PAGEOPH, Vol. 120 (1982) 0033-4553/82/020365-10$01.50 + 0.20/0

A Fourier Transform Method for the Interpretation of Self-Potential Anomalies Due to Two-Dimensional Inclined Sheets of Finite Depth Extent

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Abstract – The self-potential anomaly due to a two-dimensional inclined sheet of finite depth extent has been analysed in the frequency domain using the Fourier transform. Expression for the Fourier amplitude and phase spectra are derived. The Fourier amplitude and phase spectra are analysed so as to evaluate the parameters of the sheet. Application of this method on two anomalies (synthetic and field data) has given good results.

Key words: Fourier transform method; Self-potential anomalies.

1. Introduction

Quantitative interpretation of self-potential (SP) anomalies is usually carried out by approximating the causative source to a simple body of regular geometric shape (viz. sheet, cylinder, sphere etc.) in several of the following ways: (1) Using a few characteristic points on the anomaly curve. The methods falling in this category are developed by YUNGUL (1950), PAUL (1965), and BHATTACHARYA and ROY (1981). The main disadvantage with this method is that only a few points are used on the anomaly curve and hence the interpreted results are not reliable, if the data is contaminated by noise. But these methods are fast and suitable for rough estimates. However, there are methods (e.g. curve matching technique, method of least squares and spectral analysis) in which all the data points may be used for interpretation. (2) In the curve matching technique (MEISER, 1962) the field curve is compared to the album of pre-computed theoretical curves. This process will be cumbersome when the variables are many. (3) In the method of least squares, the initial parameters are assumed and the final solution is obtained by minimizing the difference between the observed and calculated values using the least squares procedure. (4) In spectral analysis, the data is analysed in the wave number domain using the Fourier amplitude and phase spectra.

In the present paper we have shown the applicability of the Fourier transform to interpret the SP anomaly due to an inclined sheet of finite depth extent. The Fourier amplitude, modified amplitude and phase spectra are analysed to find the parameters of the sheet.

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2. Theory

Let us consider a sheet (Fig. 1), inclined at an angle θ extending infinitely along its strike. The upper and lower edges of the sheet are situated at depths of h and H units

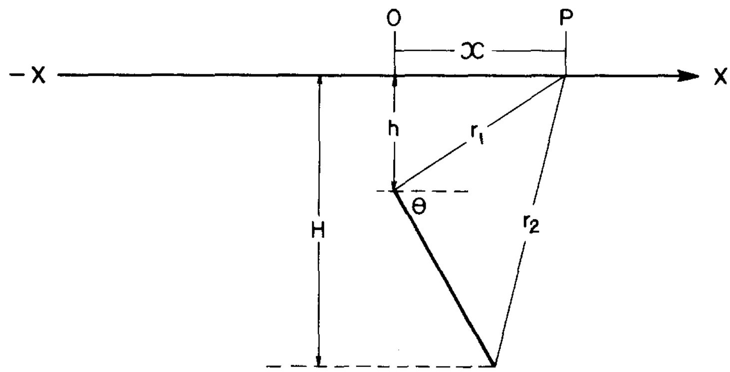

Figure 1
Cross section of an inclined sheet of finite depth extent.

respectively below the ground surface. The expression for the SP anomaly due to the sheet along a profile perpendicular to its strike is given by ROY and CHOWDHURY (1959)

$$V(x) = \frac{I\rho}{2\pi} \ln \frac{r_1^2}{r_2^2} \tag{1}$$

where I is the current per unit length ρ is the resistivity of the surrounding medium and r_1 and r_2 are the distances of the edges of the sheet from the point of observation.

Expressing r_1 and r_2 in terms of x (the distance of the point of observation from the origin) h and H we have

$$V(x) = \frac{I\rho}{2\pi} \ln \frac{x^2 + h^2}{(x-a)^2 + H^2} \tag{2}$$

in which

$$a = (H - h)/\tan\theta$$

The SP anomaly due to a sheet with $h = 2$ m, $H = 5$ m, $\theta = 60°$ and $I\rho/2\pi = 100$ mV is computed using equation (2) and shown in Fig. 2.

If V_{max} and V_{min} are the maximum and minimum amplitudes of $V(x)$ it is easy to prove from equation (2) that

$$V_{max} + V_{min} = \frac{I\rho}{\pi} \ln \frac{h}{H} \tag{3}$$

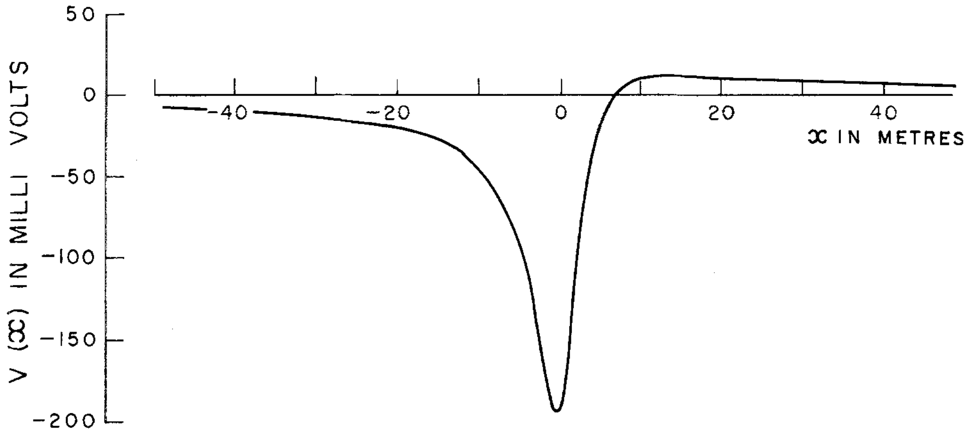

Figure 2
The SP anomaly $V(x)$ over an inclined sheet of finite depth extent with $h = 2$ m, $H = 5$ m, $\theta = 60°$ and $I\rho/2\pi = 100$ mV.

2.1 *Fourier transform of $V(x)$*

The Fourier transform, $F(\omega)$ of a function $F(x)$ is given by

$$F(\omega) = \int_{-\infty}^{\infty} F(x) \exp(-j\omega x)\, dx \tag{4}$$

using equation (4) the Fourier transformation $F(\omega)$ of $V(x)$ may be written as

$$F(\omega) = \int_{-\infty}^{\infty} \left(\frac{I\rho}{2\pi} \ln \frac{x^2 + h^2}{(x-a)^2 + H^2}\right) \exp(-j\omega x)\, dx \tag{5}$$

since

$$\int_{-\infty}^{\infty} \tfrac{1}{2} \ln(x^2 + h^2) \exp(-j\omega x)\, dx = \frac{-\pi}{\omega} \exp(-\omega h) \tag{6}$$

and

$$\int_{-\infty}^{\infty} F(x-a) \exp(-j\omega x)\, dx = \exp(-j\omega a) \int_{-\infty}^{\infty} F(x) \exp(-j\omega x)\, dx \tag{7}$$

We have from equation (5)

$$F(\omega) = R(\omega) + jX(\omega) \tag{8}$$

where $R(\omega)$, the real part of $F(\omega)$ is given by

$$R(\omega) = \frac{I\rho}{\omega} \left[\exp(-\omega H) \cos a\omega - \exp(-\omega h)\right] \tag{9}$$

and $X(\omega)$, the imaginary part of $F(\omega)$ is given by

$$X(\omega) = \frac{-I\rho}{\omega} \exp(-\omega H) \sin a\omega \tag{10}$$

The amplitude $A(\omega)$ and phase $\phi(\omega)$ of $F(\omega)$ are given by

$$A(\omega) = [R^2(\omega) + X^2(\omega)]^{1/2} \tag{11}$$

and

$$\phi(\omega) = \arctan [X(\omega)/R(\omega)] \tag{12}$$

using equations from (9) to (12) we write

$$A(\omega) = \frac{I\rho}{\omega} [\exp(-2\omega h) + \exp(-2\omega H) - 2\exp(-\omega\overline{H+h}) \cos a\omega]^{1/2} \tag{13}$$

and

$$\phi(\omega) = \arctan \left[\frac{\exp(-\omega H) \sin a\omega}{\exp(-\omega h) - \exp(-\omega H) \cos a\omega} \right] \tag{14}$$

The amplitude $A(\omega)$ and phase $\phi(\omega)$ may be analysed to evaluate the parameters of the sheet in the following way.

2.2 *Analysis of $A(\omega)$ and $\phi(\omega)$ to evaluate body parameters*

From equation (13) we have that

$$\operatorname*{Lt}_{\omega \to 0} A(\omega) = \frac{I\rho(H-h)}{\sin\theta} = C_1 \text{ (say)} \tag{15}$$

Defining the modified amplitude $A_1(\omega) = \omega A(\omega)$ we have

$$A_1(\omega) = I\rho\, [\exp(-2\omega h)) + \exp(-2\omega H) - 2\exp(-\omega\overline{H+h}) \cos a\omega]^{1/2} \tag{16}$$

Since the depth H to the lower pole is always greater than the depth h to the upper pole, the term $\exp(-\omega H)$ decays much faster than the term $\exp(-\omega h)$ and at sufficiently high frequencies the effect of the terms containing $\exp(-\omega H)$ in equation (13) is negligible. Hence at higher frequencies we have

$$A(\omega) = \frac{I\rho}{\omega} \exp(-\omega h) \tag{17}$$

and

$$A_1(\omega) = I \cdot \rho \cdot \exp(-\omega h) \tag{18}$$

Taking natural logarithms on both sides we have

$$\ln A_1(\omega) = \ln C_2 - \omega h \tag{19}$$

where

$$C_2 = I\rho \tag{20}$$

Hence, for higher values of ω the plot of $\ln A_1(\omega)$ versus ω will be a straight line (Fig. 3) whose negative slope is equal to h, the depth to the upper pole of the sheet. The intercept of this line on the $\omega = 0$ axis is equal to C_2.

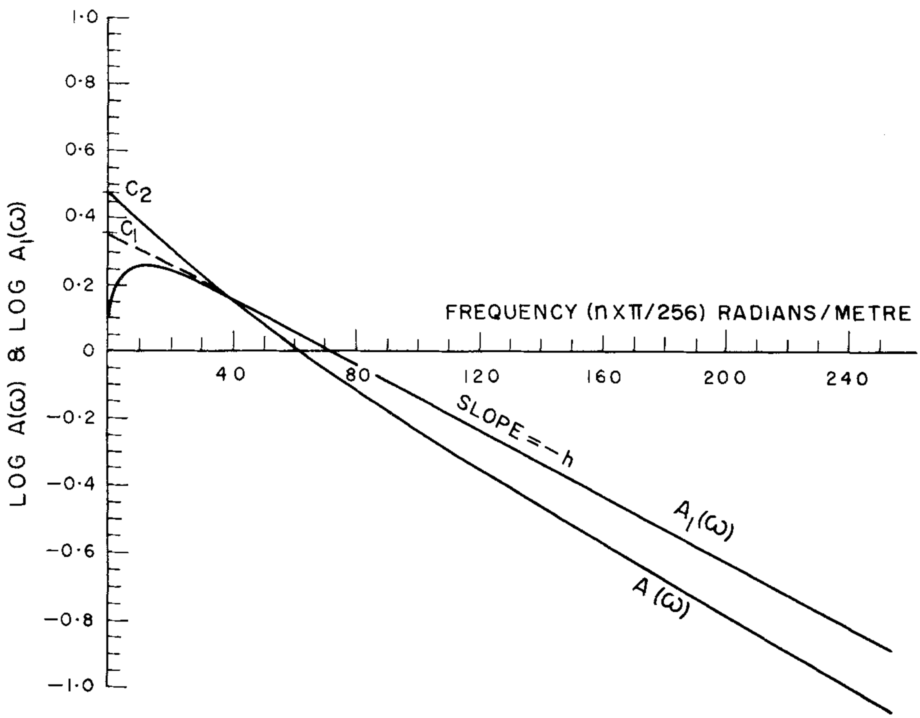

Figure 3
Theoretical amplitude $A(\omega)$ and the modified amplitude $A_1(\omega)$ spectra for the SP anomaly shown in Fig. 2.

From equation (14) we have that

$$\operatorname*{Lt}_{\omega \to 0} \phi(\omega) = \frac{\pi}{2} - \theta \tag{21}$$

Hence, using the phase spectrum, the dip θ of the sheet may be found. The depth H to the lower pole may be evaluated from the constant C_1 using equation (15) since all other terms are known. Without going to the phase spectrum, H and θ may be evaluated as follows. Since h and $I\rho$ may be evaluated from $A_1(\omega)$, H may be found from equation (3) using the following relation

$$H = \frac{h}{\exp\left[\frac{\pi}{I\rho}(V_{\min} + V_{\max})\right]} \tag{22}$$

The dip θ of the sheet may be found from equation (15) and (20) using the relation

$$\theta = \sin^{-1}\left[C_1 \frac{H-h}{C_2}\right] \tag{23}$$

Generally, the dip θ of the sheet may also be assumed either from the shape of $V(x)$ curve or from geology in which case H may be evaluated using the relation

$$H = h + C_1 \sin\theta / Ip \tag{24}$$

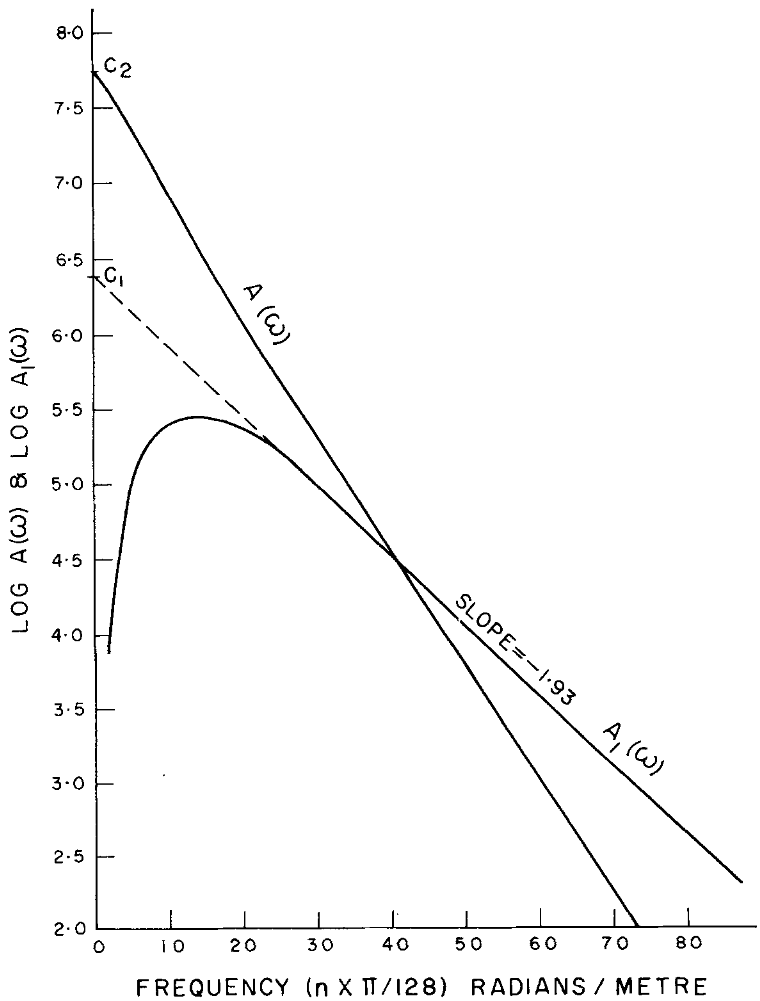

Figure 4

Computed amplitude $A(\omega)$ and the modified amplitude $A_1(\omega)$ spectra for the SP anomaly in Fig. 2.

Location of the origin

Usually, for field data, the position of the origin is not known. This may be determined from the phase spectrum in the following way.

For field data, the spectrum will be computed starting from one end of the profile. In this case, the phase spectrum will be a straight line for higher values of ω whose slope gives the displacement of the origin from the centre of the profile.

The origin may also be located on $V(x)$ curve using the following criteria. By examining a number of theoretical curves it has been concluded that the origin is very close to X_{min} (the absicca of V_{min}) with a slight displacement towards X_{max}.

Examples

Synthetic example: The SP anomaly due to a sheet with $h = 2$ m, $H = 5$ m, $\theta = 60°$ and $I\rho/2\pi = 100$ mV is generated using equation (2) over a length of 128 m on either side of the origin at 1 m intervals, and its Fourier transform is computed using the F.F.T. algorithm. The amplitude $A(\omega)$ and the modified amplitude $A_1(\omega)$ spectra are computed and shown in Fig. 4. The Fourier amplitude and modified amplitude spectra are analysed. It may be seen from Fig. 4 that the plot of $A_1(\omega)$ versus ω is a straight line for large values of ω. The depth to the upper pole of the sheet is evaluated from the slope of this straight line. The intercept of this line on $\omega = 0$ axis gave the $I\rho$ value. Then on $V(x)$ curve, V_{max} and V_{min} are found and the depth H to the bottom is evaluated using equation (22). The dip of the sheet is found from equation (23). The results of our interpretation are given in Table 1.

Table 1

Results of analysis of a synthetic SP anomaly shown in Fig. 2

Parameter	Assumed	Evaluated
Depth h to top in metres	2.00	1.98
Depth H to bottom in metres	5.00	5.10
Dip θ in degrees	60	62

Field example: The proposed technique has been used to interpret an SP anomaly (Fig. 5) taken across a mineralized belt in Kalava fault zone, 52 km south of Kurnool in Cuddapah basin, Andhra Pradesh, India (Sanker Narayan *et al.*, 1982). Earlier drilling over some anomaly locations in this area by the Geological Survey of India encountered carbonaceous shales with sulphide mineralization. These might be the

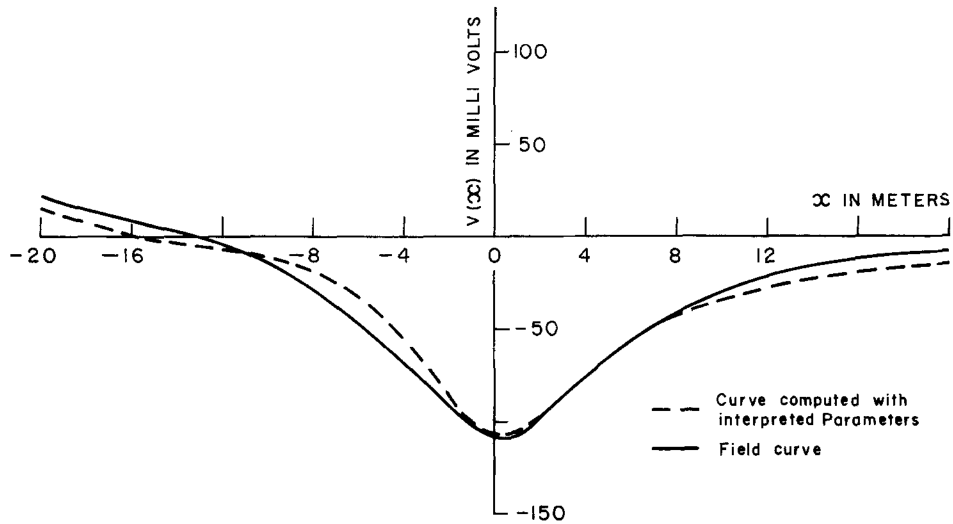

Figure 5
SP anomaly (continuous line) over a sulphide body in the Kalava fault zone (Cuddapah basin, India). The theoretical curve generated using the interpreted parameters is shown as the broken line.

Table 2

Results of analysis of the SP anomaly over a sulphide body in the Kalava fault zone, Cuddapah basin, India

Parameter	By our method
Depth h to top in metres	15.9
Depth H to bottom in metres	41.2
Dip θ in degrees	110
Ip in millivolts	72

sources causing the SP anomaly. The SP profile shown in Fig. 5 has been digitized over a length of 255 m at 1 m intervals and subjected to spectral analysis. The amplitude and modified amplitude spectra are shown in Fig. 6. The spectra are interpreted using the method proposed in this paper and the results are presented in Table 2. Using the interpreted parameters, the theoretical profile has also been generated and shown in Fig. 5 for direct comparison. There exists a close match between the observed and computed anomalies. The depth to the target obtained by SANKER NARAYAN *et al.* (1982) using ROY's (1966) continuation method is 17 m and agrees well with our interpretation.

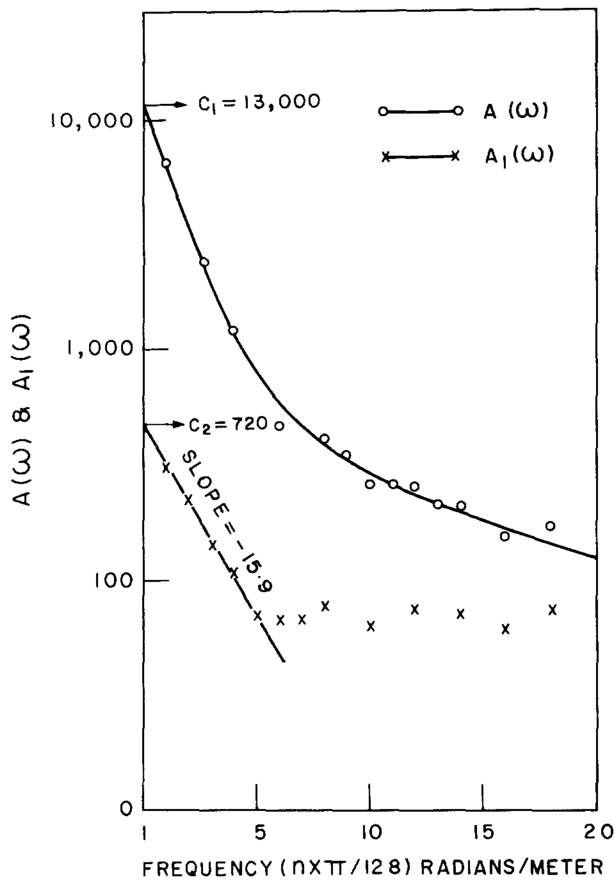

Figure 6
Amplitude $A(\omega)$ and modified amplitude $A_1(\omega)$ spectra for the SP anomaly shown in Fig. 5.

Acknowledgments

We are grateful to Dr P. V. Sanker Narayan, Assistant Director, National Geophysical Research Institute, for his encouragement throughout this work. We thank the Director, NGRI for his kind permission to publish this work.

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(Received 9th September 1981, revised 13th May 1982)