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Analysis of Gravity Anomalies over an Inclined Fault with Quadratic Density Function

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Abstract—An attempt is made to interpret the gravity anomalies over an inclined fault with variable density contrast. The decrease of density contrast with depth in sedimentary rocks is approximated by a quadratic function. The anomaly equation of an inclined fault is derived with the quadratic density function. The constants a_0, a_1 and a_2 of the quadratic density function can be found from the known density-depth values. A synthetic anomaly profile of the fault model is interpreted by the non-linear optimisation technique using the Marquardt algorithm. The distances are measured from an arbitrary reference point and thus the origin of the fault model is also treated as an unknown parameter. For the assumed values of the constants a_0, a_1 and a_2, the various parameters of the fault model are found by the non-linear optimisation technique. The convergence of the method is shown by plotting the values of the objective function, lamda, and the parameters of the fault model with respect to iteration number. The two parameters inclination and origin are found to be correlated. The same program is used to interpret the gravity anomalies with different density contrasts. Finally, the use of modelling with the quadratic density function is discussed.

Key Words: Gravity anomalies; Inclined fault; Quadratic density function.

Introduction

Many methods are available to interpret the gravity anomalies over a fault with constant density contrast (SHARMA and GELDART, 1968; RADHAKRISHNA MURTHY and BHASKARA RAO, 1980). There is ample evidence to show that the density of the sedimentary rocks increases with depth. ATHY (1930) observed that the density of sediments increases with depth exponentially. CORDELL (1973) has reviewed three cases where the decrease of density contrast in sedimentary basins could be approximated by an exponential function. However, the representation of density contrast by an exponential function is not convenient in the analysis of gravity anomalies, as the anomaly equations are not obtained in closed form. The exponential decrease of density over a small interval of depth is approximated by a linear function by MURTHY and RAO (1979). They have divided the side of a polygon body into several segments and calculated the contribution of each segment with linear decrease of density contrast. AGARWAL (1971) has considered the decrease

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of density contrast linearly with depth. BHATTACHARYA and CHAN (1977) and GENDZWILL (1970) have also developed methods with variable density contrast.

Here, an attempt is made to approximate the decrease of density contrast with depth in sedimentary rocks by a quadratic function. This facilitates deriving the anomaly equation for an inclined fault in closed form. A method has been developed to interpret the parameters of this model by a non-linear optimisation technique using the Marquardt algorthm. The origin is also treated as an unknown parameter and is found by the Marquardt algorithm. A synthetic anomaly profile is interpreted to test the proposed method.

Anomaly Equation

The decrease of density contrast in sedimentary rocks could be approximated by a quadratic function (Bhaskara Rao, 1985), as

$$\Delta\rho(Z) = a_0 + a_1 Z + a_2 Z^2 \tag{1}$$

where Z represents the depth measured positive in downward direction, a_0 represents the extrapolated value of density contrast at the surface, and a_1 and a_2 are the constants of the quadratic function.

The equation for a gravity anomaly of an inclined fault, with inclination i and depths to top and bottom Z_1 and Z_2 respectively, is derived by integrating the gravity effect of an elementary mass (Fig. 1). Thus,

$$\Delta g(x) = 2\gamma \int_{Z=Z_1}^{Z_2} \int_{u=-(z-z_1)\cot i}^{\infty} \frac{\Delta\rho(Z) Z\, dZ\, du}{(u-x)^2 + Z^2} \tag{2}$$

γ is the gravitational constant.

Substituting the value of $\Delta\rho(Z)$ in equation (2), we have

$$\Delta g(x) = 2\gamma a_0 \int_{Z=Z_1}^{Z_2} \int_{u=-(z-z_1)\cot i}^{\infty} \frac{Z\, dZ\, du}{(u-x)^2 + Z^2}$$

$$+ 2\gamma a_1 \int_{Z=Z_1}^{Z_2} \int_{u=-(z-z_1)\cot i}^{\infty} \frac{Z^2\, dZ\, du}{(u-x)^2 + Z^2}$$

$$+ 2\gamma a_2 \int_{Z=Z_1}^{Z_2} \int_{u=-(z-z_1)\cot i}^{\infty} \frac{Z^3\, dZ\, du}{(u-x)^2 + Z^2}$$

After evaluating these integrals, the anomaly equation might be written incorporating the origin (D) also. D is measured from an arbitrary reference to the point 0 (Fig. 1). This is achieved by putting $x = x\text{-}D$. This means that the distances are measured from an arbitrary reference point instead of from the origin. The origin is at a distance

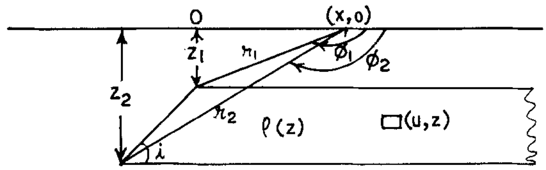

Figure 1
Fault Model.

of D from the reference point. Thus, the relevant equation for the gravity anomaly of an inclined fault is given by,

$$\Delta g(x) = 2\gamma a_0 \left[F_1 \left\{ \sin i \ln \frac{r_2}{r_1} + \cos i \, (\phi_2 - \phi_1) \right\} + Z_2 \phi_2 - Z_1 \phi_1 \right]$$

$$+ \frac{2\gamma a_1}{2} \left[Z_2^2 \phi_2 - Z_1^2 \phi_1 + F_1 (Z_2 - Z_1) \sin i - F_1^2 \left\{ \sin 2i \ln \frac{r_2}{r_1} \right.\right.$$

$$\left.\left. + \cos 2i (\phi_2 - \phi_1) \right\} \right]$$

$$+ \frac{2\gamma a_2}{3} \left[Z_2^3 \phi_2 - Z_1^3 \phi_1 + F_1 (Z_2^2 - Z_1^2) \frac{\sin i}{2} - F_1^2 (Z_2 - Z_1) \sin 2\, i \right.$$

$$\left. + F_1^3 \left\{ \sin 3\, i \ln \frac{r_2}{r_1} + \cos 3\, i \, (\phi_2 - \phi_1) \right\} \right] \tag{3}$$

where $r_1^2 = (x - D)^2 + Z_1^2$

$$r_2^2 = [(Z_2 - Z_1) \cot i + x - D]^2 + Z_2^2$$

$$\phi_1 = \frac{\pi}{2} + \arctan \frac{x - D}{Z_1}$$

$$\phi_2 = \frac{\pi}{2} + \arctan \frac{x - D + (Z_2 - Z_1) \cot i}{Z_2}.$$

and $F_1 = (x - D) \sin i - Z_1 \cos i$.

It may be noted that the programming of equation (3) is not difficult, as it involves the same arctan and logarithmic terms, and the time taken will not be much longer than for the case of constant density contrast. Also, equation (3) is conveniently given in three parts representing the constant density contrast and the first and second order decrease of density contrast with depth. Thus, the same programme can be used for any of the three cases by assigning proper values to a_0, a_1 and a_2.

Method of interpretation

In the interpretation of real anomalies, the values of the constants a_0, a_1 and a_2 are deduced by the least squares fitting of the density-depth values to the quadratic function. The density-depth values are taken from the measured density values in bore-holes, or densities deduced by seismic methods, or densities deduced otherwise. Thus for given values of a_0, a_1, and a_2, the parameters to be interpreted are the origin, i, Z_1, and Z_2.

The anomaly equation is too lengthy to develop any simple method of interpretation. However, this could be easily programmed to a digital computer. Here, the non-linear optimisation technique using the Marquardt algorithm is adopted to find the unknown parameters. The basis of the method is that an initial model is assumed and the various parameters are successively improved so as to minimize the objective function defined by

$$F = \sum_{i=1}^{N_{\text{obs}}} [\Delta g_{\text{obs}}(x_i) - \Delta g_{\text{cal}}(x_i)]^2 \tag{4}$$

where

$$\Delta g_{\text{obs}} = \text{observed anomaly}$$

$$\Delta g_{\text{cal}} = \text{calculated anomaly}$$

$$\text{and } N_{\text{obs}} = \text{total number of observations.}$$

The method of the Marquardt algorithm is discussed by many authors such as COLES (1976), RADHAKRISHNA MURTHY and BHASKARA RAO (1982), etc. The necessary system of simultaneous equations is given by

$$\sum_{K=1}^{4} \sum_{i=1}^{N_{\text{obs}}} \frac{\partial \Delta g(i)}{\partial P_j} \frac{\partial \Delta g(i)}{\partial P_k}(1 + \delta_{jk}\lambda) dP_k$$
$$= \sum_{i=1}^{N_{\text{obs}}} [\Delta g_{\text{obs}}(i) - \Delta g_{\text{cal}}(i)] \frac{\partial \Delta g(i)}{\partial P_j} \tag{5}$$
$$\text{for } j = 1 \text{ to } 4$$

where

$$\delta_{jk} = 1 \text{ for } j = k$$
$$= 0 \text{ for } j \neq k$$

and P_k represents one of the parameters Z_1, Z_2, i, and D. λ is a constant to be selected, by trial and error, so that the objective function decreases. Using the notation,

$$F_2 = (Z_2 - Z_1) \cot i + x - D$$

$$F_3 = \sin i \ln \frac{r_2}{r_1} + \cos i(\phi_2 - \phi_1)$$

$$F_4 = \sin 2i \ln \frac{r_2}{r_1} + \cos 2\, i(\phi_2 - \phi_1)$$

$$F_5 = \sin 3i \ln \frac{r_2}{r_1} + \cos 3\, i\, (\phi_2 - \phi_1)$$

$$F_6 = Z_1 \cot i - (x - D)$$

$$F_7 = -Z_2(Z_2 - Z_1) \operatorname{cosec}^2 i$$

$$F_8 = \frac{F_2 \cot i}{r_2^2} + \frac{Z_1}{r_1^2}$$

$$F_9 = \frac{Z_2 \cot i}{r_2^2} - \frac{x - D}{r_1^2}$$

$$F_{10} = F_2 \cot i + Z_2$$

$$F_{11} = (x - D) \cos i + Z_1 \sin i$$

$$F_{12} = -\frac{F_2(Z_2 - Z_1) \operatorname{cosec}^2 i}{r_2^2}$$

$$F_{13} = -\frac{F_2}{r_2^2} + \frac{x - D}{r_1^2}$$

$$F_{14} = \frac{Z_2}{r_2^2} - \frac{Z_1}{r_1^2}$$

$$F_{15} = \cos i \ln \frac{r_2}{r_1} - \sin i\, (\phi_2 - \phi_1)$$

$$F_{16} = 2 \cos 2\, i \ln \frac{r_2}{r_1} - 2 \sin 2i(\phi_2 - \phi_1)$$

and $$F_{17} = 3 \cos 3i \ln \frac{r_2}{r_1} - 3 \sin 3i\, (\phi_2 - \phi_1)$$

the partial derivatives in equation (5) are given by

$$\frac{\partial \Delta g(x)}{\partial Z_1} = 2\gamma a_0 \left[-F_3 \cos i - F_1(F_8 \sin i + F_9 \cos i) - \phi_1 - \frac{Z_2^2 \cot i}{r_2^2} + \frac{Z_1(x - D)}{r_1^2} \right]$$

$$+ \frac{2\gamma a_1}{2} \left[-\frac{Z_2^3 \cot i}{r_2^2} + \frac{Z_1^2(x - D)}{r_1^2} - 2\, Z_1 \phi_1 - F_1 \sin i \right.$$

$$-(Z_2-Z_1)\sin i\cos i+2F_1F_4\cos i+F_1^2(F_8\sin 2i+F_9\cos 2i)\Big]$$

$$+\frac{2\gamma a_2}{3}\left[-\frac{Z_2^4\cot i}{r_2^2}+\frac{Z_1^3(x-D)}{r_1^2}-3Z_1^2\,\phi_1-\frac{\sin i\cos i}{2}(Z_2^2-Z_1^2)\right.$$

$$-Z_1F_1\sin i+2F_1(Z_2-Z_1)\cos i\sin 2i$$

$$\left.+F_1^2\sin 2i-3F_1^2F_5\cos i-F_1^3(F_8\sin 3i+F_9\cos 3i)\right]$$

$$\frac{\partial\Delta g(x)}{\partial Z_2}=2\gamma a_0[\{F_1(F_{10}\sin i+F_6\cos i)+F_6Z_2\}/r_2^2+\phi_2]$$

$$+\frac{2\gamma a_1}{2}[2Z_2\phi_2+F_1\sin i+\{F_6Z_2^2-F_1^2(F_{10}\sin 2i+F_6\cos 2i)\}/r_2^2]$$

$$+\frac{2\gamma a_2}{3}[3Z_2^2\phi_2+Z_2F_1\sin i-F_1^2\sin 2i$$

$$+\{Z_2^3F_6+F_1^3(F_{10}\sin 3i+F_6\cos 3i)\}/r_2^2]$$

$$\frac{\partial\Delta g(x)}{\partial i}=2\gamma a_0\left[F_3F_{11}+F_1\left(F_{15}+F_{12}\sin i+\frac{F_7\cos i}{r_2^2}\right)+\frac{Z_2F_7}{r_2^2}\right]$$

$$+\frac{2\gamma a_1}{2}\left[\frac{F_7Z_2^2}{r_2^2}+(Z_2-Z_1)(F_1\cos i+F_{11}\sin i)-2F_1F_4F_{11}\right.$$

$$\left.-F_1^2\left(F_{16}+F_{12}\sin 2i+\frac{F_7\cos 2i}{r_2^2}\right)\right]$$

$$+\frac{2\gamma a_2}{3}\left[\frac{F_7Z_2^3}{r_2^2}+\frac{(Z_2^2-Z_1^2)}{2}(F_1\cos i+F_{11}\sin i)\right.$$

$$-2F_1(Z_2-Z_1)(F_1\cos 2i+F_{11}\sin 2i)$$

$$\left.+3F_1^2F_5F_{11}+F_1^3\left(F_{17}+F_{12}\sin 3i+\frac{F_7\cos 3i}{r_2^2}\right)\right]$$

$$\frac{\partial\Delta g(x)}{\partial D}=2\gamma a_0\left[-F_3\sin i+F_1(F_{13}\sin i-F_{14}\cos i)-\frac{Z_2^2}{r_2^2}+\frac{Z_1^2}{r_1^2}\right]$$

$$+\frac{2\gamma a_1}{2}\left[-\frac{Z_2^3}{r_2^2}+\frac{Z_1^3}{r_1^2}-(Z_2-Z_1)\sin^2 i+2F_1F_4\sin i\right.$$

$$\left.-F_1^2(F_{13}\sin 2i-F_{14}\cos 2i)\right]$$

$$+\frac{2\gamma a_2}{3}\left[-\frac{Z_2^4}{r_2^2}+\frac{Z_1^4}{r_1^2}-\frac{\sin^2 i}{2}(Z_2^2-Z_1^2)\right.$$

$$+ F_1 \sin i\{2(Z_2 - Z_1)\sin 2i - 3F_1F_5\}$$

$$\left. + F_1^3(F_{13}\sin 3i - F_{14}\cos 3i)\right] \tag{6}$$

These equations are not further simplified, as the above representation is also convenient for interpretation with constant density contrast and first or second order decrease of density contrast with depth.

The initial values of Z_1, Z_2, and i are deduced by characteristic curves (RAO and RADHAKRISHNA MURTHY, 1978) for $a_1 = a_2 = 0$. The origin is assumed in the vicinity of the inflection point. The gravity anomaly is calculated for the initial model and the objective function $F(1)$, as defined by equation (4), is found. λ is initially given a value of 0.5. The derivatives in equation (5) are calculated with the help of equations (6). Equations (5) are solved for the increments dZ_1, dZ_2, di and dD. If Z_1', Z_2', i', and D' are the initial values of Z_1, Z_2, i, and D, then the improved values Z_1^*, Z_2^*, i^* and D^* are given by

$$Z_1^* = Z_1' + dZ_1$$

$$Z_2^* = Z_2' + dZ_2$$

$$i^* = i' + di$$

$$\text{and } D^* = D' + dD.$$

The objective function $F(2)$ is calculated again using these values. If $F(2) \leqslant F(1)$, then the step is a success and the value of λ is decreased by a factor of 1/2 and the value of $F(2)$ is assigned to $F(1)$. The method is iterated until some convergence criteria are met. If $F(2) > F(1)$ at any stage, the value of λ is doubled and the equations (5) are solved for dZ_1, dZ_2, di and dD. These increments are added to the values of Z_1, Z_2, i and D at the previous successful step and the objective function $F(2)$ is calculated. This procedure is repeated until $F(2) \leqslant F(1)$. The usual convergence criteria are that the value of the objective function is below a certain tolerance limit, or that the increments of various parameters are negligible.

Example

A synthetic anomaly of 30 Km length at an interval of 1 Km for $Z_1 = 1$ Km, $Z_2 = 5$ Km, $i = 60°$, $D = 15$ Km, $a_0 = -0.4$, $a_1 = 0.08$ and $a_2 = -0.005$ is interpreted by this method. The gravity anomaly for this model is shown in Fig. 2a. The exact values of a_0, a_1 and a_2 are assumed. Initially, the origin is assumed in the vicinity of the inflection point. The initial values for Z_1, Z_2, and i are obtained from the characteristic curves method (RAO and RADHAKRISHNA MURTHY, 1978) for $a_1 = a_2 = 0$. The initial model and the corresponding gravity anomaly are also shown by a

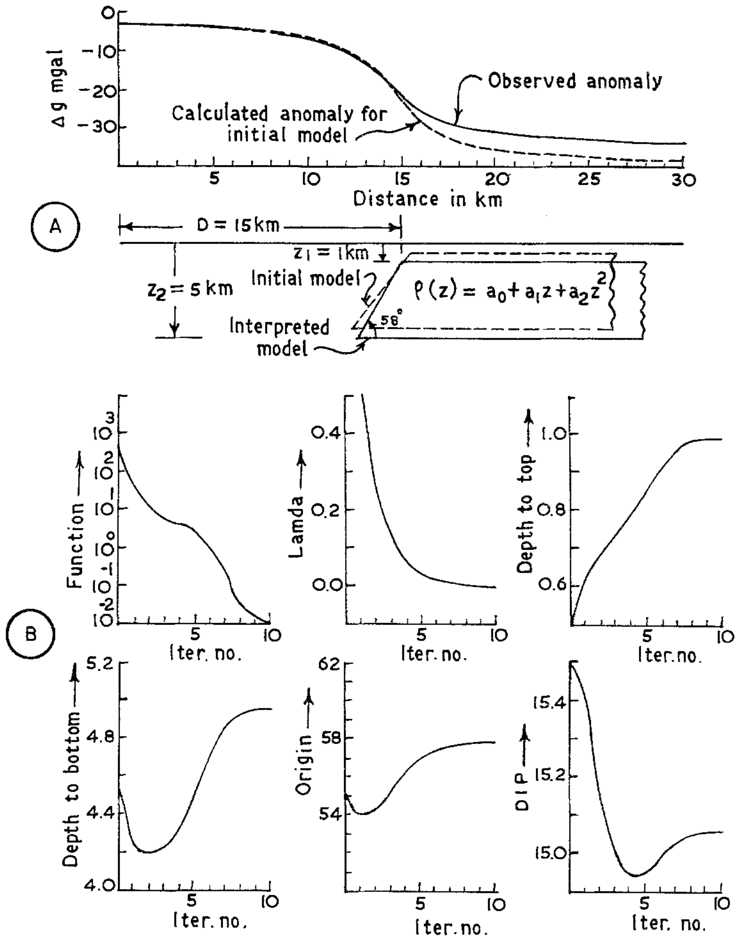

Figure 2
Interpretation of gravity anomalies over an inclined fault with quadratic density function.

dotted line in Fig. 2a. The convergence of the method is illustrated in Fig. 2b by plotting the values of the objective function, lamda, and the parameters Z_1, Z_2, i, and D with respect to iteration number. The values of various parameters at the 10th iteration are as follows: $Z_1 = 0.99$ Km, $Z_2 = 4.96$ Km, $i = 57.9°$, and $D = 15.06$ Km. The program is terminated at 10th iteration since the residuals are well below 0.01 mgals and the value of the objective function is 0.01. To study the correlation that exists between various parameters, the gravity anomalies are interpreted for different initial models. Though the interpreted values of Z_1 and Z_2 are nearly equal to the actual values in all the cases, the values of i and D are quite different from their actual values (Table 1). This shows that there is a strong correlation between the parameters i and D. This means that many values of i and D could satisfy the

Table 1

Interpreted models for different initial values of the parameters

S.No.	Model	Z_1	Z_2	i	D	*Function*	λ
1	Initial model	1.5	4.5	45.0	14.0	934.3	—
	Interpreted model	0.9506	4.835	51.25	15.26	0.15	0.0625
2	Initial model	1.5	4.5	45.0	16.0	1293.0	—
	Interpreted model	0.9160	4.727	47.36	15.39	0.3726	0.0078
3	Initial model	0.5	4.5	50.0	15.5	324.7	—
	Interpreted model	0.9652	4.881	53.95	15.17	0.08	0.0009
4	Initial model	0.5	4.5	55.0	15.5	301.9	—
	Interpreted model	0.9887	4.960	57.87	15.06	0.01	0.0009
5	Initial model	1.5	4.5	75.0	14.0	1044.0	—
	Interpreted model	1.050	5.207	78.67	14.54	0.55	0.0312
6	Initial model	1.5	4.5	75.0	16.0	1592.0	—
	Interpreted model	1.000	5.190	75.70	14.62	0.52	0.0039
7	Initial model	1.5	4.5	55.0	15.5	1254.0	—
	Interpreted model	0.9886	4.960	57.72	15.06	0.01	0.0313

Table 2

Interpreted models for different density contrasts

S.No.	Density function	Z_1	Z_2	i	D	*Function*	λ
1	$\Delta\rho(Z) = -0.4 + 0.08Z - 0.005Z^2$	0.9886	4.960	57.72	15.06	0.0104	0.03125
2	$\Delta\rho_0 = -0.325$	1.309	3.916	51.90	15.10	0.3369	0.0009
3	$\Delta\rho_0 = 0.225$	0.8088	4.574	62.09	15.04	0.0028	0.0009

interpretation within a given tolerance of error. Thus there is a trade-off between the errors and the interpreted values of the parameters i and D. Therefore, in the interpretation of a real anomaly, it is necessary to fix one of these parameters in order to deduce the other accurately.

The anomaly is again interpreted for different constant density contrast, and the values of the interpreted parameters are shown in Table 2. The same program is used to interpret the gravity anomaly with constant density contrast by assigning $a_1 = a_2 = 0$. Thus we can see from Table 2 that the interpretation of the anomaly profile with the density contrast at the top (-0.325 gm/cc) or with the average density contrast (-0.225 gm/cc) can not represent the true model, although the residuals are below 0.1 mgal in both the cases.

Discussion

A synthetic gravity anomaly profile over an inclined fault wherein the density contrast decreases with depth is interpreted. The decrease of density contrast in many sedimentary rocks could be satisfactorily approximated by a quadratic function. The anomaly equation could easily be programmed as it contains simple arctan and logarithmic terms. The method converges rapidly and gives accurate values for Z_1 and Z_2 as all the anomaly points are taken into account. Any reasonable values of Z_1 and Z_2 could be taken for the initial model as these are ultimately modified. However, the parameters i and D are correlated. Different interpretations could be arrived at for these parameters within a given tolerance of error. Thus, in the interpretation of real anomalies, it is necessary to fix one of these parameters in order to deduce the other accurately. We have also seen that different models could be deduced for different density contrasts. However, it is more advantageous to interpret the anomalies using the density function rather than using different densities. The density values known from bore wells or by seismic methods might be used to construct a quadratic density function. The decrease of density contrast in many sedimentary rocks could be approximated by a quadratic function, and the anomaly equation could be derived in closed form. The method proposed here is very useful to interpret the gravity anomalies over faulted sedimentary rocks where the densities of the sediments at different depths are reasonably known. Although it is not considered here, this method could be used even in the presence of first or second order regional components, as these can also be solved by the Marquardt algorithm. This program could also be used without any modification to interpret the gravity anomaly over outcropping faults by giving very small values to Z_1.

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