PAGEOPH, Vol. 124, Nos. 1/2 (1986)

0033-4533/86/020365-09$1.50+0.20/0

Fault Dimensions, Displacements and Growth

JUAN WATTERSON[1]

Abstract—Maximum total displacement (D) is plotted against fault or thrust width (W) for 65 faults, thrusts, and groups of faults from a variety of geological environments. Displacements range from 0.4 m to 40 km and widths from 150 m to 630 km, and there is a near linear relationship between D and W^2. The required compatibility strains (e_s) in rocks adjacent to these faults increases linearly with W and with $\sqrt{D}$ and ranges from $e_s = 2 \times 10^{-4}$ to $e_s = 3 \times 10^{-1}$. These are permanent ductile strains, which compare with values of $e_s = 2 \times 10^{-5}$ for the elastic strains imposed during single slip earthquake events, which are characterised by a linear relationship between slip (u) and W.

The data are consistent with a simple growth model for faults and thrusts, in which the slip in successive events increases by increments of constant size, and which predicts a relationship between displacement and width of the form $D = cW^2$. Incorporation of constant ductile strain rate into the model shows that the repreat time for slip events remains constant throughout the life of a fault, while the displacement rate increases with time. An internally consistent model with $e_s = 2 \times 10^{-5}$, giving repeat times of 160 years and instantaneous displacement rates of 0.02 cm/yr, 0.2 cm/yr, and 2.0 cm/yr when total displacement is 1 m, 100 m, and 10 km, and slip increasing by 0.5 mm with each event, gives a good approximation of the data. The model is also applicable to stable sliding, the slip rate varying with ductile strain rate and with W^2.

Key words: Fault, displacement, seismic slip, strain, strain rate, displacement rate.

Introduction

The problem addressed concerns the relationship between fault dimensions and the total displacement accrued through numerous slip events or stable sliding, and the consequent ductile and elastic strains required to maintain compatibility in rocks adjacent to the fault. The nomenclature used is illustrated in Figure 1, which shows the essential geometry of an ideal simple normal fault which does not intersect a free surface, i.e. a blind fault. The slip surface is taken to be an elliptical plane with the slip vector parallel to the short axis of the ellipse; the dimension of the slip surface parallel to the slip vector is the length L. The dimension normal to the slip vector within the slip plane is the fault width W. Displacement varies from a maximum at

[1] Department of Geological Sciences, Liverpool University, P.O. Box 147, Liverpool L69 3BX, United Kingdom.

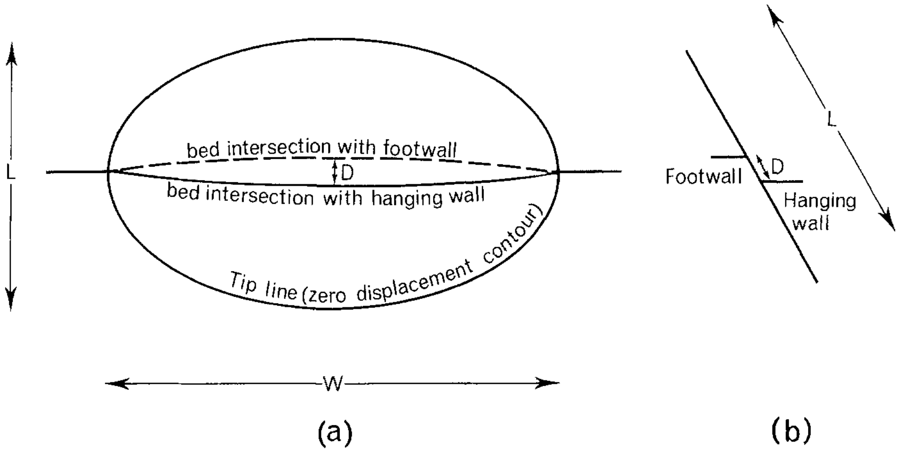

Figure 1
Essential geometry of an idealised simple normal fault with elliptical tip line, or zero displacement contour: (a) viewed along normal to slip surface; (b) side elevation. Disposition of an originally horizontal horizon on the fault plane is shown for footwall and hanging wall. L is length (parallel to slip direction), W is width, and D is maximum displacement.

the centre of the elliptical surface to zero at the edge of the slip surface which, following thrust terminology (BOYER and ELLIOTT, 1982), is an elliptical tip line. The fault is assumed to grow by radial extension of the tip line with no migration of the point of maximum displacement.

The maximum compatibility strains required by this geometry are, for total fault displacement D, length L, and width W, as follows.

1. In rocks adjacent to the fault surface and lying along the minor axis of the ellipse (parallel to the slip direction), an average linear strain $e_s = D/L$: this is the strain arising from the edge component of the fault dislocation, referred to by MURAOKA and KAMATA (1983) as the slip-parallel strain.

2. In rocks adjacent to the fault surface and initially lying along the major axis of the ellipse (perpendicular to the slip direction), an average shear strain $\gamma = D/W$: this is the strain arising from the screw component of the fault dislocation. For comparative purposes a more useful quantity is the linear strain of the long axis of the rotational strain ellipse e_s corresponding to the shear strain (RAMSAY and HUBER, 1983), which for the comparatively small strains ($\gamma < 0.2$) involved in faulting is $e_s = \gamma/2$, or $e_s = D/2W$.

The average strains can thus be simply expressed in terms of the maximum displacement and the principal dimensions of the fault surface. A more rigorous definition of the strains is not possible without knowledge of the distribution of displacements across and normal to the fault surface.

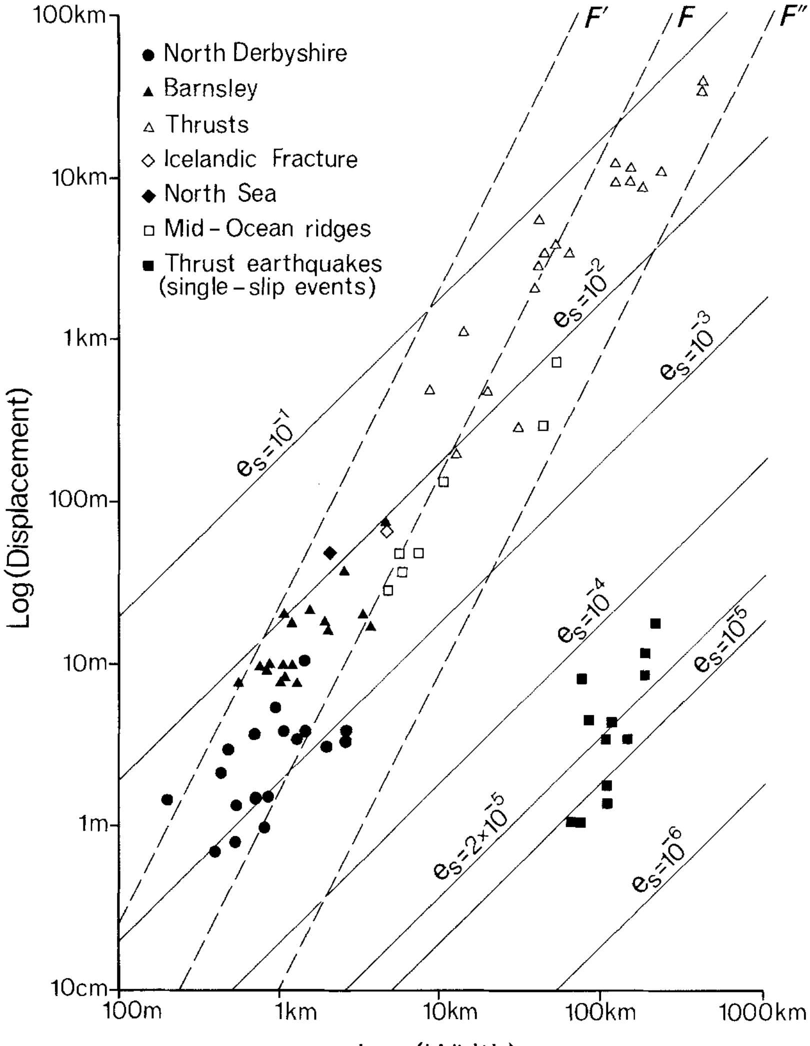

Figure 2
Width versus maximum displacement for faults, thrusts, and groups of faults (coal field faults from Barnsley and N. Derbyshire areas distinguished) and slip versus width for seismic events; e_s is linear strain required for compatibility, and *F*, *F'*, and *F''* are model growth curves. See text for explanation.

Fault and thrust data

The data available are sufficient only for examining the relationship between total displacement and fault width, and this is shown in Figure 2 for a range of faults and thrusts from different geological environments and covering a wide range of scale. The systematic distribution is in part an artifact of the log–log scale required by the range of size. Sources for the data are given below.

1. Mine data are for normal faults encountered during deep mining operations in British coal fields. Where several worked coal seams intersect the same fault at different levels and the displacement on the fault has been recorded at several places in each seam, sufficient data points may exist for contours of displacement to be drawn over the whole, or a substantial part, of the fault surface (RIPPON, 1985). This allows both the maximum displacement and the principal dimensions of the fault to be identified. Single fault surfaces generally approximate the geometry shown diagrammatically in Figure 1a, i.e., elliptical tip lines with displacement parallel to the short axis of the ellipse. The data plotted are derived either from faults where the complete fault plane has been defined or from faults for which sufficient information is available for definition of the maximum displacement and the half-width.

2. Thrust data are from ELLIOTT (1976) for thrusts in low-grade rocks in the foot hills and front ranges of the Canadian Rockies. The thrusts are not blind, and the assumption is made that the maximum widths and maximum displacements are those at the leading edge.

3. A single data point is included for a normal fault scarp in Iceland (GUDMUNDSSON, 1980), for which the same assumptions are made as for the thrusts.

4. Seven data points refer to normal fault scarps from several ocean floor areas (SEARLE, 1983; LAUGHTON and SEARLE, 1979) adjacent to spreading centres: the original data are of mean values for length and height of several scarps within each area. The plotted data may therefore underestimate the maximum displacements on these faults. The same assumptions are made as in Paragraphs 2 and 3.

5. A single data point represents two faults of similar size and displacement reconstructed from high-resolution seismic-reflection data; the faults are located in the North Sea.

Also plotted in Figure 2 are contours of the corresponding linear strains e_s. Not included in the plotted data are two faults of lengths 1.3 and 2.5 m, with displacements of 13.5 and 40.0 cm (MURAOKA and KAMATA, 1983). The displacements on these faults are an order of magnitude greater than would be expected if the linear trend in Figure 2 were to be extended to smaller scales, and differences between length and width are unlikely to reduce the anomaly significantly. These faults occur in Quaternary lacustrine sediments, which may have very different mechanical properties to the host rocks of the other faults for which data are available.

Interpretation

The immediate conclusions to be drawn from the data are as follows.

1. The strains required to accommodate these faults and thrusts, just less than $e_s = 0.1$, or 10% strain in the case of the major structures, are much too great to be

accommodated elastically and must therefore represent permanent ductile strains. In this context 'ductile' means strain in which material continuity is maintained, or in which discontinuities are on a very small scale relative to that of the strained volume.

2. Strain e_s increases with the size of the fault or thrust.
3. The relationship between total displacement and fault width is not linear.

A linear relationship between fault length and the mean slip u for individual large earthquake events has been demonstrated (SCHOLZ, 1982) for both strike slip and thrust movements, the characteristic values of e^1 being 1.25×10^{-5} for strike-slip faults and 2×10^{-5} for thrusts. The width of strike-slip faults is limited by the thickness of the seismogenic layer, and no relationship between width and seismic slip is expected or found (SCHOLZ, op. cit.). This limitation does not apply to deep thrust displacements, and the slips and fault widths for twelve such events listed by SCHOLZ (op. cit.) are plotted in Figure 2. The stress drop of small earthquakes is known to be nearly constant (HANKS, 1977) and independent of fault dimensions; as the stress drop is proportional to mean slip per fault radius, a linear relationship between u and W is expected, *pace* length per slip for large earthquakes. A slip per width ratio of 4×10^{-5} for single slip events, corresponding to $e_s = 2 \times 10^{-5}$, will be taken as the representative value. This is the strain change or strain drop (KANAMORI and ANDERSON, 1975). The earthquake data are not exactly comparable, since the values of total displacement and of e_s for the inactive faults refer to the maximum value of displacement on a fault, whereas the values for single slip earthquake events refers to mean values over the slip surface; as the mean slip is likely to exceed 0.8 of the maximum value (ESHELBY, 1957), no correction is necessary for present purposes.

From the data, and the assumed knowledge of the representative value of e_s associated with single slip events, an estimate can be made of the size of the final slip event on each of the faults for which the total displacement is known; e.g., a fault of width 100 km with a total displacement of 10 km will have had a final slip of 4 m, given the assumptions discussed previously. This allows some constraints to be placed on the manner in which faults grow and on the number of slip events which are required to achieve the total displacement on a seismic fault. Some absolute limitations are immediately apparent. If a fault is the product of only a single slip event, i.e. $D = u$, then it would plot somewhere along the line corresponding to $e_s = 2 \times 10^{-5}$; this is clearly not the case for any of the faults for which data are available. If every slip event on a fault were of the same size, then, given a linear relation between u and W, the fault would not increase in width, and the growth curve of the fault would parallel the y axis (log D) of Figure 2; such a relationship would require complete relaxation of all elastic compatibility strains between successive slip events, and there would be no systematic relationship between width and total displacement in a sample of faults. This is not the case for the sample available. On the other hand, if no relaxation of elastic compatibility strains took place between successive slip events, the established linear relationship between u and W for earthquake events would not obtain. The real case must lie between these extremes, and it requires relaxation between successive seismic faulting events of a proportion of the elastic strain, which is replaced by an equivalent permanent strain. It is evident that in the general case a fault will increase in width with time and that individual displacements of successive slip events must also increase.

Fault growth model

It might be expected that a model for the increase of successive slip events would require a complicated growth law, but this seems not to be the case. A model reasonably consistent with the data is one in which the amount of slip in individual events increases in simple arithmetic progression. Where successive slips increase by a constant increment,

$A = u/2$ and $D = N . A$, so $D = N . u/2$.
$u = N . k$, so $N = u/k$, and therefore $D = u^2/2k$.
But $u \propto W$, so, for constant k, $D = c . W^2$, or $\log D = c + 2 \log W$.

Here D is total displacement, u is last slip, W is fault width, A is average slip, N is number of slip events, and k is the increment by which successive slip events increase, and c is a constant.

A fault growing according to this model will have a growth curve which plots as a straight line on a log–log plot with a slope of 63.5 ($\tan^{-1} = 2$). The position of the growth curve is determined by the values of k and u/W. A representative growth curve F, Figure 2, corresponds to values of $k = 0.5$ mm and $u/W = 4 \times 10^{-5}$. Most of the data are included in the range bounded by the two lines F' and F'', which correspond to $k = 0.03$ and $k = 8$ mm respectively for $u/W = 4 \times 10^{-5}$. Alternatively, F' and F'' correspond to a u/W of 8×10^{-5} and 5×10^{-6} respectively at constant $k = 0.5$ mm.

If $D = cW^2$, the value of the constant c is 1.6×10^{-3} for line F, 2.5×10^{-2} for F', and 1×10^{-4} for F''. Given the range in type and scale of the faults and thrusts in the data set, and the range of material properties of the rocks in which they occur, the required range of values for the constant and for either k or u/W is sufficiently small for the model to be a potentially useful approximation.

The conclusions drawn here conflict in some respects with the assumptions of WILLIAMS and CHAPMAN (1983), who accept the popular concept of a 'ductile bead' at the propagating tip line of a fault. This concept requires that accommodation strains are imposed only adjacent to the current tip line, thereafter remaining unmodified during subsequent evolution and growth of the fault. This requirement is a consequence of the unstated assumptions (op. cit.) that (a) total maximum displacement is directly proportional to fault dimension (length or width), and that (b) the displacement gradient from fault tip to point of maximum displacement is linear.

Table 1

Characteristics of Fault Growth with Constant Values $e_s = 5 \times 10^{-15}$ sec^{-1}, $u/W = 4 \times 10^{-5}$, and u Incrementing by 0.5 mm with each Slip Event

Total maximum displacement (km)	0.001	0.01	0.1	1.0	10	50
Width (km)	1.6	5	15.8	25	158	173
Time (yr $\times$ 10^3)	11	33	107	327	1070	2356
Average displacement rate (cm/yr)	0.01	0.03	0.1	0.32	1.0	2.19
Instantaneous displacement rate (cm/yr)	0.02	0.06	0.2	0.63	1.98	4.3
Last slip (cm)	3.16	10	31.6	100	316	692
Number of slip events	63	200	633	2000	6329	14451
Repeat time (yr)	160	160	160	160	160	160

The first of these assumptions is inconsistent with the data given here, and the second assumption is doubtful, given the radial distribution of slip required by the ESHELBY (1957) model for single slip events. The increase in strains with fault dimensions demonstrated in Figure 2 strongly suggests that rocks adjacent to the whole slip surface undergo strain changes with each increment of slip, rather than just those adjacent to the propagating tip line, although this cannot be demonstrated conclusively, given that e_s is defined as the mean strain.

Rates of strain and of displacement

Taking the growth curve F as representative, the simple growth model can be used in examining the relationship between rates of strain, rates of displacement, and repeat times for seismic events. During the growth of a fault the most likely constant factor is the linear ductile strain rate, and this may in fact be the parameter which determines all other rates.

Taking $\dot{e}_s = 5 \times 10^{-15}$ sec^{-1} as the constant ductile strain rate, the other changes during growth of a fault or thrust to a total displacement of 50 km are listed in Table 1. This shows the model to predict constant repeat times for seismic events throughout the life of an individual fault, regardless of size attained, and to predict an increase in both average and instantaneous displacement rates with increase in size of the fault. The value of 5×10^{-15} was chosen for $\dot{e}_s$ in order to give a reasonable repeat time of 160 years, but it is within the range expected within the seismogenic layer. The ductile strain rates expected below the seismogenic layer range from 10^{-12} sec^{-1} to 10^{-14} sec^{-1} and are likely to decrease by approximately one order of magnitude for each 100° reduction in temperature (see data for quartz in WHITE, 1976, Fig. 7).

Although the model has been expressed in terms of seismic, i.e. intermittent, fault displacement, it is also applicable to faults which are characterised by stable sliding.

If stable sliding takes place with constant elastic strain, i.e. plastic yielding *ss*, the ductile strain rate is constant and the relationships between total displacement, width, displacement rates, and linear ductile strain rate are the same as for seismic faults.

The model is directly applicable only to growing faults in which each slip event, or stable sliding, occurs over the entire fault surface, and in which the fault surface is extended either with each seismic event or continuously during stable sliding. It may prove possible to adapt the model to slip events that affect only a part of an existing fault surface, but it cannot be applied to kinematically controlled faults, where the displacement rate on a single slip surface is determined directly by plate motions; such faults in any case terminate in transfer structures rather than tip lines, and length and width as defined here cannot be measured.

Acknowledgments

Much of the coal field fault data plotted in Figure 2 was provided by John Rippon and Jim Barnett of the British National Coal Board; I am grateful to these colleagues and to John Mortimer and John Walsh of the Liverpool Fault Analysis Group for advice and assistance, and to Britoil plc for access to seismic-reflection data.

This work is supported by Grant GR/4719 from the British Natural Environment Research Council and contracts from the National Coal Board (YCE.330/19572) and the U.S. Geological Survey Earthquake Hazards Reduction Program (14-08-0001-22072). Publication is with the permission of the National Coal Board.

REFERENCES

BOYER, S. E. and ELLIOTT, D. (1982), *Thrust systems.* Amer. Assoc. Petr. Geol. *66*, 9, 1196–1230.

ELLIOTT, D. (1976), *The energy balance and deformation mechanisms of thrust sheets.* Phil. Trans. R. Soc. London A *283*, 289–312.

ESHELBY, J. D. (1957), *The determination of the elastic field of an ellipsoidal inclusion and related problems.* Proc. Roy. Soc. London A *241*, 376–396.

GUDMUNDSSON, A. (1980), *The Vogur Fissure Swarm, Reykjanes Peninsula, SW Iceland.* Jokull *30*, 43–64.

HANKS, T. C. (1977), *Earthquake stress drops, ambient tectonic stress, and stresses that drive plate motions.* Pure Appl. Geophys. *115*, 441–458.

KANAMORI, H. and ANDERSON, D. L. (1975), *Theoretical basis of some empirical relation in seismology.* Bull. Seism. Soc. Am. *65*, 1073–1095.

LAUGHTON, A. S. and SEARLE, R. C. (1979), *Tectonic Processes of Slow Spreading Ridges, in Deep Drilling Results in the Atlantic Ocean: Ocean Crust.* M. Talwani, C. G. Harrison, and D. E. Hayes (eds.), Maurice Ewing Ser., Vol. 2, AGU, Washington, D.C.

MURAOKA, H. and KAMATA, H. (1983), *Displacement distribution along minor fault traces.* J. Struct. Geol. *5*, 483–495.

RAMSAY, J. G. and HUBER, M. I. (1983), *The techniques of modern structural geology. Vol. 1, Strain Analysis.* Academic Press, London, 307 pp.

RIPPON, J. H. (1985), *Contoured patterns of the throw and hade of normal faults in the coal measures (Westphalian) of north-east Derbyshire.* Proc. Yorkshire Geol. Soc. *45*, 147–161.

SCHOLZ, C. H. (1982), *Scaling laws for large earthquakes: Consequences for physical models.* Bull. Seism. Soc. Am. *72*, 1–14.

SEARLE, R. C. (1983), *Gloria survey of the East Pacific Rise near 3.5°S: Tectonic and volcanic characteristics of a fast spreading mid-ocean rise.* Tectonophys. *101*, 319–344.

WHITE, S. (1976), *The effects of strain on the microstructures, fabrics, and deformation mechanisms in quartzites.* Phil. Trans. R. Soc. London A *283*, 289–312.

WILLIAMS, G. and CHAPMAN, T. (1983), *Strains developed in the hanging walls of thrusts due to their slip/propagation rates: A dislocation model.* J. Struct. Geol. *5*, 563–571.

(Received 1st November 1985, revised 2nd February 1986, accepted 18th February 1986)