0(s)$ induces the optimal number of new technology firms but too little a number (zero) of conventional firms. Only if by chance $s = s_b$, $\tau_0(s)$ leads to first best.

3.2. Free-entry market equilibrium under permits

Consider now regulation by permits and assume that any number of permits L has been set to be auctioned off to the firms. Define

$$\bar{L} \equiv \frac{\tilde{Q}}{\tilde{q}_0}\tilde{e}_0 \qquad \text{and} \qquad \underline{L} \equiv \frac{\tilde{Q}}{\tilde{q}_I}\tilde{e}_I.$$

Note that $\bar{L}$ and $\underline{L}$ are equal to the socially optimal emission levels for $s = \underline{s}$ and $s = \bar{s}$, respectively. Denote by $q_i(L)$, $e_i(L)$ the quantities and emission levels of firm $i = 0, I$ under a permit regime with L permits. Then we can state the following result:

Theorem 3. *For each number of permits L being issued, there is a unique competitive free entry equilibrium characterized by the following properties.*

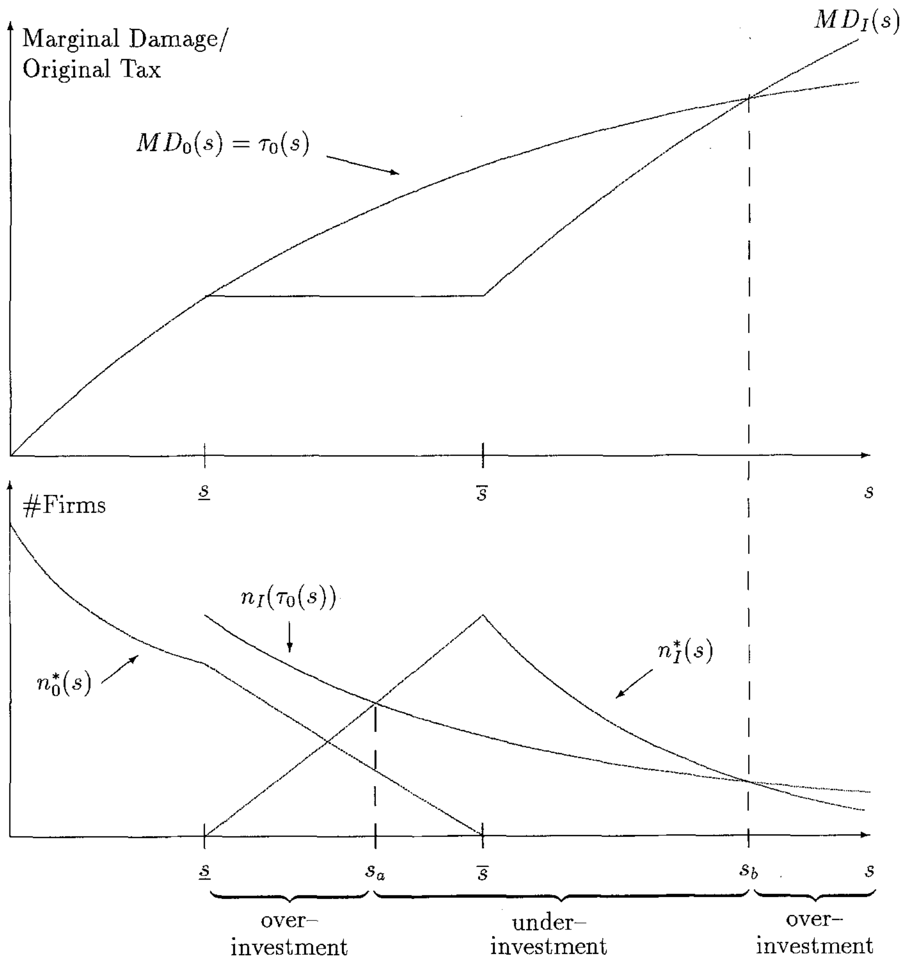

Figure 5. The upper diagram depicts socially optimal marginal damage *after* innovation, denoted by MD_I, and the originally optimal tax $\tau_0(s)$, which is equal to optimal marginal damage *before* innovation, denoted by MD_0. The lower diagram depicts the socially optimal number of firms of type 0, denoted by $n_0^*(s)$, and of type I, denoted by $n_I^*(s)$, and the number of innovators $n_I(\tau_0(s))$ under the original tax $\tau_0(s)$. Note that at point $\underline{s}$ the number $n_I(\tau_0(\underline{s}))$ can also be less than $n_0^*(\underline{s})$.

1. *If $L \geq \bar{L}$, only the conventional firms are in the market, and the market price for permits $\sigma(L)$ does not exceed $\tilde{S}$.*
2. *For all $L \in (\underline{L}, \bar{L})$, both types of firms are active, the market price is $\sigma(L) = \tilde{S}$, and there is a unique allocation of permits between the conventional and new technology firms. There will be*

$$n_0 = (L - \underline{L})\frac{\tilde{q}_I}{\tilde{q}_I\tilde{e}_0 - \tilde{q}_0\tilde{e}_I} \tag{21}$$

conventional firms holding $\tilde{e}_0$ many permits each, and

$$n_I = (\bar{L} - L)\frac{\tilde{q}_0}{\tilde{q}_I\tilde{e}_0 - \tilde{q}_0\tilde{q}_I} \tag{22}$$

new technology firms holding $\tilde{e}_I$ many permits each. Moreover, $q_i(L) = \tilde{q}_i$ is independent of L for $i = 0, I$.

3. *If $L \leq \underline{L}$, all firms run the new technology, or there is no market at all. The latter happens if the number of permits is so small that the firms' average cost exceeds the choke-off price. The market price $\sigma(L)$ does not fall short of $\tilde{S}$.*

Proof. See the appendix.

So we always get a unique equilibrium under permits. From (21) and (22) we see that n_0 and n_I vary continuously in L, even for $L \in (\underline{L}, \bar{L})$. Thus, in contrast to the tax regime, both types of firms can be active at the same time for different permit policies. Note that for $L \in (\underline{L}, \bar{L})$ the market price for permits equals $\tilde{S}$, which is equal to the tax where both types of firms produce. This should not be surprising after all since for σ less than $\tilde{S}$ the new technology firms could not compete, and for permit prices exceeding $\tilde{S}$ the conventional firms would make losses. Thus the market price alone cannot enforce uniqueness of equilibrium. However, the fixed supply of permits can match demand for permits only for a unique allocation. In other words, market clearing on the permit market leads to the unique equilibrium.

3.2.1. Welfare implications. Clearly, also under permits, the optimal policy level before innovation is not optimal any longer as soon as the new technology has been adopted. Thus, if the number of permits cannot be adjusted immediately, the allocation that arises through adoption of the new technology is not socially optimal in general. However, permits have a crucial advantage over taxes:

Corollary 4. *Under any number of permits L, adoption of a new technology never leads to a decrease in welfare.*

Proof. Total pollution is the same before and after the new technology has been available. If $L \geq \bar{L}$, no adoption happens. For $L < \bar{L}$, there will be either partial or complete adoption. But since the firms minimize costs, emissions as a scarce resource are used less inefficiently by the firms. This leads to lower abatement costs and to a higher output as before the new technology has been available. □

On the other hand, similar to the tax regime, permits also can cause too little but also too much adoption of the new technology. To see this, let us assume again that the conventional industry is regulated optimally—that is, the number of permits is given by $L_0(s) = E_0^*(s)$. Then the new technology becomes available, and the incumbents can decide to adopt it, or other firms with the new technology may enter the market. The following corollary describes what happens under the originally optimal permit policy:

Corollary 5. *If the conventional industry is optimally regulated by issuing $L_0(s)$ permits and if the new technology is available to any firm, then*

1. *For all $s \leq \underline{s}$, there will be no adoption of the new technology.*
2. *There is a parameter $\tilde{s} > \bar{s}$ such that for all $s \in (\underline{s}, \tilde{s})$ there is a unique equilibrium where both types of technologies will be employed. The price for permits is equal to $\tilde{S}$. For $L = L_0(s)$ the number of conventional firms, holding $\tilde{e}_0$ many permits each, is given by* (21), *and the number of innovators, holding $\tilde{e}_I$ many permits each, is given by* (22).
3. *For $s \geq \tilde{s}$, there will be complete adoption of the new technology.*

Proof. By Theorem 1, $L_0(s)$ exceeds $E_I^*(s)$ for $s \in (\underline{s}, \bar{s}]$ and is strictly decreasing. Define $\tilde{s}$ by $L_0(\tilde{s}) = \underline{L}$. Since $L_0(\bar{s}) > E_I^*(\bar{s}) = \underline{L}$, we get $\tilde{s} > \bar{s}$. Thus $L_0(s) \in (\underline{L}, \bar{L})$ for all $s \in (\underline{s}, \tilde{s})$. By Theorem 3 both firms are active for $L \in (\underline{L}, \bar{L})$, leading to numbers of firms given by (21) and (22). Everything else follows from Theorem 3. □

The following result yields a pattern of under- and overinvestment that is reversed to the pattern under taxes described in Corollary 3.

Corollary 6. *There is an interval (s_c, s_d), with $s_c \in (\bar{s}, \tilde{s})$ and $s_d > \tilde{s}$, such that there is*

1. *Too little adoption of the new technology, or underinvestment, for all $s \in (\underline{s}, s_c)$ and possibly for all $s > s_d$ if there is still a market for those s,*
2. *Excessive adoption, or overinvestment, for all $s \in (s_c, s_d)$.*

The result is illustrated in Figure 6. Note again, that for $s = s_c$, $L_0(s)$ induces the optimal number of innovators but too large a number of conventional firms. Only if $s = s_d$ by chance, $L_0(s)$ induces the optimal number of innovators and conventional firms (= 0).

Proof. Under policy $L_0(s)$, partial adoption happens for $s \in (\underline{s}, \tilde{s})$. For this interval part 3 of Theorem 1 implies that $L_0(s) = E_0^*(s) > E_I^*(s)$ (see Figure 6). Since the number of permits is too high, too many conventional firms stay in the market whenever partial adoption is optimal but also for a range of parameters $(\bar{s}, \tilde{s})$ for which complete adoption is optimal. This in turn implies that too few firms invest in the new technology for s greater but sufficiently close to $\underline{s}$. By continuity $L_0(s) > E_I^*(s)$ for s greater but sufficiently close to $\tilde{s}$. Hence the price for permits falls short of the optimal marginal damage and too many firms invest in the new technology if s is close to $\tilde{s}$. Finally, if there is a region where $E_I^*(s) > E_0^*(s) = L_0(s)$ and the market does not break down, we have again underinvestment for sufficiently large values of s. □

4. Concluding remarks

We have investigated a model where polluting firms compete in an output market and are regulated by either emission taxes or auctioned permits. We assumed that there are two types of potential technologies, a conventional one with relatively high marginal abatement

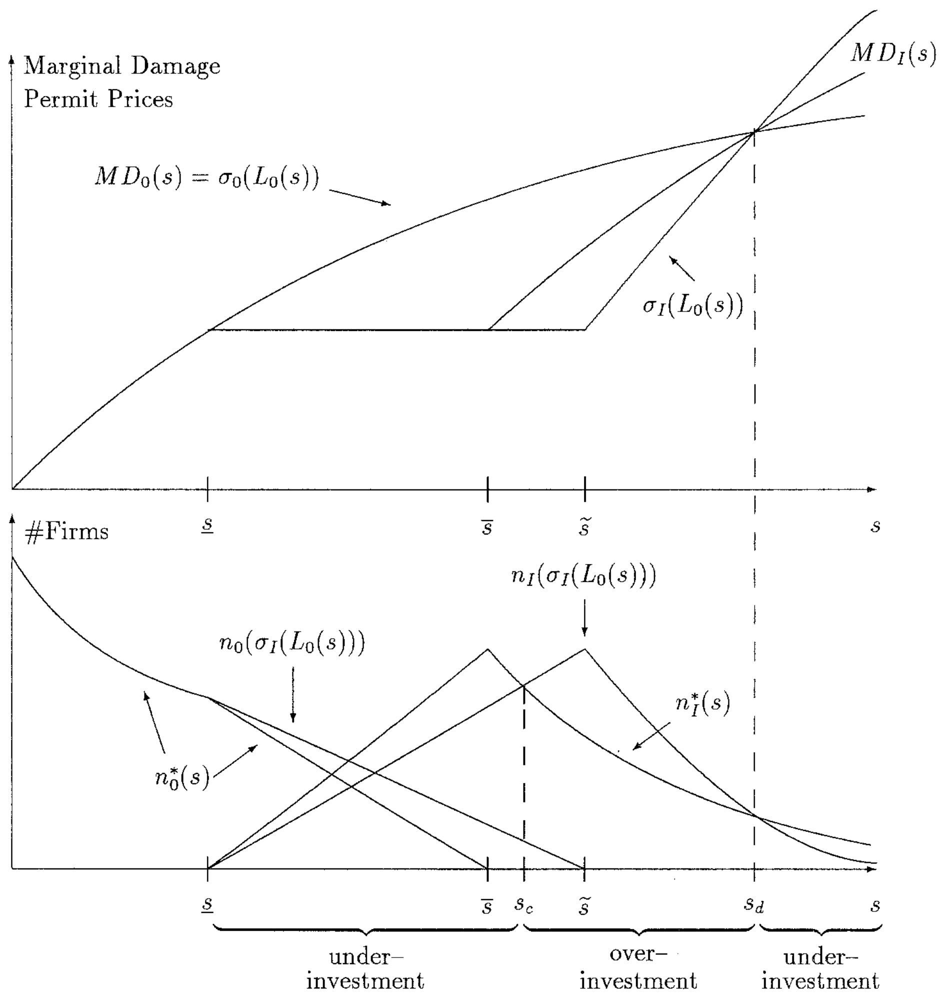

Figure 6. The upper diagram depicts the socially optimal marginal damage *after* innovation, denoted by MD_I, the originally permit price *before* innovation, denoted by $\sigma_0(L_0(s))$ (which is equal to optimal marginal damage *before* innovation), and the new permit price $\sigma_I(L_0(s))$ *after* innovation but under the original quota of permits $L_0(s)$. The lower diagram depicts the socially optimal numbers of type 0 firms $n_0^*(s)$ and of type I firms $n_I^*(s)$, and the number of conventional firms $n_0(\sigma_I(L_0(s)))$ and innovators $n_I(\sigma_I(L_0(s)))$, respectively, after innovation under the original permit policy $L_0(s)$.

costs and a new one with low marginal abatement costs but higher fixed costs. We were interested whether taxes and permits provide the correct incentives for firms to adopt the new, less polluting technology. The impact of the two policy tools turned out to be quite different. Permits allow for partial adoption, whereas taxes do not. Under the hypothesis that the conventional industry is regulated optimally before innovation and that the environmental policy cannot be adjusted immediately, both types of policies may lead to overinvestment but also to underinvestment in new technology. Taxes cause too much adoption for parameters where partial adoption is optimal but also too little adoption for parameters where complete adoption is optimal. For permits the pattern is reversed. They induce too many conventional firms to stay in the market whenever partial adoption is optimal. However, they lead to excess entry of innovators for parameters where complete adoption is optimal.

Yet permits have two crucial advantages over taxes. They never lead to a decrease in welfare whereas taxes sometimes do. Moreover, once the new technology is available, one can always implement first best ex post by adjusting the number of permits, which does not work under taxes whenever partial adoption is optimal. In other words, overinvestment under permits—if it occurs—can be considered as less severe than overinvestment under taxes.

However, even the permit system could possibly be improved by reviving a mixed system of permits, taxes, and subsidies, as has been proposed by Roberts and Spence (1976) or by a system of different permits proposed by Collinge and Oates (1982), or Henry (1989, ch. 2). Unfortunately, very little attention has been paid to those systems. The joint idea of all these systems is to make the inelastic supply of permits more elastic by approximating the marginal social damage function by different types of permits. To examine such a system under free entry is beyond the scope of this paper and is left to further research. My conjecture, however, is that such a system may lead to (almost) first best even if the regulator does not have any prior information about future technologies.

Appendix

First we state more formally the assumption on the relationship between the two cost functions.

Assumption 2. *The two cost functions C^0, and C^1 satisfy the following conditions:*

1. $$\frac{\bar{q}_0}{\tilde{C}^0(\bar{q}_0)} > \frac{\bar{q}_I}{\tilde{C}^I(\bar{q}_I)}. \tag{23}$$

2. *There are $(\hat{q}_0, \hat{e}_0) \in Y_{MAC}(C^0)$, $(\hat{q}_I, \hat{e}_I) \in Y_{MAC}(C^I)$, and $\lambda > 0$ such that*

$$\nabla C^0(\hat{q}_0, \hat{e}_0) = \lambda \nabla C^I(\hat{q}_I, \hat{e}_I), \tag{24}$$

$$\frac{\hat{e}_0}{C^0(\hat{q}_0, \hat{e}_0)} < \frac{\hat{e}_I}{C^I(\hat{q}_I, \hat{e}_I)} \tag{25}$$

and

$$\frac{\dfrac{\hat{q}_0}{C^0(\hat{q}_0, \hat{e}_0)} - \dfrac{\hat{q}_I}{C^I(\hat{q}_I, \hat{e}_I)}}{\dfrac{\hat{e}_0}{C^0(\hat{q}_0, \hat{e}_0)} - \dfrac{\hat{e}_I}{C^I(\hat{q}_I, \hat{e}_I)}} < -\frac{C_2^i(\hat{q}_i, \hat{e}_i)}{C_1^i(\hat{q}_i, \hat{e}_i)} \qquad i = 0, I. \tag{26}$$

3. *The functions h_0 and h_1 intersect only once.*

Here h_0 and h_I are the corresponding functions from Assumption 1.4 with respect to the cost function C^0 and C^I, respectively. Part 1 says that in the absence of regulation—that is, if the firms pick e such that $C_2(q, e) = 0$—the conventional firms of type 0 have the lower average costs than the innovators, who face the higher fixed costs due to installation of new technology. Or equivalently, the output-cost ratio is larger for the conventional firms. Note that the point $(\bar{q}_0/C^0(\bar{q}_0, e_0(\bar{q}_0)), e_0(\bar{q}_0)/C^0(\bar{q}_0, e_0(\bar{q}_0)))$ lies above the h_I-curve. Part 2 says that there is a region of output and emission levels where the innovative firms of type I have a cost advantage. This means that if we have two production plans for which the gradients of marginal cost of both firms point in the same direction, then the point $(\hat{q}_0/C^0(\hat{q}_0, \hat{e}_0), \hat{q}_0/C^0(\hat{q}_0, \hat{e}_0))$ lies below the h_I-curve. Note that if (25) holds, (26) is equivalent to

$$\frac{C_1^I(\hat{q}_I, \hat{e}_I)\hat{q}_0 + C_2^I(\hat{q}_I, \hat{e}_I)\hat{e}_0}{C^0(\hat{q}_0, \hat{e}_0)} < 1 < \frac{C_1^0(\hat{q}_0, \hat{e}_0)\hat{q}_I + C_2^0(\hat{q}_0, \hat{e}_0)\hat{e}_I}{C^I(\hat{q}_I, \hat{e}_I)}, \tag{27}$$

which will be needed in the proof of Theorem 1. Part 3 is made for convenience and guarantees that the cost advantages do not switch back. Note further that Parts 1 and 2 imply that h_0 and h_1 intersect at least once.

Proof of Theorem 1. The Lagrange function of the maximization problem is

$$L(\ldots) = W(q_0, q_I, e_0, e_I, n_0, n_I, s) + \lambda_0 q_0 + \lambda_I q_I + \mu_0 e_0 + \mu_I e_I + \nu_0 n_0 + \nu_I n_I,$$

where λ_0, λ_I, μ_0, μ_I, ν_0, and ν_I are the Kuhn Tucker multipliers of the nonnegativity constraints. The first-order conditions are

$$P(Q) - C_1^0(q_0, e_0) + \lambda_0 = 0, \tag{28}$$

$$P(Q) - C_1^I(q_1, e_I) + \lambda_I = 0, \tag{29}$$

$$-S_1(E, s) - C_2^0(q_0, e_0) + \mu_0 = 0, \tag{30}$$

$$-S_1(E, s) - C_2^I(q_I, e_I) + \mu_I = 0, \tag{31}$$

$$P(Q)q_0 - C^0(q_0, e_0) - S_1(E, s)e_0 + \nu_0 = 0, \tag{32}$$

$$P(Q)q_I - C^I(q_I, e_I) - S_1(E, s)e_I + \nu_I = 0. \tag{33}$$

Suppose that there is an interior solution, i.e., all the multipliers are zero. Differentiating (32) and (33) w.r. to s and employing (28) through (31) yields

$$P'(Q)Q'(s)q_0(s) - [S_{11}(E(s), s)E'(s) + S_{12}(E(s), s)]e_0(s) = 0, \tag{34}$$

$$P'(Q)Q'(s)q_I(s) - [S_{11}(E(s), s)E'(s) + S_{12}(E(s), s)]e_I(s) = 0. \tag{35}$$

Now suppose $Q'(s) \neq 0$. This implies by (32) and (33) that $[S_{11}(E(s), s)E'(s) + S_{12}(E(s), s)] \neq 0$ and hence by (32) through (35) we get

$$\frac{q_0}{q_I} = \frac{e_0}{e_I} = \frac{C^0(q_0, e_0)}{C^I(q_I, e_I)},$$

or

$$\left(\frac{q_0}{C^0(q_0(s), e_0(s))}, \frac{e_0(s)}{C^0(q_0(s), e_0(s))}\right) = \lambda \left(\frac{q_I(s)}{C^I(q_I(s), e_I(s))}, \frac{e_I(s)}{C^I(q_I(s), e_I(s))}\right)$$

for some $\lambda > 0$. But this contradicts Assumption 2, since if the two h_i curves intersect, the gradients cannot have the same direction. Hence, $Q(s) = \bar{Q}$, and $Q'(s) = 0$. But then (34) or (35) imply

$$\frac{d}{ds} S_1(E(s), s) = S_{11}(E(s), s)E'(s) + S_{12}(E(s), s) = 0. \tag{36}$$

Thus: $E' = -S_{12}/S_{11} < 0$. Differentiating (28) through (31) w.r. to s, and using (36) we get a homogeneous linear system in $q_0'(s)$, $q_I'(s)$, $e_0'(s)$, and $e_I'(s)$. Hence $q_0'(s) = q_I'(s) = e_0'(s) = e_I'(s) = 0$. Next we have $\bar{Q} = n_0(s)\tilde{q}_0 + n_I(s)\tilde{q}_I$ and $E(s) = n_0(s)\tilde{e}_0 + n_I(s)\tilde{e}_I$, giving $E'(s) = n_0'(s)[\tilde{e}_0 - \tilde{e}_I \cdot (\tilde{q}_0/\tilde{q}_I)]$. This implies that $n_0(s)$ is decreasing and $n_I(s)$ is increasing, or vice versa. We will exclude below that $n_0(s)$ is increasing.

Now let s be close to zero. Then $S_1(E, s)$ is close to zero. Since $C^0(\tilde{q}_0, \tilde{e}_0) > 0$ and $C^I(\tilde{q}_I, \tilde{e}_I) > 0$, (30) and (31) cannot be satisfied for $\mu_0 = \mu_I = 0$, and s sufficiently close to zero. On the other hand, for s close to zero we have

$$P(Q(s)) = C_1^0(q_0(s), e_0(s)) \approx \frac{C^0(q_0(s), e_0(s))}{q_0(s)} \approx \frac{\bar{C}^0(\bar{q}_0)}{\bar{q}_0}$$

or

$$P(Q(s)) = C_1^I(q_I(s), e_I(s)) \approx \frac{C^I(q_I(s), e_I(s))}{q_I(s)} \approx \frac{\bar{C}^I(\bar{q}_I)}{\bar{q}_I}.$$

Since we have $\bar{q}_0/\bar{C}^0(\bar{q}_0) > \bar{q}_I/\bar{C}^I(\bar{q}_I)$, by Assumption 2 we conclude $\nu_I > 0$, $\nu_0 = 0$, and hence $n_0 > 0$ and $n_I = 0$.

Next observe that since $Q_I(s) = \bar{Q}$, $q_0(s) = \tilde{q}_0$, and $q_I(s) = \tilde{q}_I$ are constant for those s for which there is an interior solution, $n_0(s)$ and $n_I(s)$ must be bounded. Since also $e_0(s) = \tilde{e}_0$, and $e_I(s) = \tilde{e}_I$ are constant and greater than zero, $E(s)$ must be greater than 0 for those s. But this implies that for s sufficiently high $S_1(E(s), s)$ must exceed $-C_2^0(\tilde{q}_0, \tilde{e}_0)$, and $-C_2^I(\tilde{q}_I, \tilde{e}_I)$. Thus (30) and (31) cannot be satisfied for $\mu_0 = \mu_I = 0$ and s sufficiently high. This means that only one type of firm can be active, hence $\nu_0 > 0$ or $\nu_I > 0$. Since

$MRS(s) \equiv S_1(E(s), s)/P(Q(s))$ is increasing (we omit a proof here, see Requate, 1994), and since it becomes greater than $\tilde{R} = \tilde{S}/P(\tilde{Q}) = -C_2^i(\tilde{q}_i, \tilde{e}_i)/C_1^i(\tilde{q}_i, \tilde{e}_i)$ for $i = 0, I$, the inequalities (27) must hold. Now assume there is a solution with $P(Q) = C_1^I(q_I, e_I)$ and $S_1(E, s) = -C_2^I(q_I, e_I)$. Then (32) and (33) become

$$C_1^I(q_I, e_I)q_0 - C_2^I(q_I, e_I)e_0 - C^0(q_0, e_0) + \nu_0 = 0, \tag{37}$$

$$C_1^I(q_I, e_I)q_I - C_2^I(q_I, e_I)e_I - C^I(q_I, e_I) + \nu_I = 0. \tag{38}$$

This implies $\nu_I = 0$, and $\nu_0 > 0$ by the first inequality of (27). Assuming $P(Q) = C_1^0(q_0, e_0)$ and $S_1(E, s) = -C_2^0(q_0, e_0)$, this would lead to $\nu_I < 0$ by the second part of (27), which is impossible.

One can show that our Assumptions on $P(\cdot)$, $S(\cdot, \cdot)$ and on the cost functions imply that the objective function of the social planner is strictly concave. Thus the solution is unique. Since all involved functions are continuous in s, the solution must be continuous in s. Hence there must exist parameters $\underline{s}$ and $\bar{s}$ such that $n_0(s) > 0$ and $n_I(s) = 0$ for $s < \underline{s}$ and $n_0(s) = 0$ and $n_I(s) > 0$ for $s > \bar{s}$.

A proof for the comparative statics results from Part 2 can be obtained by the author on request (see Requate, 1994). □

Proof of Theorem 3. We proceed indirectly again. One can easily show that the aggregate factor demand for permits is strictly decreasing if the price for permits rises, if the output market clears, and if only one type of firm is around. This implies that the price for permits is unique and rises if the supply of permits goes down. Hence, if only conventional firms are around and $L = \bar{L}$, then $\sigma(\bar{L}) = \tilde{S}$. If only new technology firms are around, and $L = \underline{L}$, then also $\sigma(\underline{L}) = \tilde{S}$.

Now suppose $\sigma > \tilde{S}$. Then by the same arguments as for the emission tax (in Section 3), only type I firms can be active, and $L > \underline{L}$. Similarly, suppose $\sigma < \tilde{S}$. Then only type 0 firms can be active, and $L > \bar{L}$. Now suppose $L \in (\underline{L}, \bar{L})$. Then $\sigma(L) = \tilde{S}$, otherwise L would be smaller than $\underline{L}$ or greater than $\bar{L}$. But if $\sigma(L) = \tilde{S}$, both types of firms can be active with $q_i(L) = \tilde{q}_i$, and $e_i(L) = \tilde{e}_i$ for $i = 0, I$. Contrary to the tax solution, however, there are unique numbers of firms $n_0(L)$, and $n_I(L)$, determined by the two (linearly independent) equations: $\tilde{Q} = n_0(L)\tilde{q}_0 + n_I(L)\tilde{q}_I$ and $L = n_0(L)\tilde{e}_0 + n_I(L)\,\tilde{e}_I$. Solving for $n_0(L)$, and $n_I(L)$ yields (21) and (22). □

Acknowledgments

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Notes

1. Laffont and Tirole (1994a, p. 2): "Stand alone spot markets (in which the government sets at the beginning of each period the number of permits for that period) create excessive incentives for investment. . . .This incentive can be reduced by the introduction of a futures market."
2. Laffont and Tirole (1994b, p. 2): "While spot markets destroy incentives for innovations, futures markets bring limited improvement."
3. This is a standard assumption. It guarantees that the second-order conditions are satisfied.
4. On the border we assume $S(0, s) = S_1(0, s) = 0$ for all $s \geq 0$.
5. Note that the steepness of the damage function matters also in related models—for example, for the choice between price versus quantity regulation in Weitzman's (1974) seminal paper on regulation under imperfect information. See also Adar and Griffin (1976) and Fishelson (1976).
6. We can consider our firms as multiproduct firms producing the consumption good and emissions (or abatement). In the language of Baumol, Panzar, Willig (1982), all production plans in (4) have scale elasticity 1.
7. One can show that h is concave if the third derivatives of the cost function are sufficiently bounded. To write down explicit conditions, however, is tedious and does not yield further insight.
8. A cost function with the properties of Assumption 1 can be derived from a Cobb-Douglas production function where one input is energy, which has a nonzero factor price, and pollution is proportional to the use of energy. (I am grateful to Cees Withagen for asking about underlying production functions.)
9. This implies that complete bypass as assumed in Laffont and Tirole (1994a, 1994b) is not possible.
10. We cannot, however, show that emissions go up in general if s is sufficiently high. Due to the fixed costs it may happen that output is too low for a single firm to survive—that is, the market may break down. For a polynomial cost function of type $C(q, e) = ½[(\beta q + \alpha - e)^2 + \gamma q^2] + F$, one can show for suitable parameters α, β, γ, F, and s sufficiently high that both can happen: $E_I^*(s)$ exceeds $E_0^*(s)$ and the number of firms is greater than 1, but also that $E_I^*(s)$ exceeds $E_0^*(s)$ in a region where the number of firms falls short of 1 (if we treat it as a continuous variable)—that is, there is no market.
11. I am grateful to Lans Bovenberg, whose comment inspired this figure.

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