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Scattering Attenuation and the Fractal Geometry of Fracture Systems

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Abstract—Scattering of seismic waves can be shown to have a frequency dependence $Q^{-1} \propto \omega^{3-\nu}$ if scattering is produced by arrays of inhomogeneities with a $3D$ power spectrum $W_{3D}(k) \propto k^{-\nu}$. In the earth's crust and upper mantle the total attenuation is often dominated by scattering rather than intrinsic absorption, and is found to be frequency dependent according to $Q^{-1} \propto \omega^{\gamma}$, where $-1 < \gamma \leq -0.5$. If D_1 is the fractal dimension of the surface of the $3D$ inhomogeneities measured on a $2D$ section, then this corresponds respectively to $1.5 < D_1 \leq 1.75$, since it can be shown that $\gamma = 2(D_1 - 2)$. Laboratory results show that such a distribution of inhomogeneities, if due to microcracking, can be produced only at low stress intensities and slow crack velocities controlled by stress corrosion reactions. Thus it is likely that the earth's brittle crust is pervaded by tensile microcracks, at least partially filled by a chemically active fluid, and preferentially aligned parallel to the maximum principal compressive stress. The possibility of stress corrosion implies that microcracks may grow under conditions which are very sensitive to pre-existing heterogeneities in material constants, and hence it may be difficult in practice to separate the relative contribution of crack-induced heterogeneity from more permanent geological heterogeneities.

By constrast, shear faults formed by dynamic rupture at critical stress intensities produce $D_1 = 1$, consistent with a dynamic rupture criterion for a power law distribution of fault lengths with negative exponent D. The results presented here suggest empirically that $D_1 \simeq \frac{1}{2}(D + 1)$, thereby providing the basis for a possible framework to unify the interpretation of temporal variations in seismic b-value ($b \simeq D/2$) and the frequency dependence of scattering attenuation (γ).

Key words: Scattering attenuation, fractal dimension, subcritical crack growth, rock fracture.

Introduction

Fluid-filled fractures play an important and often understated role in geological processes and geophysical observations. For example the formation of ore-bearing veins in fracture systems relies on the presence of saline fluids to act as agents for mineral transport and deposition. However, this process of dissolution, transport and precipitation of minerals requires the presence of an ambient stress field to

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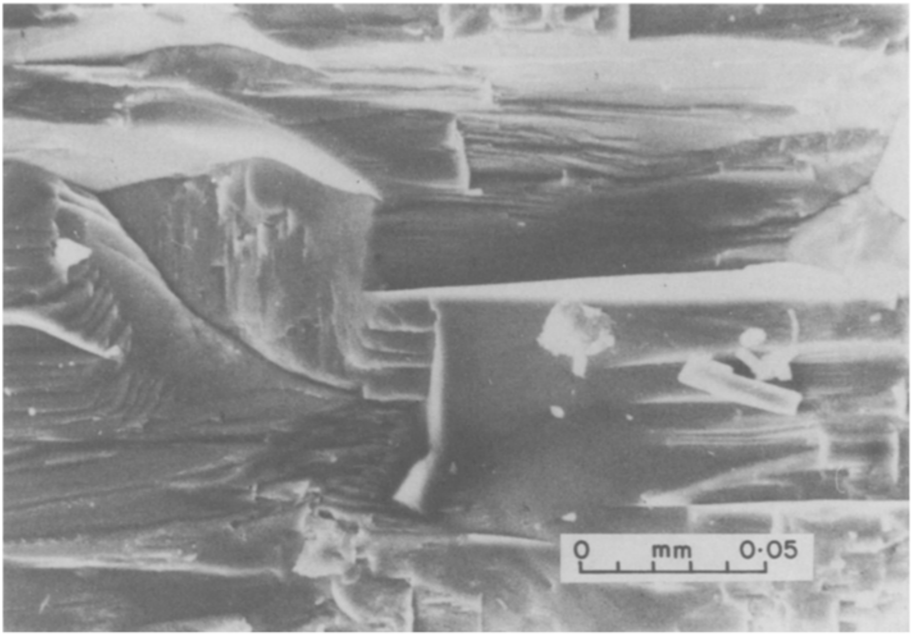
produce the fracture system as well as a suitable medium of transport. The natural roughness of the fracture surface further aids this process, by creating bottlenecks and electrochemically suitable sites for mineral deposition.

Most fractures propagate not only as the result of the purely mechanical effects of an ambient stress field, but also due to the combined effects of a stress concentration at the crack tip and the chemical weakening of molecular bonds ahead of the crack tip by the mechanism of stress corrosion (ATKINSON, 1984; ATKINSON and MEREDITH, 1987). The single most important factor in this process is the mere presence of a chemically active fluid, even in trace amounts, and the most common mechanism cited for the chemical weakening process is the hydrolytic weakening of Si-O or Al-O bonds. It is well-known that water is present in the earth's crust in hydrothermal systems (connected fractures) and as fluid inclusions (isolated fractures). Water is a natural product of every prograde metamorphic reaction, and is also incorporated into the crust by subduction of wet sediments and hydrated minerals. Furthermore, large fracture systems throughout the crust, such as those around faults, are known to be hydrated compared to country rock (SIBSON, 1977), and are probably major channels for fluid transport (SIBSON, 1981). More direct geological evidence for the importance of stress corrosion in natural fracture systems is documented in KERRICH *et al.* (1981), ETHERIDGE (1983) and SEGALL and POLLARD (1983). In summary there is a welter of laboratory results, theoretical modelling and, most importantly, geological observation to support the hypothesis that stress corrosion is an important mechanism for distributed fracture in the earth's crust (ATKINSON, 1987).

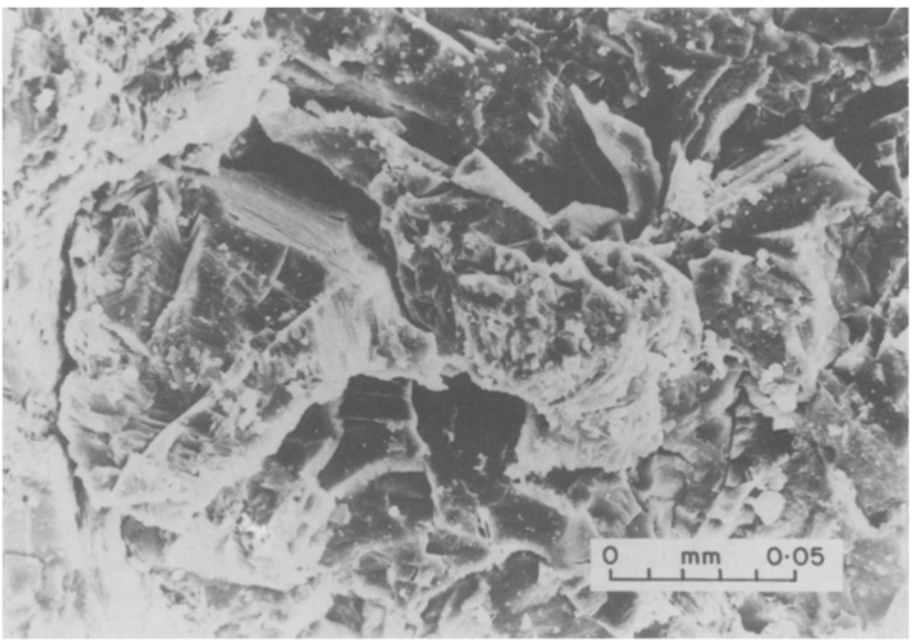
The combined effect of stress concentration and chemical corrosion at a crack tip is to promote stable, quasi-static crack growth at velocities much lower than the near-sonic velocities associated with dynamic rupture and purely mechanical fracture. Figure 1 shows that rapid mechanical fracture tends to be less sensitive to the polycrystalline microstructure than subcritical crack growth, producing much smoother fracture surfaces. Time-dependent subcritical crack growth produces rougher surfaces because at low crack velocities weaknesses such as grain boundaries, which are prime access routes for moisture, can stress corrode preferentially (MEREDITH and ATKINSON, 1983). Thus stress corrosion can lead to cracking with similar heterogeneity to the host medium.

Figure 1

Scanning electron micrographs of the fracture surfaces of Whin Sill dolerite double torsion specimens. a) Rapid mechanical fracture at high crack velocity ($> 10\ ms^{-1}$) during a fracture toughness test carried out at ambient pressure in air at 20°C. Note the dominance of smooth transgranular fracture, cleavage steps and hackle marks. b) Subcritical crack growth at low velocity (approx. $10^{-7}\ ms^{-1}$) in water at ambient pressure at 20°C. The crack surface is dominated by rough, grain-boundary microcracking even though some transgranular fracture is still evident. The double torsion technique is described in MEREDITH and ATKINSON (1983).



(a)

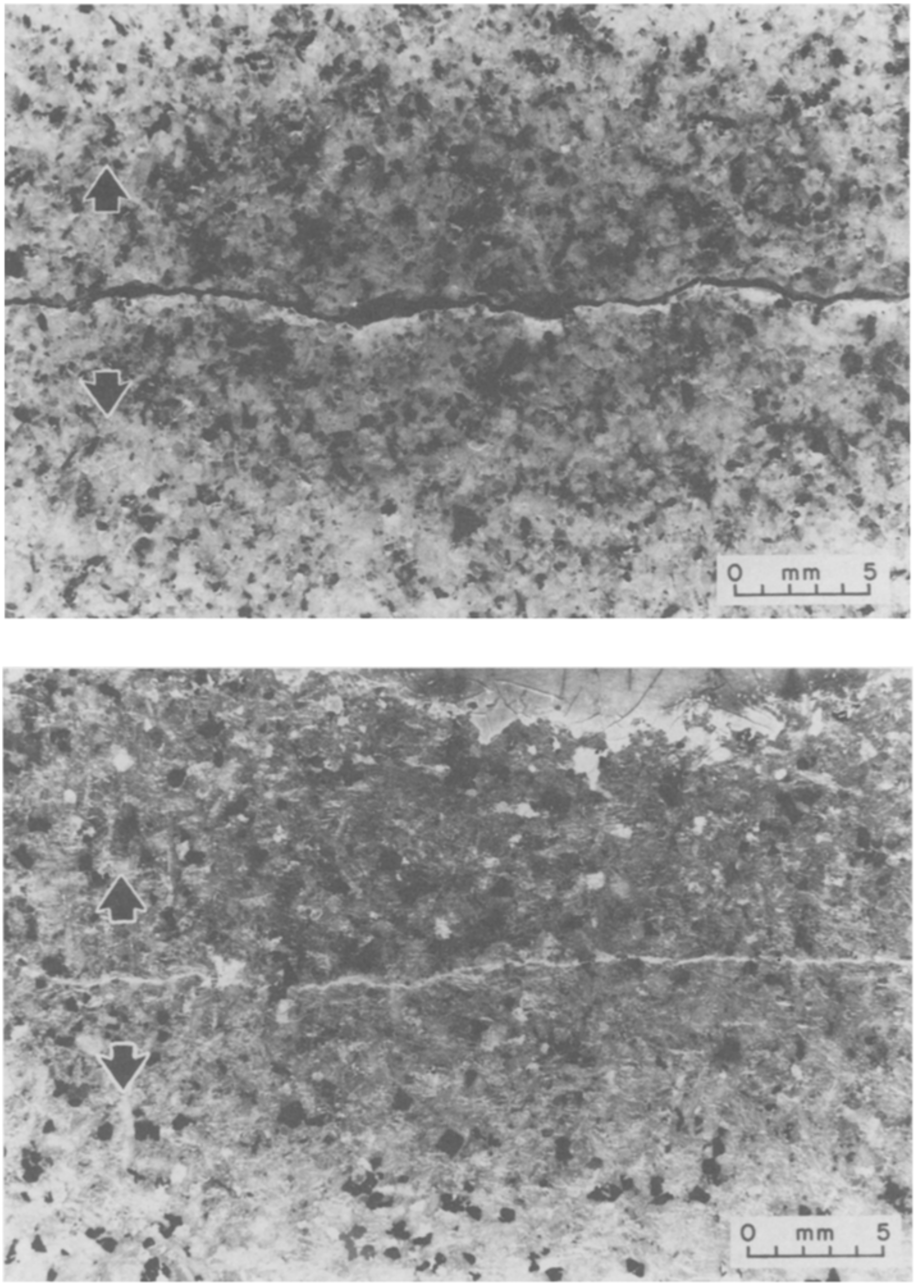


(b)

This characteristic roughness is currently being quantified by the use of a fractal dimension D_1 which measures the strictly self-similar increase in length with decrease in ruler size, first developed to describe the geometry of coastlines, and now applied to many natural systems (MANDELBROT, 1977, 1982). Consistent with MEREDITH and ATKINSON'S (1983) qualitative results, this quantitative measure of surface roughness is found to be larger for slow-forming fractures such as joints ($1.0 \le D_1 \le 1.5$; BROWN and SCHOLZ, 1985) than for dynamic features such as the San Andreas fault ($1.0008 < D_1 < 1.0191$; AVILES *et al.*, 1987).

Stress corrosion has a further effect on the geometry of fracture systems. For example Figure 2b shows that a macrocrack, produced in Westerly granite by tensile subcritical crack growth under the influence of water at room temperature and pressure, is surrounded by extensive microcracking aligned perpendicular to the maximum tensile stress. More rapid crack growth produces relatively fewer subsidiary microcracks owing to the rapid relief of stress on the macrocrack (Figure 2a). This heterogeneity in the chemical composition and mechanical properties of the host medium will exert a strong control over the density of microcracking from place to place and hence result in crack-induced heterogeneity in seismic velocities and density which may be strongly correlated with pre-existing heterogeneity. Thus the density distribution of arrays of microcracks formed by stress corrosion is strongly dependent on pre-existing geological heterogeneity, in the same way that fracture roughness is correlated to heterogeneity on a smaller scale.

The pervasiveness of such aligned microcracks produced at slow, subcritical macrocrack velocities has an important influence on wave propagation effects such as scattering attenuation and seismic anisotropy—particularly shear-wave birefringence. Both effects are now known to be time-dependent in active tectonic zones (e.g., JIN and AKI, 1986; PEACOCK *et al.*, 1988), a phenomenon more likely to result from changes in crack geometry or fluid content than changes in other more permanent types of geological heterogeneity. This holds out the possibility of distinguishing the relative contribution of either in active tectonic zones. The effect of aligned fluid-filled microcracks on velocity anisotropy (including shear-wave splitting) and scattering attenuation has been considered in the long wavelength limit for low crack densities by HUDSON (1981, 1986) and CRAMPIN (1978, 1981, 1984), using a single scattering approximation and including viscous losses. These studies show that the theory (which is second order in velocities but only first order in attenuation) is adequate for interpretation of the velocity anisotropy resulting from such a structure, but has so far been unable to reproduce either the observed magnitude or the frequency dependence of Q factors found in the earth. This may be because intrinsic attenuation has wrongly been attributed to scattering (HUDSON, 1988, pers comm.) or because the theory is often applied to sparse arrays of smooth, elliptical, discrete microcracks of identical size, shape (aspect ratio) and orientation in a homogeneous medium. (In general the theory is applicable to a wider range of crack distributions and geometry.) From the discussion above we can see that these assumptions are rarely applicable to the



(a)

(b)

Figure 2

Optical reflection micrographs of samples of Westerly granite fractured in double torsion specimens. Tensile stress was applied in the direction of the arrows. a) Rapid mechanical fracture at high crack velocity ($> 10\ \mathrm{ms}^{-1}$) during a test carried out in air at 20°C. Note the presence of a small number of aligned tensile microcracks. b) Subcritical crack growth at a low velocity (about $10^{-7}\ \mathrm{ms}^{-1}$) in water at ambient pressure at 20°C. Pervasive microcracking aligned perpendicular to the applied tensile stress is shown throughout the sample (a thin section in this case).

earth, except perhaps that aspect ratios tend to be more constant than crack lengths (e.g., Figure 2b).

In an attempt to model the observed scattering attenuation WU (1982) and WU and AKI (1985) have applied the Born approximation to a distribution of heterogeneities which may be caused by variations in geology or crack density, and where the seismic wavelengths are of the same order as the size of the heterogeneities. Although this has sometimes been called 'multiple' scattering WU'S (1982) approach is based on calculating the forward and backscattered energy at each of the multiple interfaces, and then assuming that all of the backscattered energy and none of the forward-scattered energy is lost, before considering the effect of the next interface. Clearly this is not a complete multiple scattering formulation, and furthermore the Born approximation violates the principle of conservation of energy during scattering. Nevertheless the observed magnitude and frequency dependence of scattering attenuation in the earth at frequencies in the range 1–25 Hz clearly validates the predictions of WU'S (1982) theory. The aim of the present paper is to demonstrate, with the aid of WU and AKI'S (1985) theory and observations, that the observed frequency dependence of scattering attenuation in the earth at frequencies in the range 1–25 Hz is consistent with the presence of relatively dense arrays of microcracks formed at relatively slow crack velocities in the presence of fluids, and hence formed under conditions sensitive to pre-existing crustal inhomogeneity.

Frequency Dependence of Q

Attenuation of seismic waves above that caused by geometric spreading is caused by intrinsic absorption (viscous damping) or by scattering. Scattering involves no energy loss, but produces a more extended, lower amplitude wavetrain by the resulting interference. Which of these two mechanisms dominates in any given situation depends on the relative wavelengths of the seismic wave and the acoustic heterogeneities of the fracture system. LERCHE and PETROY (1986) have noted that fractures are much more efficient than porosity caused by grain-packing in promoting second-order variations (i.e., those dependent on azimuth and frequency) in velocity and attenuation. Furthermore the more crack-like the pores (i.e., the smaller the aspect ratio) the greater is the effect of fluid content on anisotropy and attenuation.

If the seismic wavelengths λ are much greater than the size a of the scatterers, and the number of scatterers is small enough to ignore multiple scattering, then Q is found to be frequency-dependent according to

$$Q^{-1} \propto \omega^3, \tag{1}$$

where ω is an angular frequency.

This classic behaviour is known as Rayleigh scattering. For example HUDSON

(1981) applied the theory of scattering to a sparse fracture system of unconnected, fluid-filled microcracks embedded in an otherwise homogeneous continuum. CRAMPIN (1984) summarises these results for the special case of identical, perfectly aligned, smooth elliptical cracks. If the crack density is $\varepsilon = Na^3/V$, where N is the number of elliptical cracks of semi-axes a, a and c, V is the volume in which the cracks are embedded, and each crack has a small aspect ratio ($c \ll a$), then it can be shown for example that P waves are attenuated by scattering according to the formula

$$Q^{-1} = [(\alpha\varepsilon)/(15\pi\beta)](\omega a/\alpha)^3\{X \sin^2 2\theta + Y[(\alpha/\beta)^2 - 2\sin^2\theta]^2\}, \tag{2}$$

where α is the P wave velocity, β is the S wave velocity, θ is the incidence angle of the wave to the crack normal and X, Y are constants which depend on α, β and conditions on the crack faces. Similar expressions are also given (HUDSON, 1981; CRAMPIN, 1984) for the two split shear waves. These formulae imply (a) that Q is frequency dependent, according to the Rayleigh scattering case, and (b) that Q is dependent on the angle between the propagation direction and the preferred crack alignment. In order to model geological conditions the anisotropy is often expressed as a superposition of aligned and randomly oriented cracks, with only a small proportion of aligned cracks. In spite of this the Rayleigh scattering theory is unable to reproduce either the frequency dependence or the magnitudes of Q observed in the earth.

HUDSON (1981, 1986) also presents theories for seismic velocity in a homogeneous cracked medium, which show that velocity depends on the crack density and aspect ratio but not on the absolute value of the crack radius, as long as it remains small compared to the seismic wavelength. This may explain why velocity perturbations due to the arrays of self-similar cracks believed to exist in active tectonic zones can be reproduced by models with cracks of a single radius and aspect ratio (e.g., CRAMPIN and BOOTH, 1985), even though the magnitude and frequency dependence of scattering attenuation are not accounted for.

HUDSON'S (1981, 1986) single scattering theory is valid for the long wavelength limit ($ka \ll 1$, where k is the wavenumber), and depends directly on the geometry of the aligned cracks. In contrast recent attempts to model the observed scattering in the earth for the case $ka \approx 1$, WU (1982) and WU and AKI (1985) use gradually changing effective elastic properties which may be caused either by clusters of microcracks (i.e., heterogeneity in the microcrack density distribution) or pre-existing lithospheric heterogeneity. Thus the sharp changes in elastic properties due to individual cracks invoked by HUDSON (1981, 1986) are replaced by smoothly varying functions due to spatial variations in geological composition, crack density or fluid content. In the case of a smoothly varying heterogeneous distribution of small cracks, the dimension that determines the scattering attenuation is not the dimension of an individual crack but the length scale of the heterogeneities in crack density and geometry.

LERCHE (1985) and LERCHE and PETROY (1986) have attempted to model the case of multiple scattering of a dense array of microcracks with a range of sizes. For a Gaussian distribution of microcrack radii they found $Q^{-1} \propto \omega^3$, similar to Rayleigh scattering. However, if the $3D$ power spectrum of the inhomogeneities in velocity produced by the cracks is given by a power law

$$W_{3D}(k) \propto k^{-\nu}, \tag{3}$$

where k is the wavenumber, then the frequency dependence of Q is given by

$$Q^{-1} \propto \omega^{\gamma}$$
$$\gamma = 3 - \nu. \tag{4}$$

The main objection to their approach is that the imaginary terms representing the attenuation are derived from a function which is completely real (HUDSON, 1987, pers. comm.). Nevertheless the same result was obtained by WU and AKI (1985), who showed that if

$$W_{3D}(k) = \frac{-2\pi}{k}\frac{d}{dk} W_{1D}(k) \tag{5}$$

(TARTARSKII, 1971), where W_{1D} is the $1D$ power spectrum of random heterogeneities, then

$$W_{1D}(k) \propto k^{-2+m} \tag{6}$$

follows if the frequency-dependence of the Q due to the essentially single scattering of high frequency S waves is also

$$Q^{-1} \propto \omega^{\gamma}$$
$$\gamma = m - 1. \tag{7}$$

Therefore, using completely different approaches, LERCHE (1985) and WU and AKI (1985) obtain the same result, indicating that the relationship of the frequency dependence of scattering attenuation to the distribution of heterogeneities is fairly robust.

WU and AKI (1985) report empirical measurements of m in the range $m = 0$ (upper mantle) to $m = 0.54$ (crust) compared with a theoretical prediction of $m = 4$ from Rayleigh scattering. Using a wider body of data, JIN *et al.* (1985) also report $0.39 < m < 0.54$ for 22 local earthquakes distributed around the globe. These measurements are based on frequencies in the range 1–25 Hz, the lower bound being determined by surface wave contamination, and the upper bound being determined by random background noise. This implies that the dimensions of the inhomogeneities responsible for the observed scattering range from a few hundred m to a few km. SEGALL and POLLARD (1983) report field observations, in granitic rocks of the Sierra Nevada, of near-parallel dilatant cracks which they deduced were formed by quasi-static subcritical crack growth in the presence of fluids. This strongly supports the postulate that stress corrosion operates in the earth as well as in the laboratory. The lengths of individual fractures ranged from 1 cm to 100 m, so the observed scattering attenuation is not likely to be due individually to any but the largest cracks of the type reported by SEGALL and POLLARD (1983), although the smaller cracks will significantly affect the velocity anisotropy. Thus the long wavelength theory of HUDSON (1981, 1986) is applicable to the velocity anisotropy in this frequency range, but not the scattering, which is dominated by heterogeneities of similar size to the seismic wavelength, and is best modelled by WU's (1982) approach. Any time-dependent variations in scattering attenuation in the frequency range 1–25 Hz are not due to changes in geological heterogeneity, but may be caused instead by changes in the density, geometry or fluid content of clusters of dilatant microcracks associated with shear faulting on the scale of a few hundred m to a few km.

Thus WU and AKI (1985) and LERCHE (1985) both propose a power-law frequency-dependence of Q where

$$\nu = 4 - m \tag{8}$$

follows from comparing (3) with (4), (6) and (7). In the next section we shall see that ν or m can also be related to the fractal dimension D_1 produced by the intersection of these inhomogeneities with the surface of the earth. In principle that implies that D_1 may be measured from geological mapping or air photographs after image processing, in areas where the basement is exposed and the seismic heterogeneities have an optical signature.

Fractal Fracture Systems

Most geologists are familar with the scale-invariance of many geological structures, otherwise it would not be necessary to attach scale bars to photographs of field exposures or thin sections. ALLÈGRE *et al.* (1982) showed that rock fracture is also scale-invariant, and proposed a renormalisation group approach to the scaling of fractures, which TURCOTTE (1986) applied to the fragmentation process. In terms of the microcracks under discussion here, scale-invariance implies (a) that all the cracks have the same aspect ratio, (b) that the distribution of crack lengths is a power law of negative exponent D, and (c) that the fracture surfaces have the same roughness. Since there will always be an upper and lower bound to the scale at which D applies (MANDELBROT, 1977), it is important to specify these where possible. This geometrical requirement is similar to the physical condition that a finite range of frequencies is necessary to avoid violation of Kramers-Kronig relations for a power-law frequency dependence in attenuation.

SHAW and GARTNER (1986) present convincing evidence for the scale-invariance of natural fault systems, and showed for example that $D = 1.76$ for the

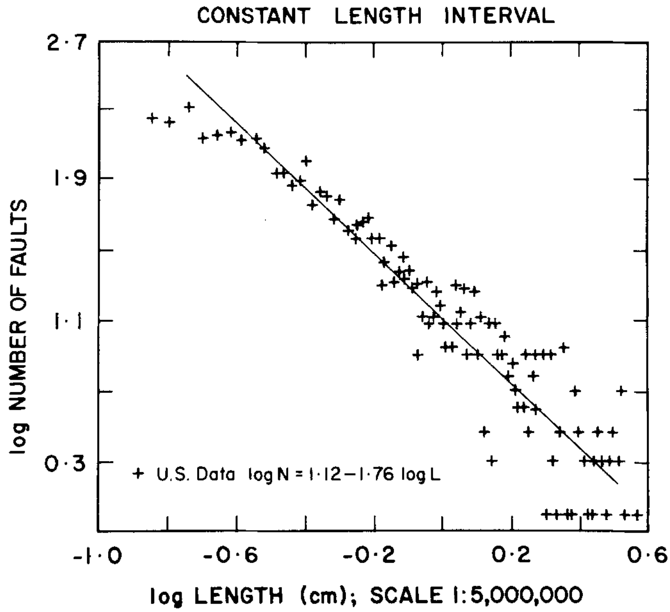


Figure 3
Discrete frequency-length distribution of active surface faults in the U.S. mapped by HOWARD *et al.* (1978) on the scale 1:500,000; after SHAW and GARTNER (1986). The data are consistent with a power law distribution of fault lengths with exponent 1.76, equivalent to the slope of the best-fitting line.

distributed system of active shear faults in the size range 10–200 km in the contiguous US mapped by HOWARD *et al.* (1978) (see Figure 3). If deformation in an area is instead concentrated on a main shear fault which crosses the entire area and a few branches, then SHAW and GARTNER (1986) showed that $D \sim 1$ for a range of sizes, from a shear box clay deformation experiment (25 μm–80 mm) through to a small fault in Iran (7–400 m) (all from TCHALENKO, 1970) and the San Andreas system (10–1000 km) (HOWARD *et al.*, 1978) (see Figures 4 and 5, after SHAW and GARTNER, 1986). Thus the Cantor set (MANDELBROT, 1982) appears to be a reasonable mathematical approximation to natural fault systems produced by the surface expression of earthquake rupture (SHAW and GARTNER, 1986). Such a Cantor set comprises one main branch surrounded by a cascade of smaller, self-similar, unconnected branches, with fractal dimension $D = D_1 = 1$ and plotted on a $2D$ Euclidean space.

Shear faults in the earth are split into characteristic segments, and compressional and tensional areas exist at the ends of segments, notably in fault jogs, fault bends and in echelon offsets (KING and NÁBĚLEK, 1985; SIBSON, 1985). The relatively high-stress intensities in these areas will produce near-fault microcracking and dilatancy accompanied by fluid channelling, and hence influence the scattering of seismic waves (e.g., JIN and AKI, 1986). In addition to these local effects at

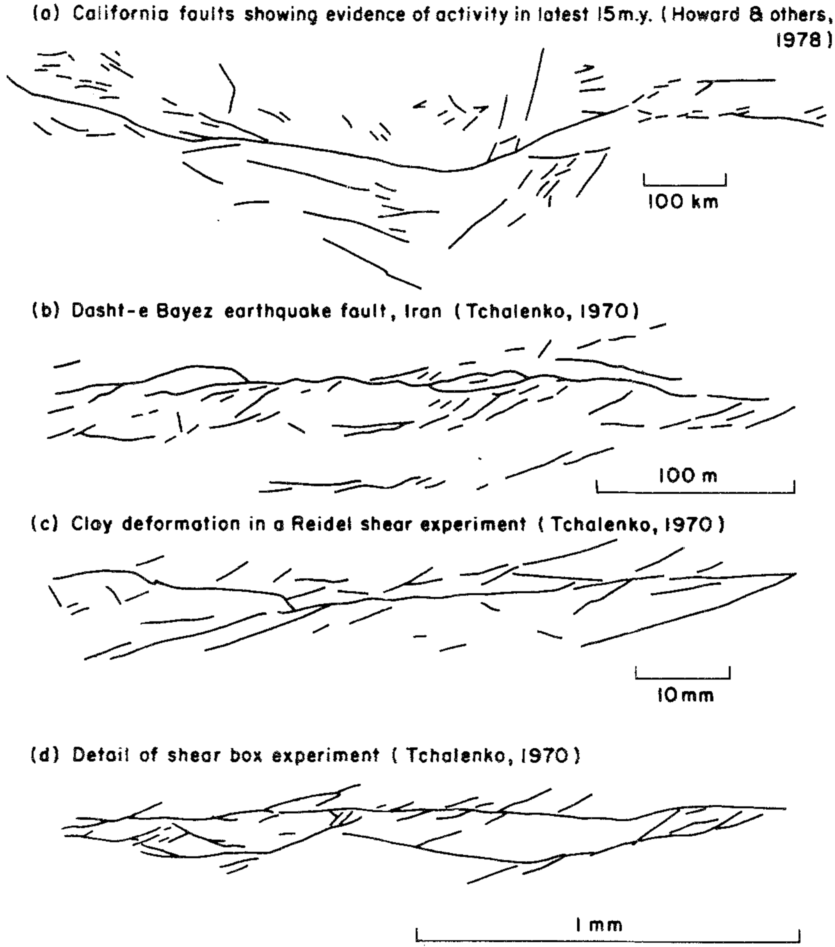


Figure 4
Geometry of shear faulting over a range of scales, after SHAW and GARTNER (1986). On the scales shown, each example is dominated by one major throughgoing fault.

relatively high stress concentrations, pervasive microcracking at low stresses due to stress corrosion may be present in the volume surrounding a major fault. SCHOLZ *et al.* (1973) have proposed a model for earthquake precursors based on local high-stress dilatancy in the fault nucleation zone, and CRAMPIN *et al.* (1984) a model based on extensive low-stress dilatancy. More recently MAIN and MEREDITH (1989) and MEREDITH *et al.* (1990) have combined the effects of a remotely applied stress and the local stress concentration caused by a pre-existing fault to provide a model capable of explaining most aspects of the earthquake cycle, including phases of elastic deformation, strain hardening, strain softening and dynamic rupture. Earthquakes occur as a result of stresses applied remotely from a fault (ultimately by two tectonic plates moving at their terminal velocities), and in response to the

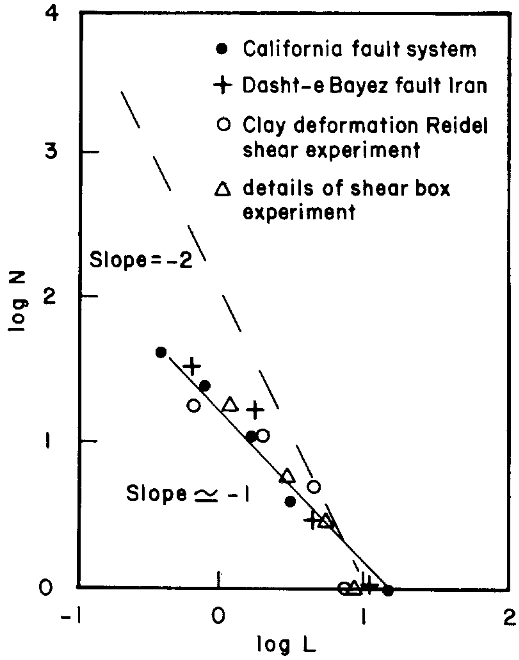


Figure 5
Normalised discrete frequency-length distribution of the faults shown in Figure 5. All four sets of experimental data are consistent with a slope of -1 (solid line), and a dashed line of slope -2 is shown for reference.

geometry of the fracture system. It therefore seems possible to use the information given by shear wave splitting remote from the fault to monitor changes in remote microcrack geometry, in conjunction with broadband observations of scattering attenuation near the fault in order to monitor changes in the geometry of the near-fault fracture system. (Broadband recording is essential if an accurate estimate of the frequency dependence of Q is to be obtained.) We therefore see these two approaches as complementary rather than competitive, since velocity anisotropy and scattering attenuation will occur wherever a system of stress-induced microcracks exists. However, monitoring of scattering attenuation is likely to pick up the more dense clusters of dilatant microcracks near the dominant fault more effectively, and measurements of shear-wave splitting may be more effective for the more sparsely distributed microcracks remote from the stress concentration on the fault. (Although shear-wave splitting will also be seen to change near the fault, the observations would be more difficult to obtain and interpret for such localised, high density microcracking.)

Few direct observations are available as yet on the size distribution of microcracks formed during contemporaneous shear faulting in controlled laboratory conditions, though the results of HADLEY (1976) are consistent with a power law distribution of crack lengths under compression, and MEREDITH and ATKINSON (1983) provide indirect evidence for the scale-invariance of microfracture accompanying tensile macrocrack growth. Figure 2 is typical of the geometry of such microcracks, and their size distribution can be inferred indirectly from the acoustic emission they produce via the seismic b-value. Several authors (CAPUTO, 1976; AKI, 1981; KING, 1983; MAIN and BURTON, 1984a) have noted that the slope b of the log-linear frequency-magnitude distribution of earthquakes is consistent with a power law distribution of fault lengths of negative exponent D where

$$D = \frac{3b}{c} \tag{9}$$

and c is a constant which depends on the relative durations of the seismic source and the time constant of the recording system. For most earthquake studies $c = 1.5$ is appropriate, so that $D = 2b$ (KANAMORI and ANDERSON, 1975; MAIN *et al.*, 1989).

Critical, dynamic rupture occurs as a result of unstable microcrack coalescence when $D_c = 1$ (MAIN, 1988) and the observed range of b-values in laboratory trials and field examples is consistent with $D < 3$, equivalent to the system of microcracks or faults being confined within a Euclidean volume. Thus $D \sim 1$ observed for fault systems by TCHALENKO (1970), HOWARD *et al.* (1978) and SHAW and GARTNER (1986) is consistent with these fault systems being formed by dynamic, mechanical rupture at high crack velocities. In contrast it will be shown below that higher values of D, for a system dominated by one major throughgoing fault, may be produced by subcritical crack growth at slow crack velocities. Figure 6 shows how the inferred fractal dimension D varies experimentally with stress intensity during tensile subcritical crack growth by stress corrosion. In this diagram the time constants of the events and recording system are such that $c = 3$, $b = D$ (MAIN *et al.*, 1989).

Stress intensity (K) is a measure of the intensity of the concentrated stress field around at a crack tip, and it depends on the length (x) of the crack responsible for stress concentration as well as the remotely applied stress (σ). It can be shown for tensile failure (LAWN and WILSHAW, 1975) that

$$K = Y\sigma x^{1/2}, \tag{10}$$

where Y is a numerical factor depending on the loading configuration and crack geometry. Empirically the crack propagation velocity V is related to K by

$$V = V_c(K/K_c)^n \tag{11}$$

where n is known as the stress corrosion index (e.g., ATKINSON, 1984; ATKINSON and MEREDITH, 1987). This law has also been applied to earthquake rupture in shear mode by DAS and SCHOLZ (1981) and MAIN (1988). K_c is the critical stress intensity (sometimes known as the fracture toughness), at which dynamic rupture occurs at a critical velocity V_c close to the sonic velocity of the medium; and is a

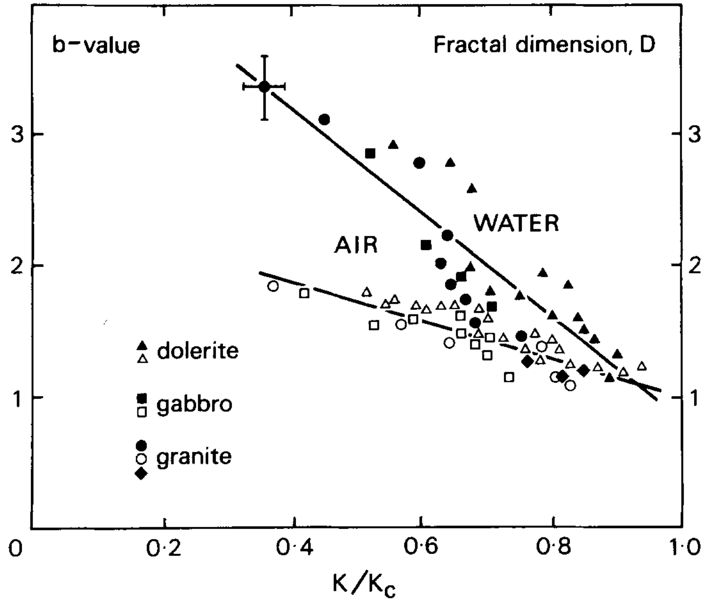


Figure 6
Synoptic diagram of the variation in seismic b-value and the inferred fractal dimension D with stress intensity K and crack tip humidity during tensile crack propagation for a variety of crystalline rocks. The data were derived from tensile crack growth experiments performed in air or water at ambient pressure and temperature using the double torsion technique, and are normalised relative to the fracture toughness K_c. Solid lines are least squares fits to the data points and converge at the point ($K/K_c = 1$, $b = 1$, $D = 1$). b depends on the time constants of the event and the recording instrument, so D is the more fundamental parameter. For these tests the relevant relation between b and D is $b = D$, but for most earthquake studies $b = D/2$ (MAIN *et al.*, 1989).

material constant in the same sense that elastic constants are. Thus Figure 6 may be thought of as a plot of D versus $(V/V_c)^{1/n}$. Note that once the data are normalised by the fracture toughness K_c for each material, results obtained from different rock types all fall on the same curve, within the experimental resolution. Figure 6 also demonstrates that, in a system dominated by one macrocrack, high values of D can be produced only by low macrocrack velocities, and are also more likely to be produced by the chemical activity of fluids and the resulting enhancement of the stress corrosion mechanism. More specifically $D > 2$ can be produced only in the presence of a partial pressure of water or water vapour, equivalent to at least the humidity of ambient laboratory air. In contrast, $D = 1$ is consistent with dynamic rupture at near-sonic velocities accompanied by the formation of a dominant throughgoing macrocrack. Overall D is confined to the range $1 \lesssim D \lesssim 3$ as expected. This difference in geometry between dynamic and subcritical crack growth, and its likely effect on scattering properties at different frequencies, has previously been pointed out by WU (1986), but the formulation in the present work is somewhat different, and based on a wider body of empirical observation of laboratory-scale fracture.

It is important to note that the power-law exponent D is not in general equal to the fractal dimension D_1, which might be obtained by spectral analysis of the expression of faults and joints at the earth's surface, since D from the length distribution ignores surface roughness and not all earthquake faults have a surface expression. Because D_1 is measured on a surface of Euclidean dimension 2, $1 \leq D_1 \leq 2$. $D_1 = 1.5$ represents a 1-dimensional Brownian motion function, and the general form of the power law spectrum of a fractional Brownian motion function (MANDELBROT, 1977) is of the form

$$W_{1D}(k) \propto k^{-(2H+1)} \tag{12}$$

where H is the Hurst number and $D_1 = 2 - H$. WU and AKI (1985) noted that $H = 0.5$ ($D_1 = 1.5$) corresponds to a random, memory-less process operating during the formation of lithospheric heterogeneities. $H < 0.5$ ($D_1 > 1.5$) corresponds to an antipersistent process (a changing of the function in the opposite sense to a previous step). At low stress intensities the stabilisation of fracture at one locality, due to strain relief by microcracking, makes it less likely that fracture will recur at that site. This negative feedback corresponds to fractal antipersistency and results in distributed damage. In contrast $H > 0.5$ ($D_1 < 1.5$) corresponds to persistency or positive feedback, most likely to occur in the earth on and around propagating macrocracks and faults at high stress intensities. The limiting case for a persistent function is $H = 1$ ($D_1 = 1$), corresponding to highly directional dynamic rupture at critical stress intensities. Thus the critical values for both D and D_1 are unity.

Similarly if the length distribution of microfractures is 'volume-filling' ($D = 3$) in the same way that a map of a river and its tributaries is 'area-filling' ($D_1 = 2$)—otherwise they would not be able to drain the catchment—then the upper and lower bounds of D and D_1 are related by

$$D_1 = \tfrac{1}{2}(D + 1). \tag{13}$$

Random background seismicity is often reported to occur around a b-value of 1 (e.g., SMITH, 1986), or $D = 2$, for $c = 1.5$, so (13) also holds approximately for at least one special intermediate case. HUANG and TURCOTTE (1988; Figure 4) also showed that D and D_1 are approximately related by this equation in the persistent fractal regime for a model of microscopically brittle failure using a fractal distribution of stress and strength. Furthermore, the seismic b-value for small events of source dimensions of a few hundred m to a few km in the New Madrid area of the eastern US is in the range $b = 1.00$–1.25 (Figure 7), corresponding to $D = 2.0$–2.5, $D_1 = 1.5$–1.75 and $m = 0.0$–0.5 by equations (6), (9), (12) and (13). This is consistent with WU and AKI'S (1985) measurement of m from scattering off the same scale of inhomogeneities in the crust of the Hindu Kush. Figure 7 shows that the small events ($m_b < 2.5$) are in the antipersistent regime ($b = 1.25$, $D_1 = 1.75$); the small and intermediate

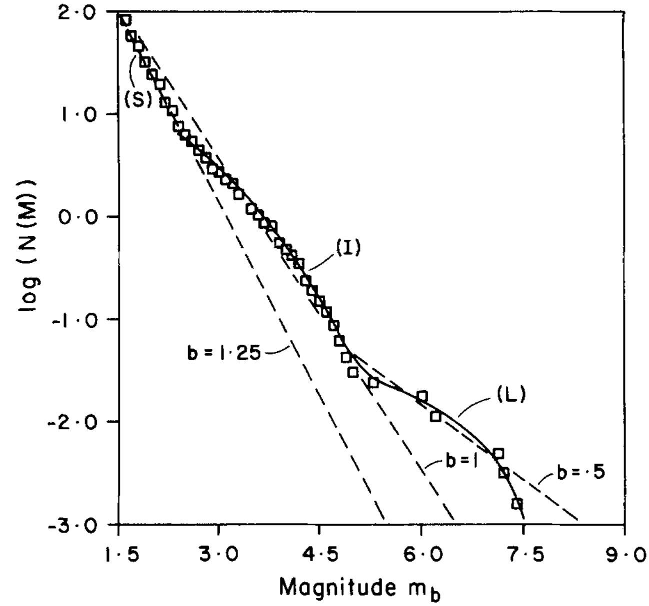


Figure 7
Cumulative frequency-magnitude distribution of the seismicity in the New Madrid zone. There appear to be three different classes of event: small (S), intermediate (I), and large (L). Intermediate and large events are best interpreted by a Weibull distribution (solid lines: MAIN and BURTON, 1984b), and dashed reference lines are drawn with $b = 0.5$, 1, 1.25 corresponding to $D = 1$, 2, 2.5 (here $D = 2b$). The small events correspond to faults of a few hundreds of metres and are consistent with $b = 1.25$. Small and intermediate events together (hundreds of m to a few km) are consistent with $b = 1$, and large events (10–100 km) are consistent with $b = 0.5$. The data are a synthesis of historical, geological and instrumental investigations of the area (JOHNSTON, 1981).

Table 1

Frequency dependence of Q^{-1} under different fractal conditions

H	D_1	D	v	m	$W_{1D}(k)$	γ	$Q^{-1}(\omega)$
0	2	3	3	1	k^{-1}	0	ω^0
0.5	1.5	2	4	0	k^{-2}	-1	ω^{-1}
1	1	1	5	-1	k^{-3}	-2	ω^{-2}

H—Hurst number; D_1—fractal dimension derived from the intersection of inhomogeneities with a surface; D—negative power law exponent of length distribution; v—exponent in formula $Q^{-1} \propto \omega^{3-v}$; m—exponent in formula $Q^{-1} \propto \omega^{m-1}$; $W_{1D}(k)$ – $1D$ spectrum of inhomogeneities; k—wavenumber; ω—angular frequency. Exponents are related by the formulae: $D_1 = 2 - H$; $v = 7 - 2D_1$, $m = 4 - v$, and $D_1 = 1/2\ (D + 1)$.

events grouped together ($m_b < 5.0$) are approximately in the random regime ($b = 1, D_1 = 1.5$) and the very largest events ($m_b > 5.0$) are in the persistent regime ($b = 0.5, D_1 = 1$). This highlights the importance of determining upper and lower limits to the scale at which a particular fractal dimension holds, and indicates that (13) represents, at worst, a reasonable linear approximation so what may be a generally nonlinear relation, with some degree of scatter in nature.

Table 1 summarises the relationships between the various parameters introduced in the text up to this point and their frequency dependence.

Fractals and Q

The observation by WU and AKI (1985) and JIN *et al.* (1985) that $m \simeq 0.5$ for the earth's crust has received further confirmation from independent measurements (REITER *et al.*, 1987). The latter results show $m = 0.6$ ($Q^{-1} \propto \omega^{-0.4}$) in the northeastern US and eastern Canada, using the standard coda decay method in the range 1 to 10 Hz. A multiple scattering interpretation gave values of scattering Q of 30 to 150 at 1 and 5 Hz respectively, consistent with $m = 0.5$. Scattering Q was found to be lower than intrinsic Q (270 to 1350 at the same frequencies), indicating that scattering is the major contributor to strong motion attenuation in this area.

If scattering dominates the total attenuation, and $0 < m \leq 0.5$, then $3.5 < v \leq 4.0$, and $1.5 < D_1 \leq 1.75$, corresponding to $2 < D \leq 2.5$ from (13). The scattering attenuation of shear waves has also been modelled by SATO (1982), using scattering by a random medium with an exponential autocorrelation function (also corresponding to rather rough heterogeneity). We have already seen that such high D values (corresponding to relatively rough heterogeneity) can be produced in the laboratory only by extremely slow macrocrack growth at low stress intensities in the presence of fluids (Figure 6). Figure 8 is a cartoon illustrating the point that the observed frequency dependence of attenuation in the earth could have been produced by arrays of microcracks filled or partially filled with fluid and formed under conditions of low-stress intensity predominantly in the antipersistent fractal regime. This presupposes that the fracture systems in the earth's crust are related to the fractal heterogeneities in elastic properties invoked by WU and AKI (1985) to explain the frequency dependence of Q in the lithosphere. However, we have already seen that under conditions of slow crack growth fracture systems have the time to exploit such heterogeneities (Figures 1, 2), and therefore it is likely that the two are also strongly correlated in the earth. WU and AKI (1985) do not consider the possibility of subcritical crack growth, and hence only note that D_1 inferred from scattering in the crust should be an upper bound to the fractal dimensions measured on the San Andreas fault, for example. While this is true, the mechanism of stress corrosion at low-stress intensities in the fluid-saturated crust allows the possibility of much higher values of D and D_1 (specifically

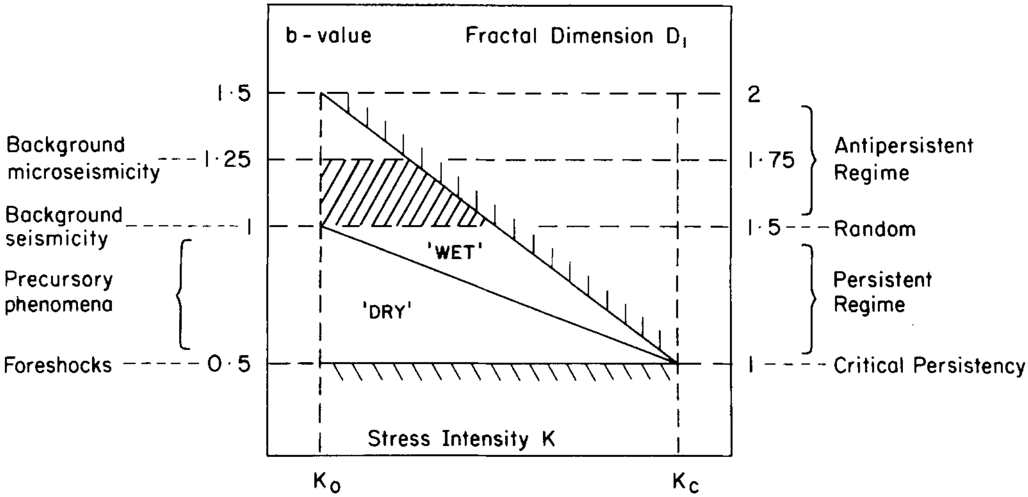


Figure 8
Sketch of the possible relationship between seismic b-values and the fractal dimension D_1 caused by the intersection of faults or microcracks on a planar surface. The diagram assumes $b = 2D$ and $D_1 = 1/2$ $(D + 1)$. The hatched area shows the permissible range of values of b and stress intensity K which are associated with the observed frequency dependence of scattering in the earth's crust. In the context of the diagram 'WET' implies water or water vapour with a partial pressure at least equivalent to that of natural humidity under ambient laboratory conditions, or else normalised to the relevant hydrostatic stress at appropriate geological depths. 'DRY' implies the presence of water in even smaller trace amounts, but only the flat hatched line ($D_1 = 1$) is strictly consistent with a purely mechanical fracture in the complete absence of a chemically active fluid.

$D > 2$, $D_1 > 1.5$) as observed indirectly from seismic b-values and the frequency dependence of scattering attenuation.

Because of the possiblity of fluid-enhanced subcritical crack growth, it has been suggested (e.g., CRAMPIN *et al.*, 1984) that anisotropy and shear-wave splitting in the earth are caused by fluid-filled microcracks formed at low stresses and aligned parallel to the maximum principal compressive stress. However, the theory used to interpret shear-wave splitting has not to date been reconciled with field observation of the magnitude and frequency dependence of attenuation. This section shows that it is possible to reconcile all the observations only when a more dense and more heterogeneous population of microcracks with a range of sizes and a power law length distribution is invoked.

Discussion and Conclusion

We have shown that the observed temporal changes in shear-wave splitting and coda-Q^{-1} in active tectonic zones are most likely caused by changes in the density of arrays of dilatant microcracks fully or partially filled with fluids. Scattering attenuation is more sensitive to the larger clusters of dilatant microcracks around shear faults in the size range of a few hundred m to a few km, whereas velocity anisotropy is more sensitive to even smaller microcracks remote from the dominant fault. Laboratory experiments show that the pervasiveness of such microcracks is greatly enhanced by stress corrosion, leading to extensive microcracking at low stress intensities in the presence of a chemically active fluid. Such microcracking at slow crack propagation velocities and low-stress intensities can exploit local weaknesses and inhomogeneities much more effectively than dynamic shear faulting at near-sonic velocities and critical stress intensities. Thus the fractal dimension of the fracture systems in the earth may be closely related to the fractal dimension of pre-existing lithospheric inhomogeneities, which have been measured to be in the range $1.5 < D_1 \leq 1.75$, with $D_1 \simeq 1.75$ for crustal heterogeneities (WU and AKI, 1985; JIN *et al.*, 1985).

Dynamic rupture at critical stress intensities produces relatively smooth fracture surfaces ($D_1 \approx 1$) with few subsidiary faults ($D \approx 1$), but subcritical crack growth can produce much rougher surfaces ($1 < D_1 < 2$) with many subsidiary faults ($1 < D < 3$). This suggests a relation $D_1 = 1/2\,(D + 1)$ holds at least approximately in the subcritical regime, where $D_1 = 1.5$ ($D = 2$) corresponds to a random, memoryless process; $D_1 > 1.5$ ($D > 2$) corresponds to an antipersistent process; and $D_1 < 1.5$ ($D < 2$) corresponds to a persistent process. Antipersistency ($D > 2$) is observed in the laboratory only at a low-stress intensities in the presence of water or water vapour at ambient partial pressures, and persistency ($D < 2$) is found to be due to more dynamic mechanical fracture at high-stress intensities, a process which is less sensitive to pre-existing material heterogeneity or the presence of a corrosive reagent.

Fracture systems in the earth are commonly fluid-filled, otherwise the ubiquity of veins in geological exposure would be difficult to explain, hence they are likely to have fractal dimensions $D > 2$ or $D_1 > 1.5$, i.e. in a range similar to the measurements of lithospheric inhomogeneity from scattering attenuation. Thus it may not be possible to distinguish other types of lithosphere inhomogeneity from fracture systems using scattering attenuation alone. In active tectonic zones the seismic b-value ($b = D/2$) may provide an independent measure of the current distribution of new fracture systems, and shear-wave splitting could in principle indicate the presence or absence of crack-induced heterogeneity.

If the relation between D_1 and D is applicable to precursory changes in seismic b-values and scattering attenuation as suggested, then this may provide a means of unifying the interpretation of two of the most statistically significant of earthquake precursors. This will require a broadband approach to the measurement of temporal variations in scattering Q in order to measure m or γ with sufficient accuracy.

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