Pageoph
Vol. 110 (1973/IX) Birkhäuser Verlag, Basel

A Computer Program for Interpreting Vertical Magnetic Anomalies of Spheres and Horizontal Cylinders

By B. S. R. Rao, I. V. Radhakrishna Murthy and C. Visweswara Rao[1])

Summary – A common computer program for the interpretation of vertical magnetic anomalies of spheres and horizontal cylinders has been developed. The input consists of the observed anomalies noted against their distances measured from an arbitrary point in the profile and a code number for each model. The program is written so that the positions and magnitudes of the maximum and minimum anomalies are located and their ratios and signs are used to define the initial parameters of the model under consideration. The errors resulting from these approximate values are derived and are solved for the increments to be given to the initial values. The process is repeated until the sum of the squares of the errors is less than 0.25 % of the sum of the squares of the observed anomalies. The method has been tested on various theoretical examples and the results justify the validity of the programme.

Introduction

The interpretation of gravity and magnetic anomalies on modern digital computers has many advantages. Computerised techniques are therefore being developed in recent years for interpreting the anomalies of even simple geometric models. Bosum [1] has employed an iterative procedure for interpreting the magnetic anomalies of horizontal cylinders, thin sheets and infinite dykes. Hall [2] has also utilised a computer for interpreting the magnetic anomalies of dykes and faults. McGrath and Hood [3] have postulated a computer curve matching technique for interpreting the magnetic anomalies of a dipping dyke. The authors of the present paper have developed two computer oriented methods [4] for interpreting magnetic anomalies of a dyke which utilise all the observed anomalies and do not require the input of the approximate parameters. In this paper one of the methods has been extended to the vertical magnetic anomalies of spheres and horizontal cylinders. As the procedures to be followed in both the models were found to be essentially similar, a generalised computer program has been developed that can be used for both the models. The input here requires only a code number that defines the shape of the body and the observed anomalies against their distances measured from an arbitrary point in the profile.

Principle of the method

In this method, approximate values of the various parameters of the body are first determined by the computer itself from the shape characteristics of the observed profile.

[1]) Department of Geophysics, Andhra University, Waltair, India.

The anomalies resulting from these approximate values are then calculated and compared with their corresponding observed values. These errors are solved for the increments to be added to the initial approximate parameters of the model. The process is repeated until the sum of the squares of the errors is less than a specified minimum, say 0.25 % of the sum of the squares of the observed anomalies.

The vertical magnetic anomaly of a sphere polarised at any angle Q is given by

$$V_k = C\left\{\frac{[2Z^2 - (X_k - D)^2]\sin Q - 3(X_k - D)Z\cos Q}{[(X_k - D)^2 + Z^2]^{5/2}}\right\}, \tag{1}$$

where D is the distance of the origin, the point immediately above the centre of the sphere, measured from an arbitrary point in the profile, Z is the depth to the centre of

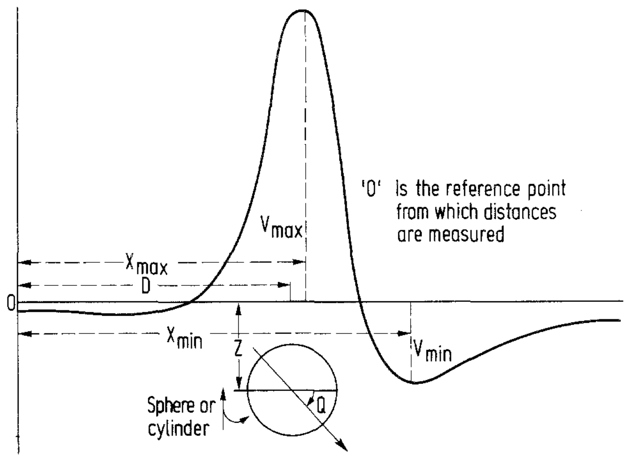


Figure 1
Parameters of the sphere or cylinder

the sphere, X is the distance of the point of observation from the arbitrary point (Fig. 1) and C is the size factor of the sphere given by

$$C = 4/3\pi R^3 I.$$

The corresponding expression for the vertical magnetic anomaly of a horizontal cylinder is given by

$$V_k = C\left\{\frac{[Z^2 - (X_k - D)^2]\sin Q - 2(X_k - D)Z\cos Q}{[(X_k - D)^2 + Z^2]^2}\right\} \tag{2}$$

where all the parameters carry their usual meanings as defined for the sphere except that the size factor C is identified as

$$C = 2\pi R^2 I.$$

We are expected to solve for C, D, Z and Q for completely defining the model. If C_1, D_1, Z_1 and Q_1 are the initial approximate values, the error in the anomaly dV at each point of observation is given by

$$\mathrm{d}V_k = \frac{\partial V_k}{\partial C_1} dC_1 + \frac{\partial V_k}{\partial D_1} dD_1 + \frac{\partial V_k}{\partial Z_1} dZ_1 + \frac{\partial V_k}{\partial Q_1} dQ_1, \tag{3}$$

where

$$\frac{\partial V_k}{\partial C_1} = \frac{[2Z^2 - (X_k - D)^2] \sin Q - 3(X_k - D) Z \cos Q}{[(X_k - D)^2 + Z^2]^{5/2}},$$

$$\frac{\partial V_k}{\partial D_1} = \frac{C}{[(X_k - D)^2 + Z^2]^{7/2}} \{[(X_k - D)^2 + Z^2] [3Z \cos Q + 2(X_k - D) \sin Q] + 5[(X_k - D)] [(2Z^2 - \overline{X_k - D}^2) \sin Q - 3(X_k - D) Z \cos Q]\},$$

$$\frac{\partial V_k}{\partial Z_1} = \frac{C}{[(X_k - D)^2 + Z^2]^{7/2}} \{[(X_k - D)^2 + Z^2] [4Z \sin Q - 3(X_k - D) \cos Q] - 5Z [(2Z^2 - \overline{X_k - D}^2)] \sin Q - 3(X_k - D) Z \cos Q]\},$$

$$\frac{\partial V_k}{\partial Q_1} = C \left\{ \frac{[2Z^2 - (X_k - D)^2] \cos Q + 3Z(X_k - D) \sin Q}{[(X_k - D)^2 + Z^2]^{5/2}} \right\}$$

for a sphere and

$$\frac{\partial V_k}{\partial C_1} = \frac{[Z^2 - (X_k - D)^2] \sin Q - 2(X_k - D) Z \cos Q}{[(X_k - D)^2 + Z^2]^2},$$

$$\frac{\partial V_k}{\partial D_1} = \{C/[Z^2 + (X_k - D)^2]^3\} \{[Z^2 + (X_k - D)^2] [2(X_k - D) \sin Q + 2Z \cos Q] + 4(X_k - D) [(Z^2 - \overline{X_k - D}^2) \sin Q - 2Z(X_k - D) \cos Q]\},$$

$$\frac{\partial V_k}{\partial Z_1} = \frac{C}{[Z^2 + (X_k - D)^2]^3} \{2[Z^2 + (X_k - D)^2] [Z \sin Q - (X_k - D) \cos Q] - 4Z [(Z^2 - \overline{X_k - D}^2) \sin Q - 2Z(X_k - D) \cos Q]\},$$

$$\frac{\partial V_k}{\partial Q_1} = C \left\{ \frac{[Z^2 - (X_k - D)^2] \cos Q + 2Z(X_k - D) \sin Q}{[Z^2 + (X_k - D)^2]^2} \right\}$$

for a horizontal cylinder.

Since as many equations of the type in (3) could be framed as the number of observations, four normal equations could be deduced from them and solved for dC_1,

dD_1, dZ_1 and dQ_1. The correct parameters of the model are now given by

$$C = C_1 + dC_1$$
$$D = D_1 + dD_1$$
$$Z = Z_1 + dZ_1$$
$$Q = Q_1 + dQ_1.$$

The process can be repeated until the sum of the squares of the errors is less than a specified minimum acceptable error.

Calculation of the initial parameters

The initial values for the two models can be found by the computer from the following rules:

1) The angle Q is determined from the ratio of the minimum and maximum anomalies as tabulated in Table 1(a).

Table 1(a)

Table for calculation of Q for a sphere

$\lvert V_{min}/V_{max}\rvert$	Angle Q
<0.05	90°
0.05–0.19	60°
0.19–0.8	30°
>0.8	0°

Table for calculation of Q for a cylinder

$\lvert V_{min}/V_{max}\rvert$	Angle Q
<0.19	90°
0.19–0.43	60°
0.43–0.8	30°
>0.8	0°

Table 1(b)

Table for values of Q in correct quadrant

Sign of		Q
$X_{max} - X_{min}$	V_{max}	
−	−	$180 + Q$
−	+	Q
+	−	$360 - Q$
+	+	$180 - Q$

2) The depth to the centre of the sphere is exactly equal to the width, defined as the distance between the two half maximum anomaly points, irrespective of the direction of magnetisation (RADHAKRISHNA MURTHY [5]), whereas the depth to the centre of the cylinder is calculated by the formula

$$Z = |\text{width}|/F_Z,$$

where F_Z is 0.972, 1.026, 1.124 and 1.528 for Q equal to 90°, 60°, 30°, and 0° respectively.

3) The origin, the point immediately above the centre, is located by knowing the positions of the maximum and minimum anomalies, the depth of the model and the direction of magnetisation, as outlined in steps 1) and 2). The origin is always between the maximum and minimum anomaly points and is located at a distance of F_D from the maximum anomaly point in the direction of minimum anomaly. This is achieved by writing

$$D = X_{\max} - F_D \frac{|X_{\max} - X_{\min}|}{X_{\max} - X_{\min}},$$

where the values of F_D are constructed in Table 2 for different values of Q.

Table 2

Tables for the values of F_D for locating the origin

Angle	Value of F_D	
	Spheres	Cylinder
90°	0	0
60°	0.135	0.177
30°	0.300	0.364
0°	0.500	0.577

4) The angle Q is then located in the correct quadrant, testing the sign of $V_{\max}$ and $X_{\max} - X_{\min}$ as given in Table 1(b).

5) The approximate parameters, thus determined in steps 1) to 4), are used to derive the shape factor V', of the anomaly given by

$$V'_k = \{[2Z^2 - (X_k - D)^2] \sin Q - 3(X_k - D) Z \cos Q\} / [(X_k - D)^2 + Z^2]^{5/2}$$

for a sphere and

$$V'_k = \{[Z^2 - (X_k - D)^2] \sin Q - 2Z(X_k - D) \cos Q\}/[Z^2 + (X_k - D)^2]^2$$

for a cylinder and the approximate value C_1 of the size factor is calculated as

$$C_1 = \sum_{k=1}^{N} |V_k| \Big/ \sum_{k=1}^{N} |V'_k|.$$

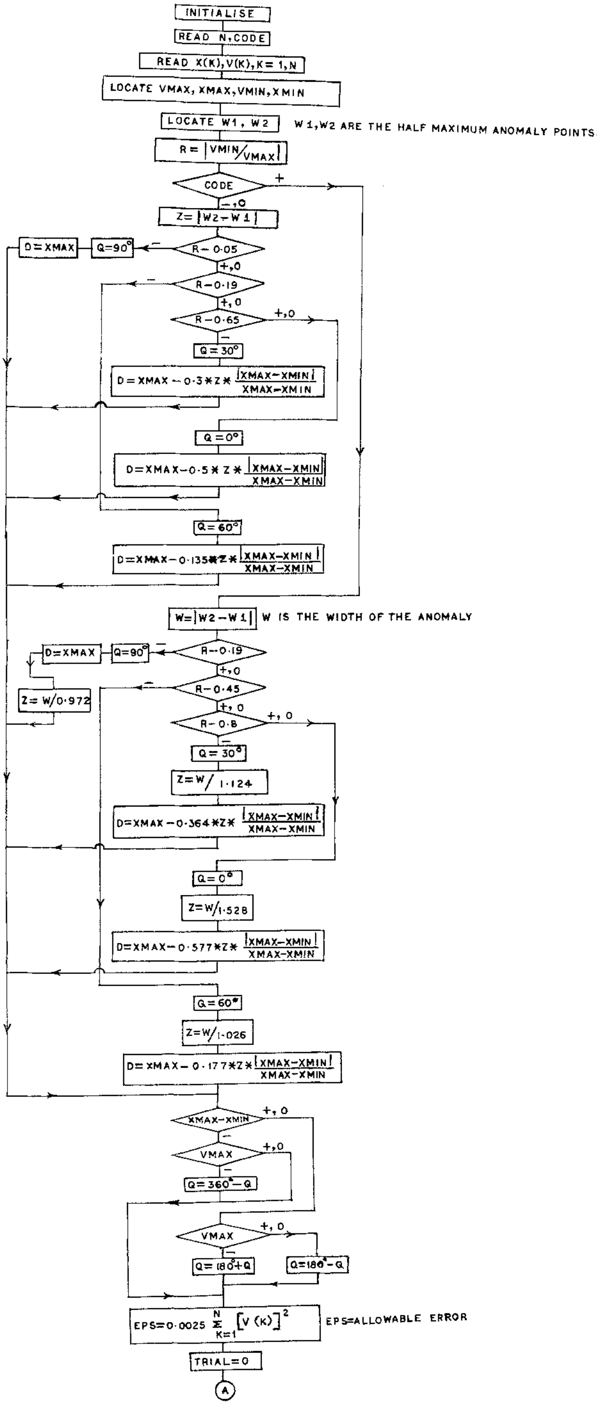


Figure 2
Generalised flow-chart for the method of interpretation

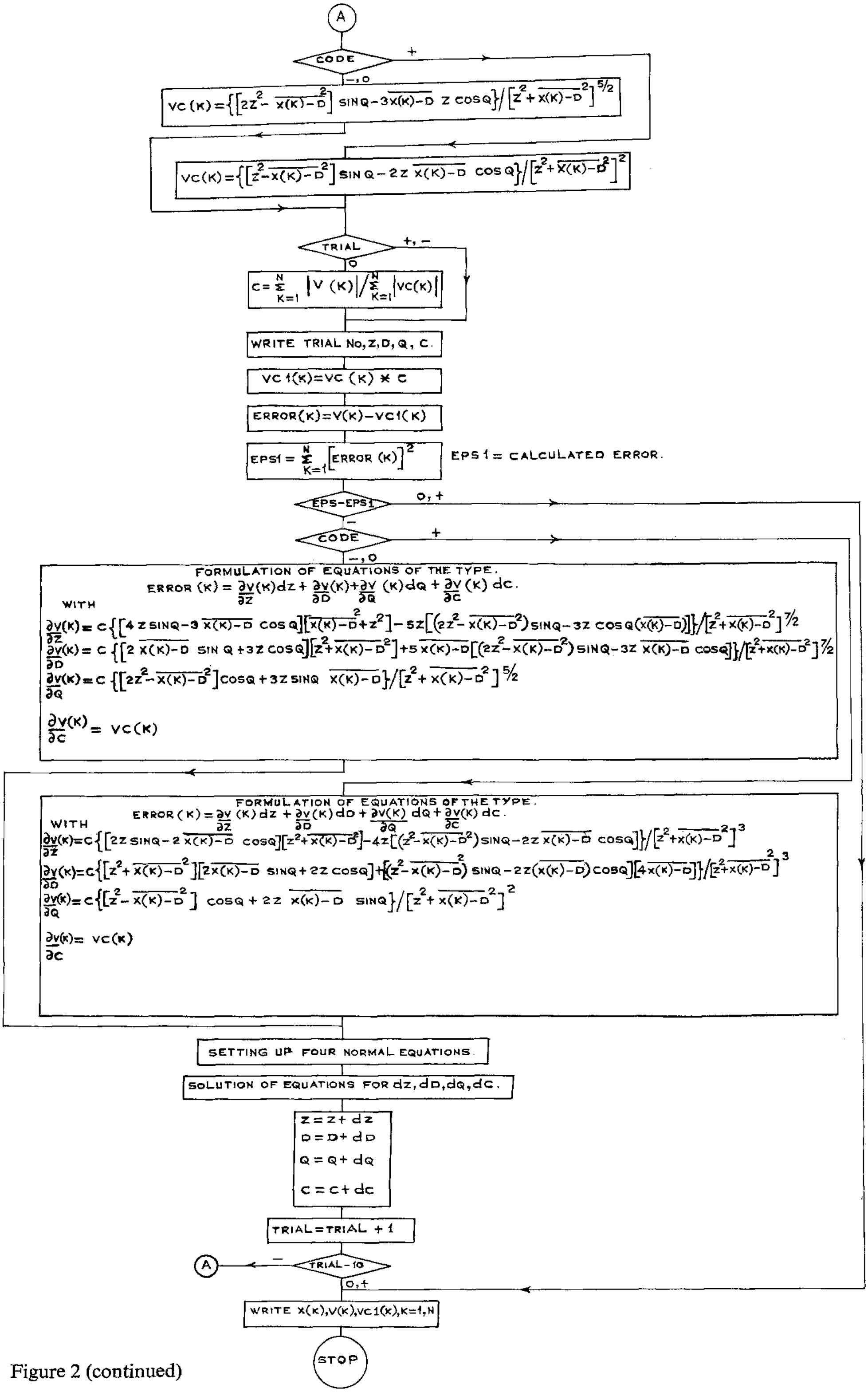


Figure 2 (continued)

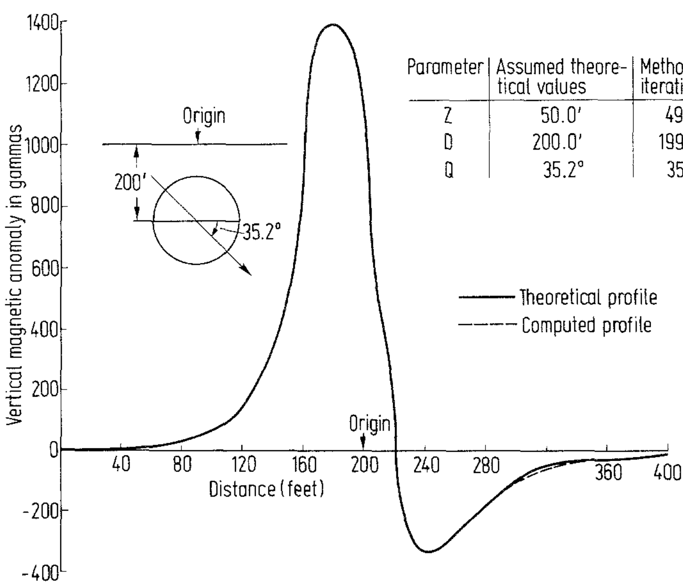
The computer program

All steps listed for calculating the initial values and their iteration are easily computerised. Also most operations for spheres and cylinders are similar, and hence a common program that could be used for both the models can be developed by ascertaining a code number to each model. The code that has been used in the present investigation is zero for a sphere and any positive number for a cylinder. The main features of the computer program to be developed for this purpose are illustrated in the flow chart in Fig. 2.

The magnetic anomalies of both the models will exhibit two minima, one on each side of a single maximum. The two minima are equal in magnitude only when the polarisation angle is 90° or 270°. In the various steps outlined above, the minimum has been defined as that having a higher magnitude in the two located or that existing to the right of the maximum anomaly when they are equal.

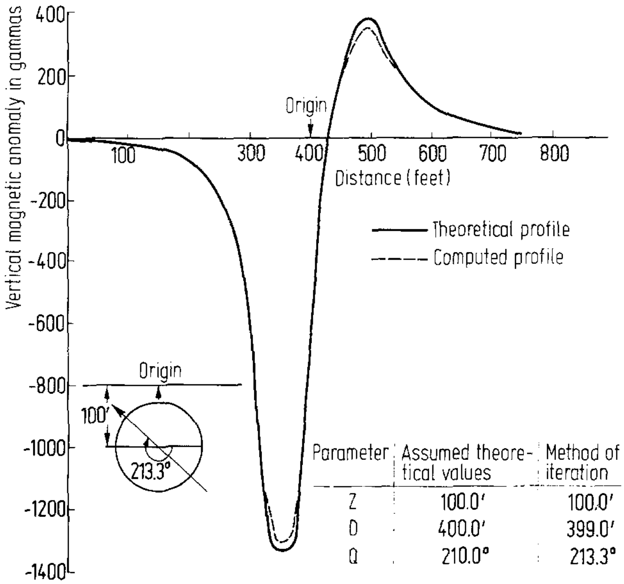
Tested examples

The proposed method has been programed to an IBM 1130 computer available in the Andhra University, Waltair (India). Successful interpretation of magnetic anomalies over spheres and cylinders with different directions of magnetisation have been worked out and four examples are presented in Figs. 3 to 6. The directions of magnetisations



Parameter	Assumed theoretical values	Method of iteration
Z	50.0′	49.7′
D	200.0′	199.9′
Q	35.2°	35.2°

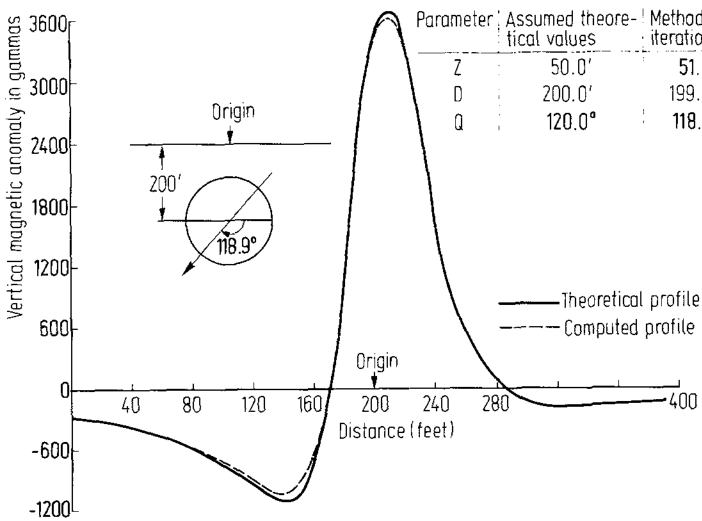
Figure 3
Solution of a theoretical profile over a sphere with direction of magnetisation 35° 12′



Parameter	Assumed theoretical values	Method of iteration
Z	100.0′	100.0′
D	400.0′	399.0′
Q	210.0°	213.3°

Figure 4

Solution of a theoretical profile over a sphere with direction of magnetisation 210°



Parameter	Assumed theoretical values	Method of iteration
Z	50.0′	51.6′
D	200.0′	199.9′
Q	120.0°	118.9°

Figure 5

Solution of a theoretical profile over a horizontal cylinder with direction of magnetisation 120°

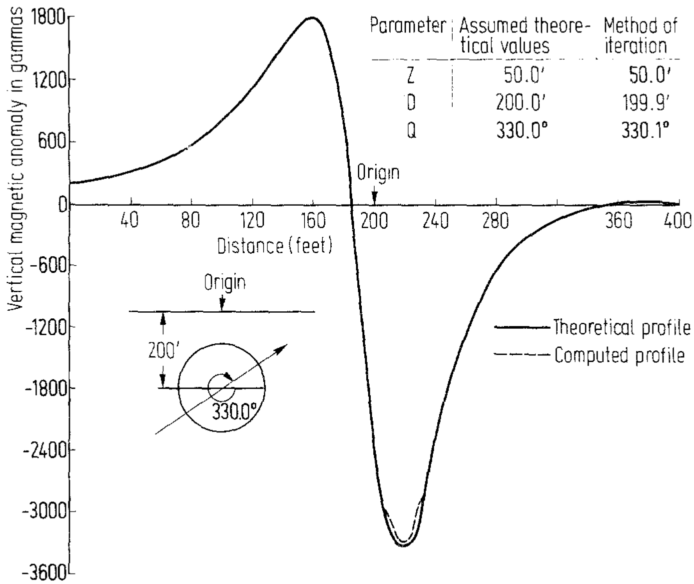


Figure 6
Solution of a theoretical profile over a horizontal cylinder with direction magnetisation 330°

selected vary widely from case to case. The computer-calculated curves in the final iteration along with the interpreted parameters are also presented in the figures. The accuracy of the interpreted values and the close fit of the calculated profile with the theoretical one establish the validity of the proposed method.

References

[1] W. Bosum, *Ein automatisches Verfahren zur Interpretation magnetischer Anomaliem nach der Methode der kleinsten Quadrate*, Geophys. Prosp. *16* (1968), 107–126.

[2] D. H. Hall, *A magnetic interpretation method for calculating body parameters for buried sloping steps and thick sheets*, Geoexploration *6* (1968), 187–206.

[3] P. H. McGrath and P. J. Hood, *The dipping dyke case—A computer curve-matching method of magnetic interpretation*, Geophysics *35* (1970), 831–848.

[4] B. S. R. Rao, I. V. Radhakrishna Murthy and C. Visweswara Rao, *Two methods for computer interpretation of magnetic anomalies of dykes*, Geophysics (1973). In Press.

[5] I. V. Radhakrishna Murthy, *Note on the interpretation of magnetic anomalies of spheres*, J. Ind. Geophys. Union *4* (1967), 41–42.

(Received 23rd November 1972)