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Hilbert Transform in the Interpretation of Magnetic Anomalies of Various Components due to a Thin Infinite Dike

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Abstract—The Hilbert transform $H(x)$ applicable to vertical (ΔZ), horizontal (ΔH), and total (ΔT) magnetic anomalies due to a thin dike of infinite depth extent is derived from the generalised expression of magnetic effect $F(x)$. The depth and dip of the dike is extracted by a simple procedure making use of $F(x)$ and $H(x)$. A modified version of the amplitude of the analytical signal is given to locate the origin. The abscissa of the point of intersection of $F(x)$ and the discrete Hilbert transform $H(1.\Delta x)$ directly yields the depth to the top. An example for each case is considered theoretically to illustrate the process. Applicability of the method is examined on the vertical component of the well-known magnetic anomaly at Kiirunavaara in northern Sweden, originally described by Von Carlheim Gyllenskjold, as well as on total magnetic anomaly of Bensons Mines, U.S.A.

Key words: Magnetic anomalies; Discrete Hilbert transform; Analytical signal; Amplitude; Abscissa.

Introduction

Quite a few papers have discussed the utility of Hilbert transform in the interpretation of magnetic and gravity anomalies (NABIGHIAN, 1972; STANLEY and GREEN, 1976; and STANLEY, 1977). MOHAN *et al.* (1982) and SUNDERARAJAN (1982) formulated a new technique for the interpretation of vertical magnetic anomalies due to two dimensional bodies. SUNDARARAJAN *et al.* (1983) have extended the technique to analyse the gravity anomalies due to two dimensional fault structures.

Basically the Hilbert transform involves a phase shift of 90 degrees, hence it would be appropriate to say that the Hilbert transform could also be used to convert the vertical component of a magnetic field into the horizontal component, or vice versa. The horizontal derivative component of the gravity field is also obtainable by such transformation of the vertical derivative of the gravity field. NABIBHIAN (1972) has stressed only the transformation of the derivatives. Nabighian's interpretation process is based on the properties of the amplitude of the analytical signal curve, whereas the present method yields the parameters in an elegantly simple mathematical way.

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Also the computational procedure for the discrete Hilbert transform is different from that of the NABIGHIAN (1972).

This paper explores the possibility that the Hilbert transform technique can be applied alike to all three components of the magnetic anomalies over the same body. The magnetic anomalies $F(x)$ of different components and their corresponding Hilbert transforms $H(x)$ are computed. We use the generalised expression of the magnetic effect due to a thin dike of infinite depth extent. Also, the discrete Hilbert transforms $H(1.\Delta x)$ of various components of $F(x)$ are computed. The simplicity and reliability of the method is adequately demonstrated by computations of synthetic and observed magnetic anomalies.

Magnetic Effect of a Thin Dike (Infinite Depth Extent)

The general expression for the magnetic effect applicable to vertical, horizontal, and total magnetic components of the field due to a thin dike of infinite depth extent Figure 1) is given by (Gay, 1963)

$$F(x) = \frac{C_F}{h} \cos\psi . \cos(\psi - \theta_F) \tag{1}$$

Expanding and simplifying the above equation we get,

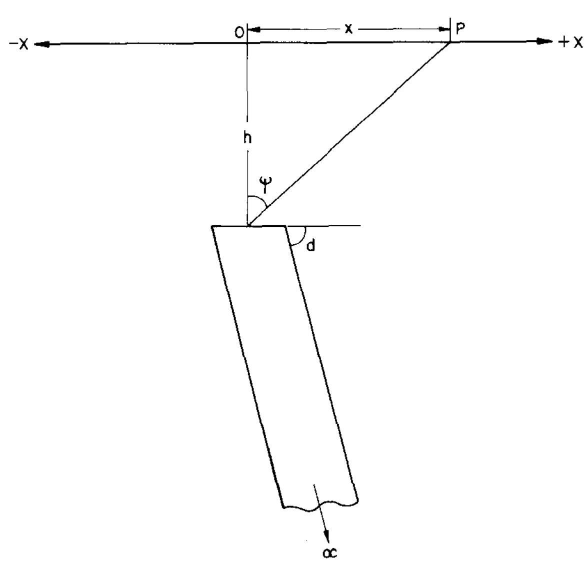

Figure 1
Geometry of thin dike.

$$F(x) = C_F\left(\frac{h.\cos\theta_F + x.\sin\theta_F}{(x^2 + h^2)}\right) \tag{2}$$

where the index parameters C_F and θ_F are defined as,

$$\begin{aligned} C_F &= 2KTt\sin d\sqrt{1-\cos^2 I.\cos^2\alpha} && \text{for } \Delta Z \\ &= 2KTt\sin d\cos d\sqrt{1-\cos^2 I.\cos^2\alpha} && \text{for } \Delta H \\ &= 2KTt\sin d(1-\cos^2 I\cdot\cos^2\alpha) && \text{for } \Delta T \\ \theta_F &= \gamma - d && \text{for } \Delta Z \\ &= \gamma - d - 90^\circ && \text{for } \Delta H \\ &= 2\gamma - d - 90^\circ && \text{for } \Delta T \end{aligned}$$

with h-depth to the top of the dike, d-dip of the dike, K-susceptibility contrast, T-undisturbed total magnetic field intensity, α-strike of the contrast measured clockwise from magnetic north, I-the inclination of the geomagnetic field, γ-$\tan^{-1}$ $(\tan I/\sin\alpha)$, ψ-$\tan^{-1}(x/h)$, and t-thickness of the thin dike.

With the analogy of the Hilbert transform of the vertical magnetic effect of a vertical sheet of infinite depth extent (MOHAN *et al.*, 1982), the Hilbert transform in the present case can be written as:

$$H(x) = -C_F\left(\frac{h.\sin\theta_F + x.\cos\theta_F}{(x^2 + h^2)}\right) \tag{3}$$

Location of Origin

The location of origin is of paramount importance in the present interpretation technique. We accomplished the location by means of simple mathematical relation defined as

$$MA(i.\Delta x) = \sqrt{(F(i.\Delta x))^2 + (H((N + I - i).\Delta x))^2} \tag{4}$$

where Δx is the station interval and N is the total number of stations considered on the profile. The origin occurs at the point where the function $MA(i.\Delta x)$ is maximum. The plot of x versus $MA(i.\Delta x)$ is known as a modified amplitude curve (SUNDARARAJAN, 1982). The concept of amplitude of the analytical signal is extremely useful in delineating geological structures of large scale geophysical data.

Extraction of Parameters

The plots of $F(x)$ and $H(x)$ intersect each other in all three cases, as shown in Figures 2, 3, and 4; at a point, for example $x = x_1$. At this point of intersection $F(x)$ and $H(x)$ are equal.

i.e.,
$$F(x) = H(x) \tag{5}$$

Further simplification leads to,

$$h = -x_1 \tag{6}$$

which implies that the depth to the top, h, of the dike is always the abscissa of the point of intersection of the magnetic anomaly and its Hilbert transform.

From equations (2) and (3), the index parameter θ_F is obtained as,

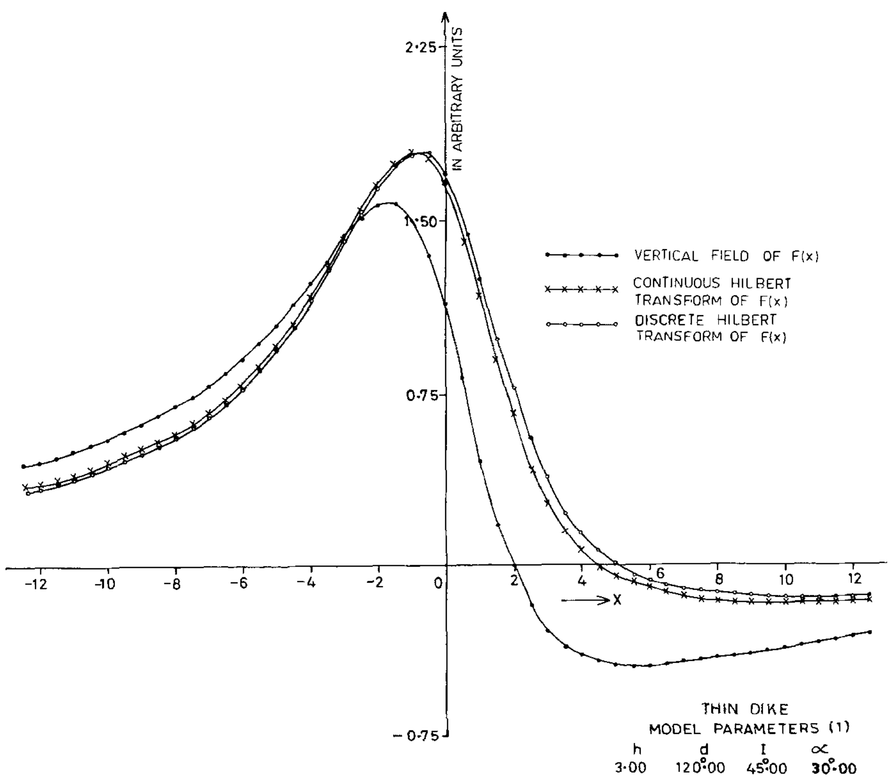

Figure 2
Computed vertical magnetic effect of thin dike, the Hilbert transform and the discrete Hilbert transform.

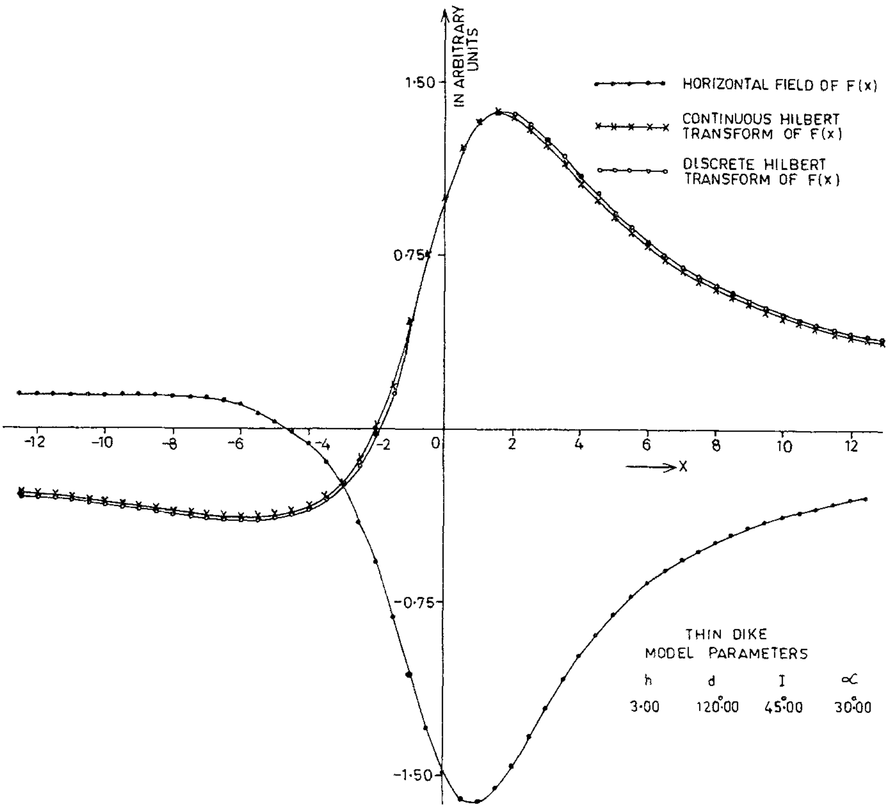

Figure 3
Computed horizontal magnetic effect of thin dike, the Hilbert transform and the discrete Hilbert transform.

$$\theta_F = \tan^{-1}\left(-\frac{x.F(x) + h.H(x)}{x.H(x) + h.F(x)}\right) \tag{7}$$

From the above equation therefore, the dip of the dike can be evaluated by assuming γ. A better estimate of θ_F or d can be obtained by considering θ_F for different values of x on either side of the origin.

The equations (2) and (3) at origin, i.e. at $x = 0$, reduce to

$$F(0) = \frac{C_F}{h}\cos\theta_F \tag{8}$$

$$H(0) = \frac{C_F}{h}\sin\theta_F \tag{9}$$

Squaring and adding equations (8) and (9) the constant terms are obtained as,

$$C_F = h.\sqrt{(F(0))^2 + (H(0))^2} \tag{10}$$

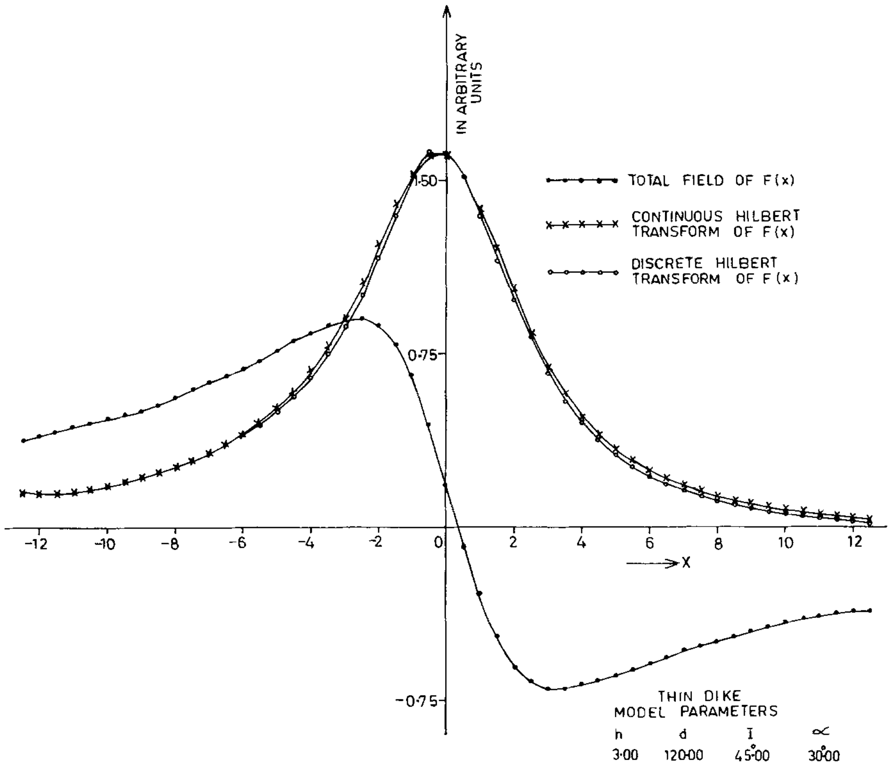

Figure 4
Computed total magnetic effect of thin dike, the Hilbert transform and the discrete Hilbert transform.

Assuming α, I, and T the susceptibility contrast K can be evaluated from the above equation.

Applications

(a) Theoretical Examples:

The procedure outlined for the interpretation of dike parameters from various components of magnetic anomalies is illustrated with one theoretical model (Table 1) in each case. Using equations (2) and (3) vertical, horizontal, and total magnetic effect of thin dike and their corresponding Hilbert transforms are computed and shown in figure 2, 3, and 4. The magnetic anomalies of various components are digitised at an interval of 0.05 units and then their discrete Hilbert transforms $H(1.\Delta x)$ are computed using the procedure outlined by MOHAN *et al.* (1982).

It is observed from the figures 2, 3, and 4 that a close identity occurs in the plots of computed and discrete Hilbert transforms. A slight deviation between the theoretical

Table 1

Theoretical Examples

Components		Parameters		
		d	h^*	C_F^*
Vertical field	Assumed values	120.00°	3.00	4.10
	Evaluated values	120.71°	2.98	4.06
Horizontal field	Assumed values	120.00°	3.00	3.55
	Evaluated values	121.73°	2.95	3.55
Total field	Assumed values	120.00°	3.00	3.24
	Evaluated values	121.23°	2.90	3.16

* In arbitrary units.

Hilbert transform and the discrete Hilbert transform is due to Gibb's phenomenon because of the use of FFT algorithm in the computation of DHT. Further, the curves of $H(x)$ and $H(1.\Delta x)$ cut the curve of $F(x)$ near the same point in all cases. The abscissa of this point of intersection of $F(x)$ and $H(1.\Delta x)$ yields the depth to the top of the dike. The dip and the constant are determined with the help of equations (7) and (10), respectively. The parameters so evaluated are presented in Table 1. It is observed that the assumed and evaluated parameters are almost the same.

(b) Field Examples

1. Vertical magnetic anomaly:

The reliability of the method is examined on the vertical component of the well-known magnetic anomaly at Kiirunavaara in northern Sweden as originally described by Von Carlheim Gyllenskjold. Its principle profile (GRANT and WEST, 1965) is shown here in Figure 5. This anomaly is due to a vein of approximately 20 percent magnetite. The magnetic inclination at Kiirunavaara is 76° and the declination of the axis of the anomaly is 18°, so that $\gamma = 85°$ in this case. The total value of the anomaly is 0.718 oersted or 71,800 gammaes.

The entire profile, approximately 674 meters in length, is digitized at intervals of nearly 10.5 meters. With the help of equations (7) and (10), the discrete Hilbert transform and the amplitude are computed and shown in Figure 5. Using the procedure outlined the parameters, namely the depth to the top, the dip, and the susceptibility contrast, are evaluated and presented in Table 2. While evaluating the susceptibility contrast the thickness of the vein is assumed as 85 metres (GRANT and WEST, 1965) and T is taken as 52024 gammas. The obtained results agree with the reported values of GRANT and WEST (1965). However in this case, the thin dike approximation is marginal.

2. Total magnetic anomaly:

The example considered here is an aerial magnetic survey profile published by the U.S. Geological Survey. Out of the three magnetic profiles so obtained, only the

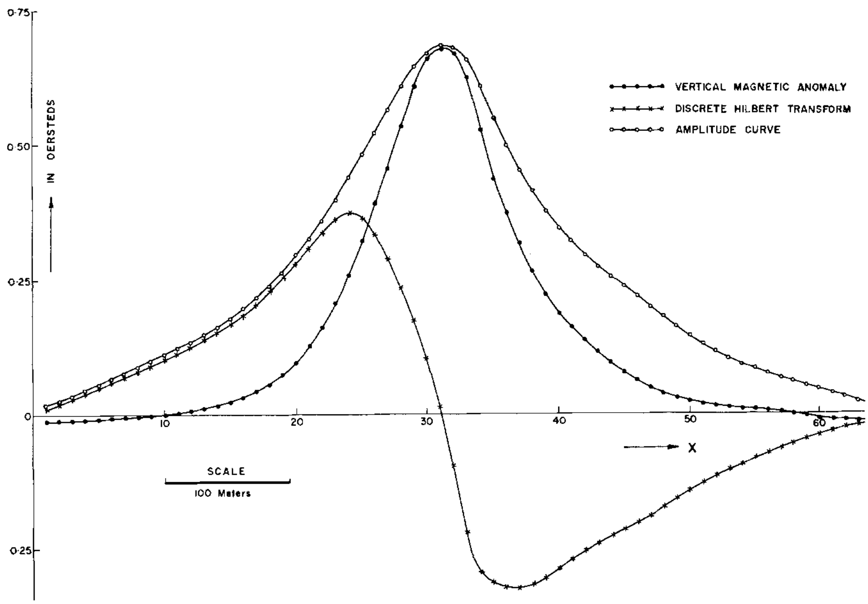

Figure 5
The vertical magnetic anomaly at Kiirunavaara, northern Sweden, the discrete Hilbert transform and the amplitude.

Table 2
Field Example

	Parameters		
	h in meters	*d* in degrees	*K* in SI units
Our method	59	86	6.29×10^{-9}
GRANT and WEST, 1965	62–63	47–75	6.79×10^{-9}

profile curve for the lowest line of flight, approximately 300 meters above sea level, is treated. This particular profile curve is due to a number of disturbing bodies. The interpretation, as described by WERNER (1953), is based wholly on the shape of the central peak of the curve, which implies the existence of two sheet-like bodies.

The profile given here, Figure 6, is the total magnetic field emanating from a sheet-like body as shown by WERNER (1953). The total length of the profile considered (10.75 km) is digitized into 128 parts each equal to 83.33 meters. The discrete Hilbert transform and the modified amplitude are computed as before, and also shown in Figure 6. In this case the depth, h, is obtained as 291.66 m, very comparable with the 270 m obtained by WERNER (1953).

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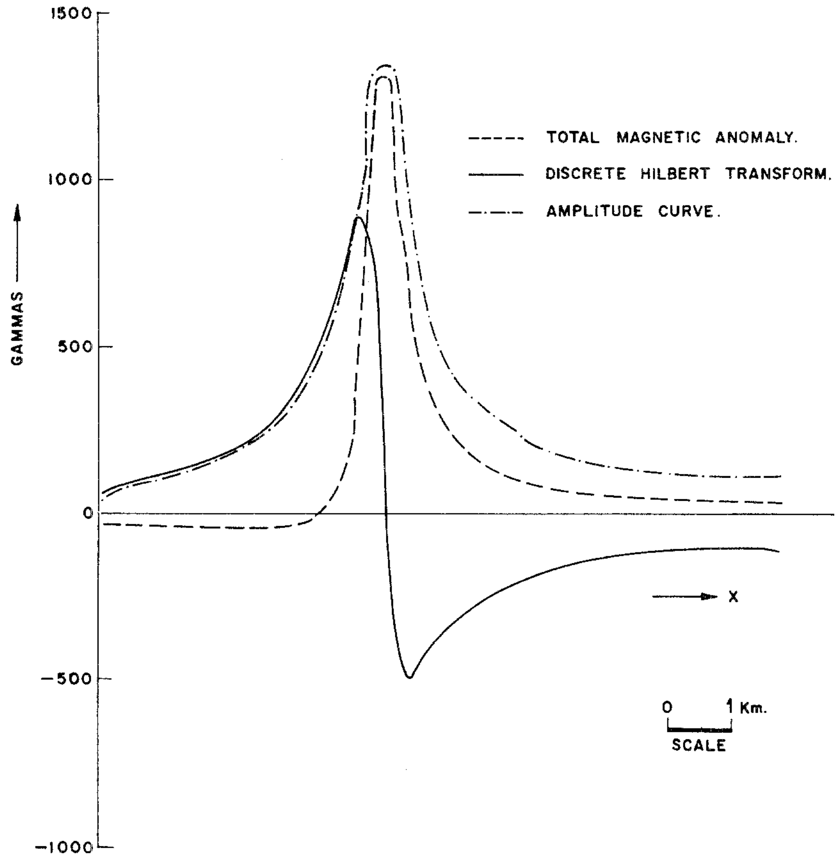

Figure 6
The total magnetic anomaly over Benson mines U.S.A., the discrete Hilbert transform and the amplitude.

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