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Array Analysis of Seismic Surface Waves: Limits and Possibilities

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Summary – The dispersion of higher modes of surface waves over a region covered by an array of stations can be measured by applying a frequency-wavenumber transform to segments of the signals in these stations, centered at a number of group-velocities. Thus, at a fixed period, modes appear as separate maxima in a display of the power spectrum on a phase-velocity vs. group velocity plane.

For regions of mild lateral heterogeneïty, the main source of error is shown to be due to the inadequacy of the array-response. Numerical experiments with synthetic signals indicate that a precision of the order of 1% can be obtained with a realistic number of stations. Improvements should be obtained by averaging data obtained from different events. The danger of misidentification of modes can be reduced by iteratively removing the sidelobes from the mode-separation diagram.

1. Introduction

In particular since the introduction of the Fast Fourier Transform (FFT) by COOLEY and TUKEY (1965), a number of methods for the analysis of surface-wave recordings in one or two stations have been developed. An excellent survey of these methods is given by DZIEWONSKI and HALES (1972). Because surface waves do not suffer from the fundamental non-uniqueness which prevents the inversion of body-wave data to unambiguous models with a low-velocity zone, they are superior to the latter in fixing the shear velocity-depth distribution in the upper 400 km of the earth's mantle.

The observational material has mainly been limited to the fundamental modes of Love and Rayleigh waves, but recently several attempts to measure clearly identified higher modes have been successful: NOLET (1975) employs a stacking technique with an array of standard long-period stations to obtain the average phase-velocities of a number of higher Rayleigh modes over Western Europe in the period range of 20–100 sec, while FORSYTH (1975) uses several events and several stations to determine those phase velocities of the fundamental and first higher Love mode that best fit the

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observed phases in a least-squares sense over an oceanic path. Earlier, MENDIGUREN (1973) and GILBERT and DZIEWONSKI (1975) used the WWSSN stations as a network to identify higher modes in the period range of the earth's eigenfunctions ($T > 80$ sec).

In this paper we describe the stacking method and analyse its resolving power by means of several numerical experiments with synthetic seismograms for different station configurations.

2. *The stacking method*

If $w(\Delta, \omega)$ denotes the spectrum of the signal $s(\Delta, t)$ in a station at Δ km from the source, then the complex wavenumber spectrum is given by a Fourier relation

$$W(k, \omega) = \int_{-\infty}^{+\infty} w(\Delta, \omega) e^{-ik\Delta} d\Delta \tag{1}$$

where k is the wavenumber and ω the circular frequency. Since we prefer to obtain a spectrum with positive wavenumbers for waves travelling in the positive Δ-direction we take the sign in the exponent of the $\Delta \to k$ transform negative, i.e. opposite to the sign in the $t \to \omega$ transform. We assume that we have N recordings in stations located at epicentral distances Δ_j ($j = 1, \ldots, N$). As the integrand of (1) is only known at a finite number of points along the Δ-axis, we cannot calculate $W(k, \omega)$ directly but will have to make an estimation:

$$\hat{W}(k, \omega) = \frac{1}{N} \sum_j w(\Delta_j, \omega) e^{-ik\Delta_j}. \tag{2}$$

The estimate (2) has the properties that it satisfies the inverse transform of (1) and that it would be an optimum estimate in a least-squares sense if we could assume that the values of $w(\Delta, \omega)$ are uncorrelated over space. Of course this is no reasonable assumption since we *know* from the theory of surface waves that the signal is a transient that can be described as a discrete sum of modes, each with their own finite wavenumber k_n at a frequency ω. But the estimate (2), which we will call the 'dirty stack' has certain advantages: it poses no computational problems and it is not critically dependent on the magnification of the seismograph, which is often poorly known. An alternative estimate would be the high resolution spectrum of CAPON (1970), but with the possible exception of the fundamental mode, the signal lengths necessary for the application of this method are not available. For these reasons we will analyse in this paper the performance of the method using (2). In section 5 we will discuss how we can use our *a priori* information to make corrections to (2). For clarity of notation, we will henceforth omit the $\wedge$-sign on W, it being tacitly understood that W is only an estimate of the true wavenumber-spectrum. Compared with the difficulties encountered in the $\Delta \to k$ transformation the transformation $t \to \omega$ is relatively easy. We use a FFT-technique, i.e. we assume that $s(t, \Delta)$ is repeated periodically over those values of t unknown to us.

The spectrum of a component of the signal in station j can be represented by

$$w(\Delta_j, \omega) = \sum_n F_{nj}(\omega) \exp \{i[k_n(\omega)\Delta_j + \phi_{nj}(\omega)]\} \tag{3}$$

where n is the mode number, which is 0 for the fundamental mode, $F_{nj}(\omega)$ the amplitude spectrum in station j and $k_n(\omega)$ the wavenumber of mode n. k_n is related to the angular order l by:

$$k_n = \frac{l + \frac{1}{2}}{a} \tag{4}$$

where a is the earth's radius, 6371 km. $\phi_{nj}(\omega)$ is the initial phase, which depends on the azimuth from the source. However, if the stations do not differ considerably in azimuth and if the array is not situated near a minimum in the radiation pattern we may neglect the variations of $\phi_{nj}(\omega)$ in the different stations (see PANZA *et al.*, 1975*a*,*b*). We will also assume that the effect of dissipation over the array is small. If necessary, we can correct for the geometrical spreading factor $\sin^{-1/2} (\Delta/a)$. Thus, we will remove the subscript j from F and ϕ. Inserting (3) into the dirty stack (2) we find:

$$W(k, \omega) = \frac{1}{N} \sum_n F_n(\omega) \exp \{i\phi_n(\omega)\} \sum_j \exp \{i[k_n(\omega) - k]\Delta_j\}. \tag{5}$$

With

$$H(k) = \frac{1}{N} \sum_j \exp (ik\Delta_j)$$

this becomes

$$W(k, \omega) = \sum_n F_n(\omega) \exp \{i\phi_n(\omega)\} H\{k_n(\omega) - k\}. \tag{6}$$

$|H(k)|^2$ is called the array response. It has a maximum of 1 in $k = 0$. If it is negligible outside this maximum, $|W(k, \omega)|^2$ will have maxima in curves (k, ω) that are parametrized by the dispersion relation $\omega = \omega_n(k)$. However, if the number of stations is small, the array response will have large sidelobes close to the main lobe that can disturb this simple picture. In that case the maximum of $|W(k, \omega)|^2$ will only coincide with the dispersion curves $\omega_n(k)$ if the number of modes that is present in the signal is small. This problem can be evaded by analysing only a segment of the signal around an arrival time τ_j determined by a chosen group velocity v_g:

$$\tau_j = \Delta_j / v_g$$

If the segment is not too long, modes with a group velocity that is markedly different from v_g will have very little energy in the segment. We do this for a series of group-velocities v_g. If we call the window function $B_j(t, v_g)$ the spectrum of the seismogram is

$$w(\Delta_j, \omega, v_g) = \int_{-\infty}^{+\infty} s(\Delta_j, t) B_j(t, v_g) e^{i\omega t} dt \tag{7}$$

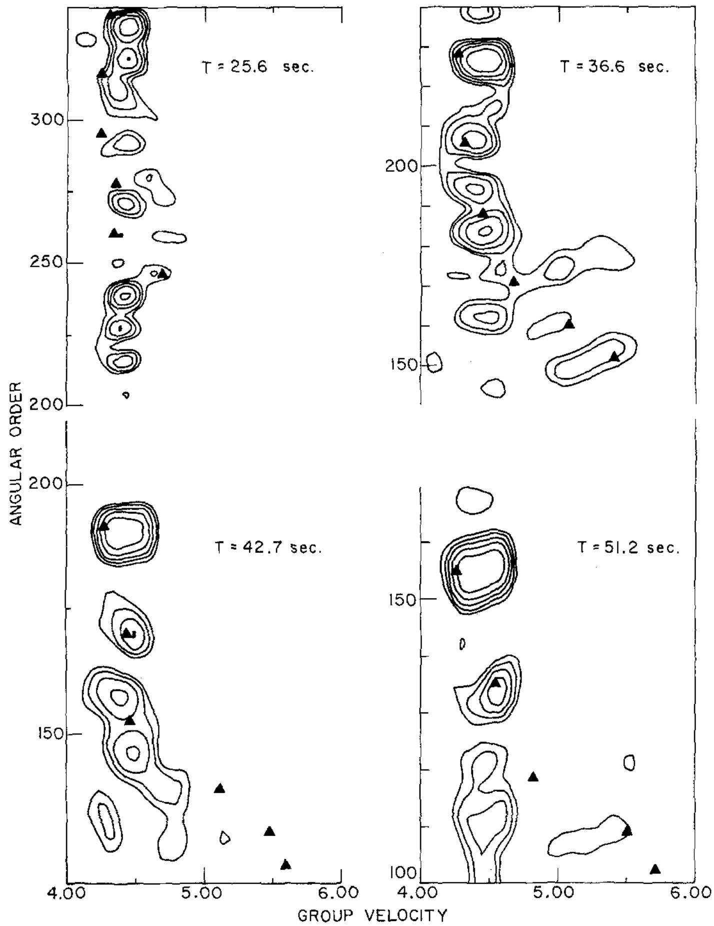

Figure 1

Mode-separation diagrams for the event at 25 october 1965, origin time $22^h34^m22^s$. Coordinates 44.21N, 145.45E, depth 159 km. These diagrams were constructed from the recordings of 9 stations in W. Europe. Black triangles denote calculated locations of higher modes for a reference mode (Gutenberg, continental) and are used for identification. The first higher mode is at the top, the others follow in order of decreasing angular order. The group velocity interval does not include the fundamental mode. The contour interval is 1 dB.

so that the stack now depends on the group velocity:

$$W(k, \omega, v_g) \leftrightarrow w(\Delta, \omega, v_g).$$

Writing a computer program, one has to choose 2 variables for a diagram of $|W|^2$. If one prints (ω, k)-diagrams for several values of v_g, the computations are very efficient but one is left to the tedious job of looking over all the diagrams and comparing to find the maximum value. It is easier to print (v_g, k) diagrams for several values of ω, as was done by CARA (1976). An example is shown in Fig. 1. We will call a diagram as in Fig. 1 a mode-separation diagram.

3. *Programming considerations*

In the present version of the program, immediately upon being read in, the digitisations are low-pass filtered using the FFT, corrected for instrument response and decimated, usually to a time interval of 8 seconds. Thus, the repeated FFT for each recording and each v_g that is necessary to calculate $w(\Delta_j, \omega, v_g)$ is done over very few points in time.

Two FFT's can be done in one by storing a second segment in the imaginary part of the array and splitting the transformed spectrum in an even and uneven part (see GOLD and RADER, 1969, p. 198).

The stack (2) is calculated for equidistant values of k so that the calculation of the complex exponent $\exp(-ik\Delta_j)$ can be replaced by a complex multiplication with $\exp(-i\delta k\Delta_j)$ which has to be calculated only once.

δk can be larger than the required accuracy since the program searches the diagram for significant maxima in the neighbourhood of specified theoretical values and finds the exact location of $k_n(\omega)$ by quadratic interpolation. As an example, to analyse the dispersion at 7 different frequencies for 41 values of v_g in a stack of 9 stations, it took one minute of computing time on a CDC 6400.

4. *Sources of errors*

Using arrays of about 10 stations in Western Europe at epicentral distances of 7000–10500 km from sources in the Far East, NOLET (1975) obtained results showing a scatter of 1–2% in the higher mode phase velocities.

In this section we will investigate the origins of this scatter both in a qualitative and

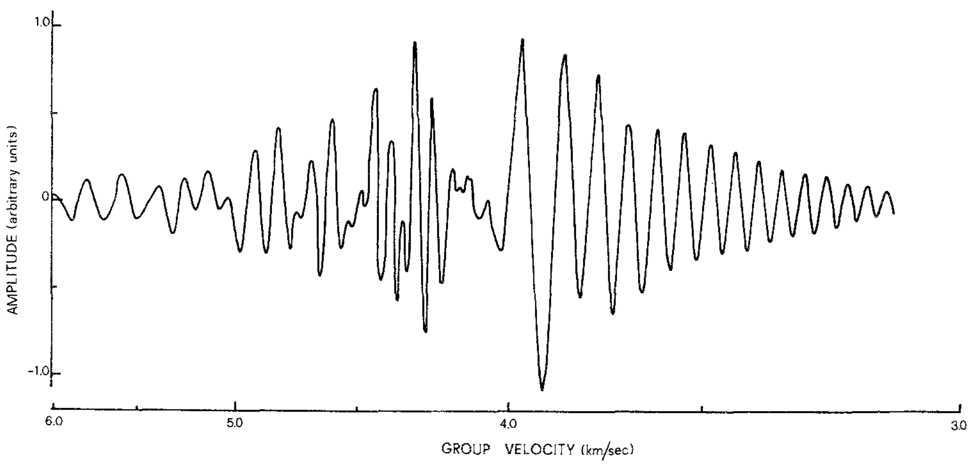

Figure 2
The synthetic seismogram that was used for the numerical tests. For details see text.

in a quantitative way. For numerical tests we make use of a synthetic seismogram, consisting of the vertical component of the fundamental and 7 higher Rayleigh modes in the period range from 9–180 seconds. The dispersion of these modes was calculated using a spherical Gutenberg-Bullen-A continental model. The excitations were calculated for a source (approximately dip–slip) at a depth of 135 km. Dissipation was calculated and taken into account. The resulting signal is shown in Fig. 2 as it would appear at an epicentral distance of 9428 km.

There are a number of sources of error that are not directly related to the method, such as digitisation errors, errors in the source location and imprecise instrument phase-corrections. Of these the latter are often the largest. If we may assume these errors to be independent their influence will decrease as $N^{-1/2}$ if we stack over N stations. Thus, we can reduce the scatter that is usually obtained in measurements of fundamental mode dispersion using the two-station method without having to take resort to heavy smoothing over the frequencies.

A second class of errors is due to the approximations in the theory. First of all, since eq. (3) arises from an asymptotic representation of a Legendre polynomial for $\Delta \gg 0$, the array should not be located too close to the source or antipode for eq. (3) to be valid. Panza *et al.* (1973) obtained the condition $k\Delta > 10$ for a 3-figure accuracy in the phases. The variation of the initial phase with azimuth is usually very small if the array is not situated near a minimum in the radiation pattern (Panza *et al.*, 1975a, b). For a regular distribution of stations over a small range of azimuths the error to be expected from the initial phase is much smaller than the other sources of error and we will not attempt to correct for it. The same applies to the phase errors due to truncating the record to a segment around τ_j. Numerous experiments with different window-sizes and shapes for the synthetic seismogram showed a phase velocity scatter of 0.03 km/sec for the 5th higher mode and usually less. In practice we use rectangular windows with a length of 2 to 5 times the period so that two different window sizes can cover at least 2 octaves in the period range of interest. A possible disturbing influence of body waves (core phases) can be minimized by a careful selection of the event type and the distance range.

Usually the largest source of errors is due to the interference of several modes of roughly equal group velocity combined with an inadequate array response. It is obviously important to know what can and cannot be achieved with a rather incomplete coverage of the region with recording stations, not only because it enables us to evaluate the quality of the measurements but also in connection with the planning and installation of new stations.

Two properties of the array response function influence the resulting stack. The width of the main lobe, largely governed by the length of the array, determines how well two or more modes, close in wavenumber, can be separated. Large sidelobes, which occur when the number of stations is small, may introduce such interference that the maxima in the mode-separation diagram are shifted from the true wavenumber $k(\omega)$. In bad cases this may give such a confusion that the main lobes cannot

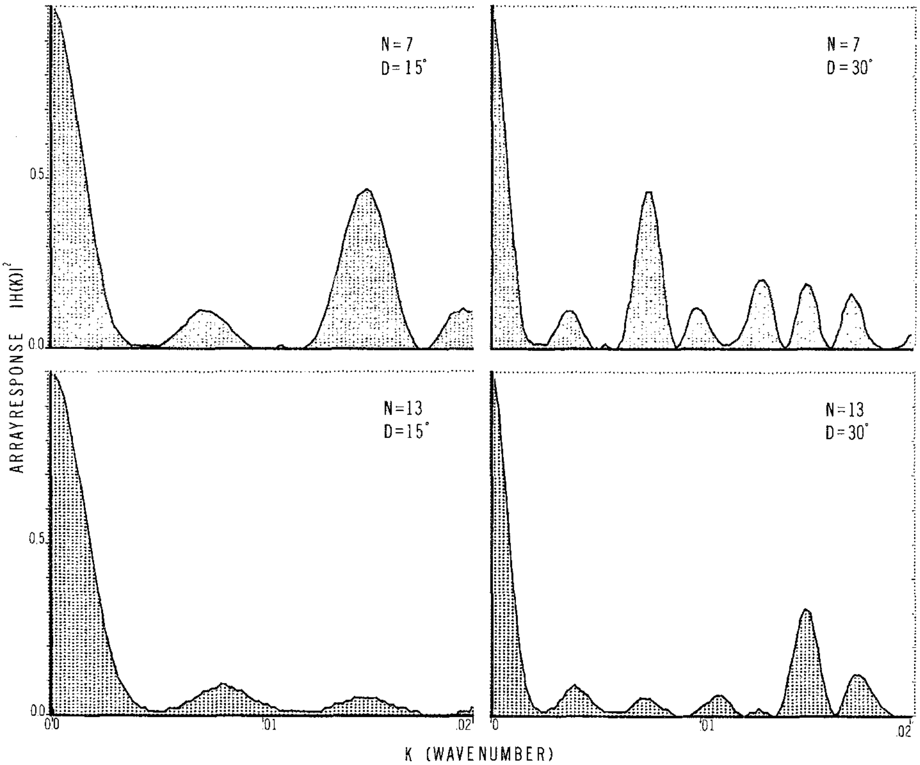

Figure 3
Array responses for different values of the number of stations (N) and the array span (D). The station distribution is given by eq. (8) in the text.

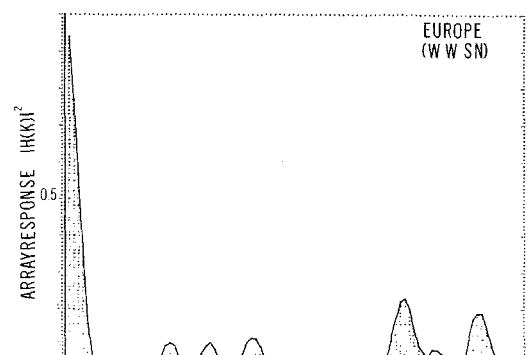

Figure 4
Array response of a network consisting of all WWSSN stations in Europe, as 'seen' by a wavetrain arriving from N. Kurile.

be distinguished clearly from the interfering lobes. Examples of array responses are shown in Figs. 3 and 4. In Fig. 3 we show the array responses for 4 imaginary arrays of N stations spanning a distance D. The distance of the i-th station from the epicentre was taken to be:

$$\Delta_i = \Delta_0 \pm \tfrac{1}{2}D\left\{1 - \cos\left(\frac{i-1}{N-1}\pi\right)\right\} \qquad i = 1, \ldots, (N+1)/2. \tag{8}$$

(Thus, there is a slight concentration towards the centre of the array). For example, at a period of 30 sec the first and second higher Rayleigh mode will be separated by 0.003 cycles/km, which is well outside the main lobe in all cases so that these modes will easily be separated. However, for the 30° array with $N = 7$ we expect interference of the first with the third mode, since their separation of 0.0064 cycles/km coincides with the large sidelobe and these modes have roughly equal group velocity at this period. With 13 stations we do not expect any trouble. The resolution that can be obtained using all available WWSSN-stations in Europe is shown in Fig. 4 for a great-circle path with an azimuth of 30°.

We used the array configuration given by (8) for a test of the method. D was taken 30°. To the synthetic seismogram of Fig. 2 we added white noise with an amplitude of 1% of the maximum to account for noise and digitising errors. For this signal we tested the method with several values of N and the average distance Δ to the source. Some of

Table 1

Higher mode dispersion measurements, corresponding to the diagrams in Fig. 5c–f.

	phase velocities					group velocities				
mode nr	theory	N = 13	error %	N = 7	error %	theory	N = 13	error %	N = 7	error %
Period 32.0 sec										
1	4.72	4.73	0.2	4.67	1.0	4.29	4.33	0.9	4.33	0.9
2	5.16	5.15	0.2	5.16	0.0	4.26	4.28	0.5	4.27	0.2
3	5.60	5.65	0.9	5.68	1.4	4.43	4.32	2.5	4.32	2.5
4	6.08	6.10	0.3	6.11	0.5	4.38	4.40	0.5	4.39	0.2
5	6.52	6.58	0.9	6.56	0.6	4.80	4.93	2.7	4.90	2.1
Period 21.3 sec										
1	4.58	4.58	0.0	4.54	0.9	4.34	4.31	0.7	4.29	1.1
2	4.82	4.80	0.4	4.90	1.7	4.25	4.22	0.7	4.29	0.9
3	5.07	5.10	0.6	5.10	0.6	4.20	4.21	0.2	4.21	0.2
4	5.36	5.37	0.2	5.37	0.2	4.29	4.29	0.0	4.29	0.0
5	5.64	5.63	0.2	5.61	0.5	4.35	4.30	1.1	4.30	1.1
6	5.97	5.98	0.2	5.96	0.2	4.43	4.42	0.2	4.41	0.5
7	6.25	6.26	0.2	6.26	0.2	4.65	4.67	0.4	4.66	0.2

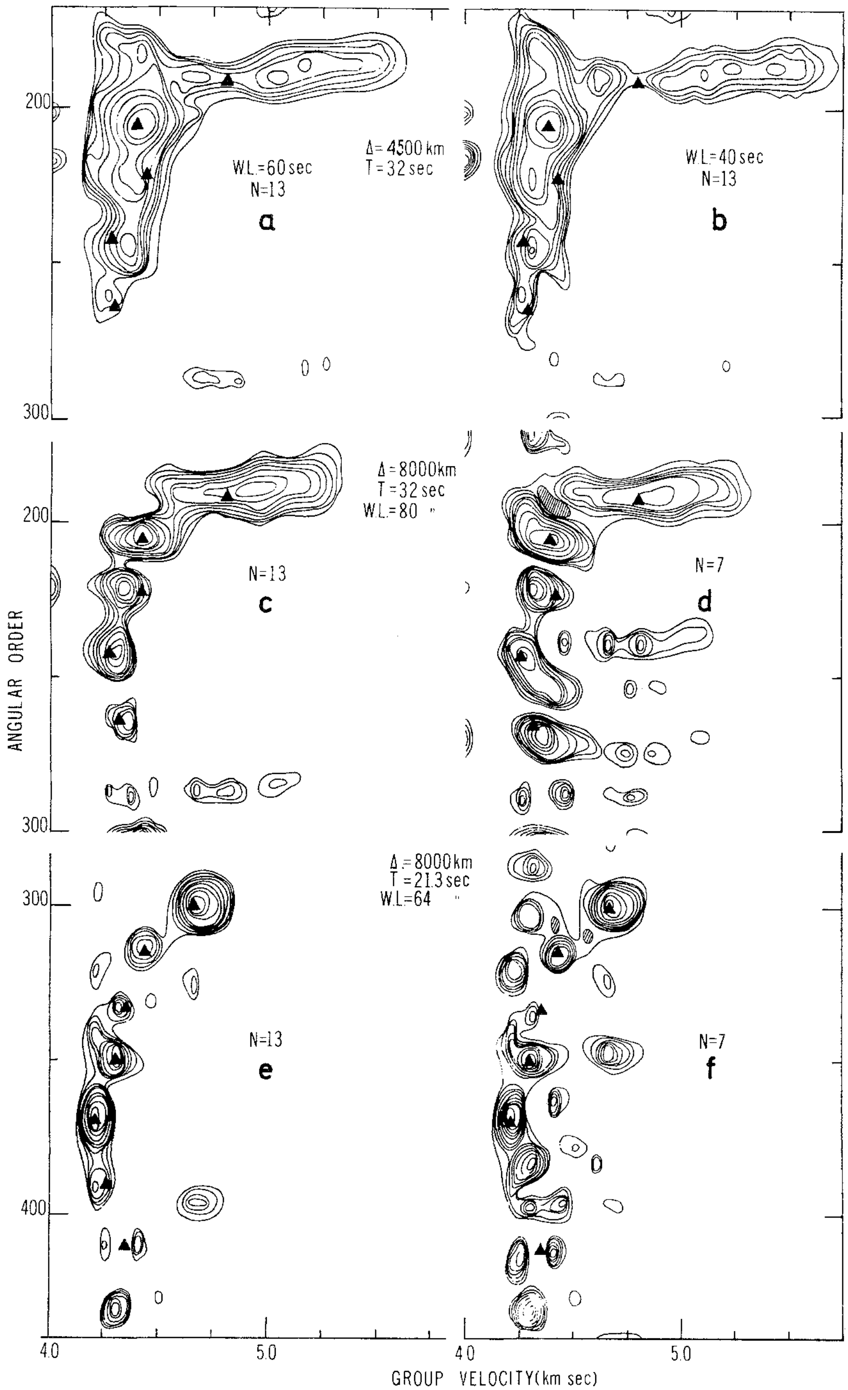

Figure 5

Mode-separation diagrams for an array with a span of 30° (3336 km) for different combinations of the period *T*, the number of stations *N* and the average distance from the epicentre △. Triangles denote the exact locations of the higher modes of the synthetic signal that is analysed. The contour interval is 1 dB. Note that the angular-order axis is reversed with respect to Figs. 1, 7, and 8. W.L. is the length of the time window used. For discussion see text.

the results are shown in Fig. 5 and Table 1. In Fig. 5 we show diagrams of $|W(k, \omega, v_g)|^2$ for fixed $\omega = 2\pi/T$. The group velocity interval is chosen so that only the higher modes are present in the picture. Contour intervals are 1 dB, the theoretical values are given by black triangles and should coincide with the maxima of the contour-pattern. The first higher Rayleigh mode has the highest angular order $l = ak - \frac{1}{2}$.

At a period of 32 seconds (Fig. 5a–d) we obtain much better results for an average Δ of 8000 km than with 4500 km, even though we work here with very short lengths of the (rectangular) time window. Obviously the small differences in group velocity can be better exploited to reduce the number of modes within the window when Δ increases. If we decrease the window size to 40 sec (Fig. 5b) the modes are better separated but the precision in the group velocity becomes worse. In general the group velocities become more accurate as the window size increases (LANDISMAN *et al.*, 1969) but our experience is that group velocity measurements with the stacking method should be viewed with caution since even in the presence of slight interference the maximum in the instantaneous amplitude may shift or break up into several maxima, each at the same value of the wavenumber. See for example the third higher mode in Fig. 5a–d or the first higher mode in Fig. 5e–f. A mild smoothing of $|W|^2$ for neighbouring values of v_g sometimes gives a small improvement in the precision of v_g. Reducing the number of stations from 13 to 7 greatly changes the picture by the introduction of side-maxima which could be wrongly interpreted. The main lobes are only slightly shifted, however. The danger of using too few stations lies primarily in the misidentification of sidelobes as modes.

Amplitudes of $|W|^2$ seem to be more influenced by the degree of interference than by the energy of the mode. Therefore, amplitude measurements with the stacking method and a realistic number of stations are not reliable.

The precision with which the phase velocities can be measured largely depends on the span of the array in relation to the wavenumber-interval under study. A rough measure of the resolving power of an array is given by the parameter:

$$\chi = 2\pi/D$$

where D is the span of the array. If χ is about one half of the difference in wavenumber between the modes, as is the case in Fig. 5, we may conclude that the errors in the phase-velocity due to the data-processing method do not exceed 1% in the case of a dense array ($N = 13$) or about 2% for a thin array ($N = 7$) and are usually much less (Table 1).

A last source of error is due to the fact that we have assumed that the earth is isotropic and laterally homogeneous. There is evidence for anisotropy just beneath the Moho-discontinuity, as was demonstrated by CRAMPIN (1967) and others. Sharp discontinuities like ocean-continent boundaries or mountain roots will give rise to multipathing. For periods larger than 10 sec, however, we expect that these disturbing effects will be mainly limited to the fundamental mode. The reason for this is that the

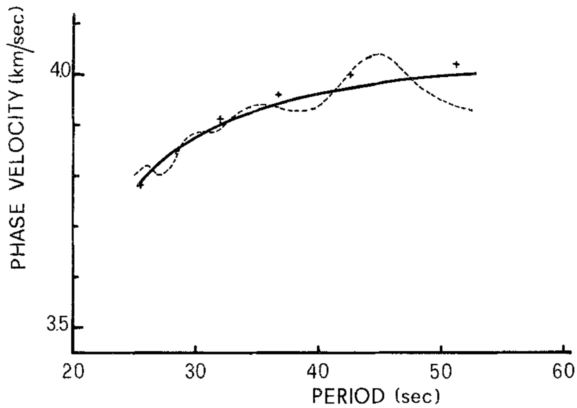

Figure 6
Analysis of a synthetic multipathed signal with the stacking method (crosses) and with the two-station method (dotted line). For details see text.

energy of the higher modes will reach much deeper, at least to the bottom of the low velocity channel if that is present. If severe heterogeneity extends below the lithosphere, the method will not work even for higher modes and the mode-separation diagrams will become blurred.

Since we assume in eq. (2) that the modes travel along a great-circle path to each of the stations the summation will never be optimal for refracted arrivals and the stacking method tends to enhance the direct arrivals relative to the (unwanted) multipathed signals, provided the array is not one-dimensional. We have tested this for the fundamental mode. To simulate a multipathed arrival we have superposed on the synthetic seismogram, calculated for a source located at 45°N, 150°E, a second signal with the same characteristics and an amplitude 20% of the first, arriving from a source at 15°N, 115°E and analysed the recordings for a small array consisting of 8 existing LP-stations in Italy. The average Δ is 9400 km, while the refracted path is on the average 500 km longer and has an azimuth deviation of 45 degrees. In view of the ray-path solutions obtained by Bungum and Capon (1974) this simulation is not unrealistic for the fundamental mode at periods of 20–40 sec. We note here that our synthetic signal is rather unfavourable for this analysis since the amplitude at low periods is rather small and grows strongly with increasing period. The errors in the results (Fig. 6) are thus on the large side. The interference disturbs the phases, such that a phase velocity analysis with the two station method would give grossly wrong results, even if we remove the higher modes from the signal, and one has to apply a heavy smoothing to obtain a reasonable result. The array analysis of this signal gives a maximum error of 0.025 km/sec or 0.6%. We conclude that small, local arrays like the Italian LP-network can be used to determine an accurate average phase velocity of the fundamental mode under the region.

5. *The cleaning algorithm*

As we mentioned in the last section, one source of possibly large errors comes from the misidentification of sidelobes as modes. As can be seen in eq. (6), the wavenumber spectrum consists of a sum of weighted and shifted array responses $H(k)$. If we fix ω and v_g, that is if we look at one 'column' in the mode-separation diagram, we can write

$$W_0(k) = \sum_n a_n H(k_n - k) \tag{9}$$

where a_n is an unknown complex amplitude. We will now try to incorporate our *a priori* knowledge that (9) is a sum over only a few terms. Since (9) is not a linear equation in k_n and since it is only an approximation to the real situation, we have to be careful with any attempt to deconvolve it.

An approach to this kind of problem is the stripping process (MACK and SMART, 1972), and a similar but slightly more elaborate scheme is used with much success in radio astronomy (HÖGBOM, 1974), where it has been labeled 'cleaning'. The complex extension of this method, that we use to remove sidelobes from the mode-separation diagram, is best described by an outline of the algorithm. The method is based on the assumption that the largest amplitude of $|W_0(k)|$ is no sidelobe but coincides with a real mode. The algorithm then proceeds as follows (we denote the iteration number with i):

(a) Find $k = k^i_{max}$ for which $|W_i(k)|$ has its maximum
(b) Subtract $W_{i+1}(k) = W_i(k) - \gamma W_i(k^i_{max}) \cdot H(k^i_{max} - k)$ where $0 < \gamma \leq 1$
(c) Store $W_i(k^i_{max})$ and k^i_{max}
(d) *If* $\int |W_{i+1}(k)| dk > \epsilon \cdot \int |W_0(k)| dk$ *go back to* (a) *else*
(e) Sum back $W(k) = W_{i+1}(k) + \gamma \sum_{l=0}^{i} W_l(k^l_{max}) \cdot \mathscr{H}(k^l_{max} - k)$

Here $0 < \epsilon \ll 1$ and $\mathscr{H}(k)$ is the main lobe of the array response $H(k)$. The loop gain γ need not be chosen too small. Clearly, if only one mode is present, $\gamma = 1$ would be most efficient. In practice we usually choose $\gamma = 0.4$. The iteration is performed separately for every value of v_g in the diagram. We work with a rather coarse grid δk, and since Re $\{H(k)\}$ and Im $\{H(k)\}$ contains oscillations with 'frequency' Δ_j, we have to be careful not to lose all precision if k_n is in between two grid points. Define:

$$W_0'(k) = e^{ik\Delta_0} W_0(k) \tag{10}$$

where Δ_0 is the average Δ of the array. From (10) we see that:

$$|W_0'(k)|^2 = |W_0(k)|^2$$

so that the mode-separation diagram does not change. But if we now apply the stacking method to calculate $W_0'(k)$ we find:

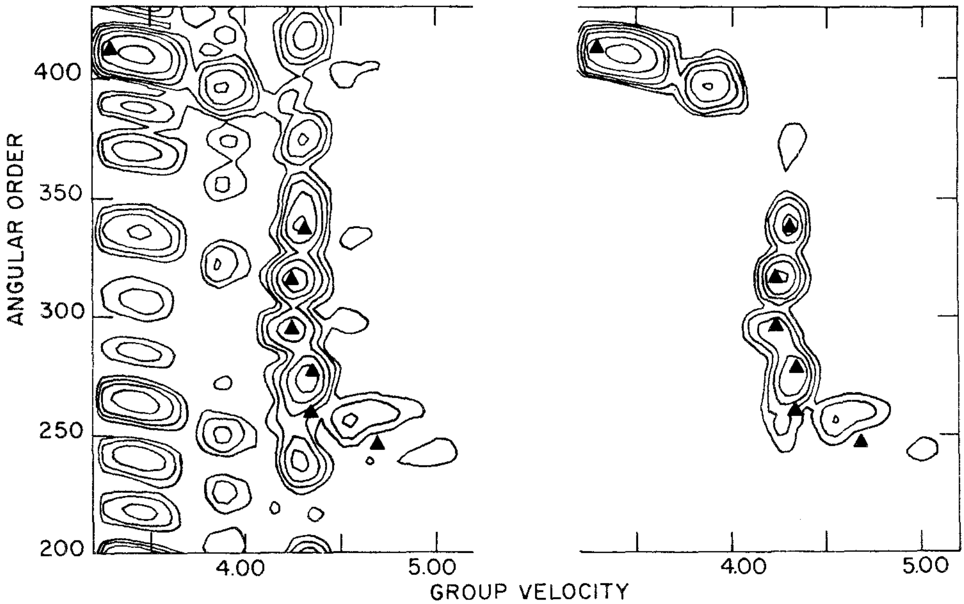

Figure 7

Mode-separation diagrams for the synthetic event before (left) and after cleaning (right) at a period of 25.6 sec. The contour interval is 3 dB. Triangles denote the exact locations of the modes of the synthetic signal under analysis.

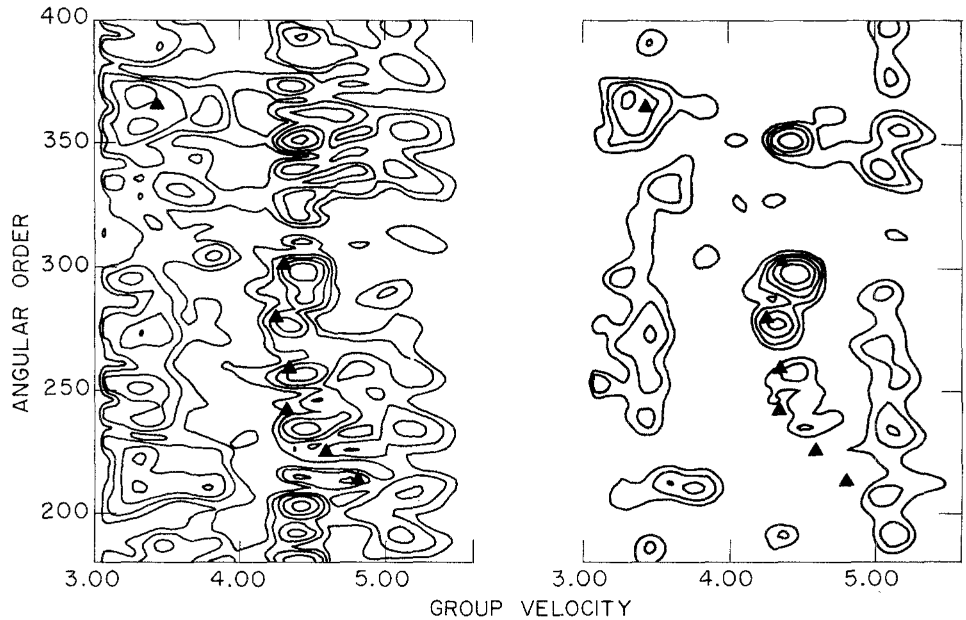

Figure 8

Mode-separation diagram for the 25 October 1965 event (see Fig. 1) at a period of 28.4 sec before (left) and after cleaning (right). Black triangles denote calculated locations of higher modes for the reference model and are used for identification. Here, as in Fig. 7, the group-velocity interval includes the fundamental mode. The contour interval is 3 dB.

$$W_0'(k) = \sum_j a_n' H'(k_n - k)$$

where

$$a_n' = a_n e^{ik_n \Delta_0}$$

and

$$H'(k) = \sum_j e^{ik(\Delta_j - \Delta_0)}$$

contains only oscillations with frequencies $(\Delta_j - \Delta_0)$ so that it behaves much smoother. One should therefore use W' for the construction of mode-separation diagrams if the cleaning algorithm is applied.

In Figs 7 and 8 we show the results of an application of the method to real and synthetic seismograms. Figure 9 show that results obtained for the synthetic seismogram as recorded in a 9-station array with a span of 2200 km. Apart from cleaning the mode-separation diagram of sidelobes for a less ambiguous interpretation of complicated responses, it is seen that the phase velocity determinations are over all slightly better. This is undoubtedly due to the fact that we remove at least partly the effect of sidelobes that interfere with main lobes.

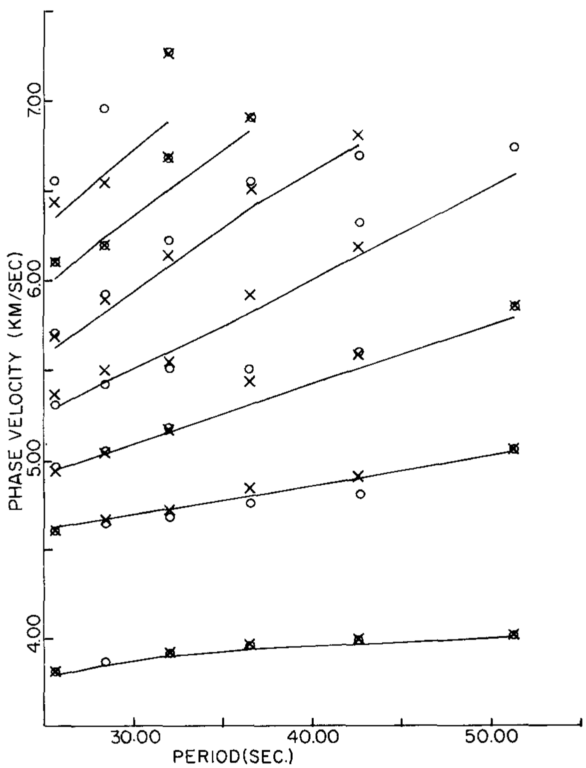

Figure 9

Combined data of an analysis of the synthetic seismogram, using a 9-station array with a span of 2200 km. Crosses denote measurements with application of the cleaning algorithm, circles without. Straight lines denote the theoretical values of the phase velocity.

6. *Conclusions*

To our knowledge, the method presented in this paper is at present the only method capable of resolving a great number of higher modes with some precision in the period range of 20–100 sec. The results are not very sensitive to noise and instrument calibration errors, because of the averaging properties of the stack, and can be used to screen refracted arrivals. These factors often spoil phase-velocity measurements of the fundamental mode with the two-station method.

The precision of the method is highly dependent on the array span and the station density. Only at a few places in the world is the WWSSN network dense enough for an optimum exploitation of the possibilities of this technique. This imposes a severe limitation to the application of the method. It would be worthwhile to install a few 'migrating' stations in regions of interest to enhance the station density temporarily, until enough data have been collected, and then move them on to other regions.

The largest errors of 1–2% due to the data-processing with limited resolving power quite satisfactorily explain the largest scatter found in real data, obtained for a purely continental region. However, no experience has yet been gained with cases where the great-circle paths cross regions of severe lateral heterogeneity.

The mutual interference of modes is a strong function of their excitation and of the array response. Thus, by averaging data obtained from events at different depths and arriving from different directions one acquires a more accurate estimate of higher-mode phase-velocities.

For a mild lateral heterogeneity, the method will give an average phase velocity over the array. In the short-period approximation (MADARIAGA, 1972) we expect that ray-theory becomes valid for surface-wave propagation, so that this phase velocity is representative for the average structure of the earth directly under the array. The depth of the mantle that can be probed depends on the span of the array, which determines at what phase-velocities the modes can be separated. Thus it is obvious that there is a trade-off between the vertical and horizontal resolution of the method. Gilbert and Dziewonski (1975) showed the possibility to average over the whole globe for angular orders lower than 70. Since we expect lateral heterogeneity to decrease with increasing depth it will be a good tactic to determine the deep structure using a large array and supplement these data with local determinations of the fundamental mode and dispersion using the two-station method.

We note here that the method might be used to attempt to obtain S-velocity structure from later arrivals and the groundroll in refraction experiments, provided the crust can locally be approximated by an average horizontal layering.

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