Gravitational Coagulation of Charged Cloud Drops in Turbulent Flow

By LEV M. LEVIN and YURII S. SEDUNOV[1])

Summary – Gravitational coagulation of particles in turbulent flow is investigated with respect to the electrostatic forces. As turbulence mixes particle trajectories at large distances, its account reduces the consideration of drawing particles together till some definite distance. Taking into account hydrodynamical forces of interaction, series converging faster than HOCKING's one and their more exact values are obtained. For electrostatic forces series with better convergence are also obtained. Equations of moment were solved numerically on electronic computer BESM–2. Results of collection efficiency computation are given.

Studying the mechanism of cloud drop formation is very important for an understanding of physical processes leading to cloud formation. Numerous investigations of this mechanism showed that at present it is very difficult to explain the formation of a wide droplet size spectrum (with diameters d from 1 to $30 \div 40$ μ) at a comparatively short period of time (about $20 \div 30$ minutes). The point is that the growth of drops by condensation must have brought to the appearance of a narrow droplet size spectrum as such a growth rate of drops rapidly decreases with the increase of their diameters.

Theoretically growth of drops by coagulation either impossible for drops with $d < 30 \div 40$ μ (purely gravitational coagulation; [1, 2, 3])[2]) or effective only for drops with $d < 5 \div 7$ μ (electrostatic coagulation at mean values of drop charges existing in clouds at the initial stage of their development: [4]). A number of investigators studied effects of turbulence on coagulation of cloud drops. They obtained the value of growth rate by coagulation with an accuracy to unknown constant coefficients [5, 6, 7, 8, 9]. Lately one at the authors made accurate calculations of mutual velocities of aerosol particles in the turbulent flow [10, 11, 12]. On these grounds precise coefficients of turbulent diffusion were determined [10, 11]. Calculations of cloud drop growth rate by the known turbulent mechanism of acceleration and by turbulent diffusion confirmed once more that velocities of cloud drop spectrum formation observed in nature could not be due only to these growth mechanism [12].

In particular, this is connected with the fact that at small distances between drops turbulent diffusion is small (it is effective at long distances) while the picture is quite

[1]) Hydrometeorological Service of the USSR, 12 Pavlik Morozov Street, Moscow, D–376, USSR.

[2]) Numbers in brackets refer to References, page 196.

the contrary for other coagulation mechanism—they are little effective at great distances.

In the present paper we tried to estimate the joint effect of turbulent, gravitational and electrostatic coagulation on cloud drop growth rate. This estimation was carried out for two-layer model. The space beyond the larger of coagulation drops is divided into two parts by the sphere the surface of which is away from the drop at a distance equal to turbulent free path length l_t[3]). Beyond the sphere the main mechanism of approach is turbulence. It provides fast approach of small drops with comparatively large relative velocities to this sphere. Within the sphere movement of particles relative to one another takes place under gravity, electrostatic and hydrodynamical interactions. As movement occurs within the sphere, when the distances between drop surfaces Δ are comparable with drop sizes. expressions for the given above forces must be thoroughly considered for correct formulation of the task.

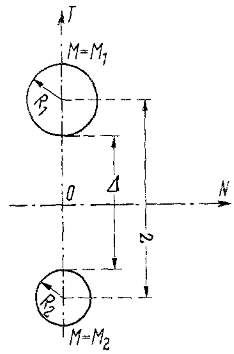

Figure 1
Bispherical coordinate system

To describe electrostatic interaction between two charged drops let us introduce bispherical co-ordinate system in which surfaces of both drops correspond to the co-ordinate values

$$\mu = \mu_1 \quad \text{and} \quad \mu = \mu_2 (\mu_1 > 0 ; \quad \mu_2 < 0) .$$

Then

$$2 r R_1 c h \mu_1 = r^2 + R_1^2 - R_2^2 ; \quad 2 r R_2 c h \mu_2 = r^2 - R_1^2 + R_2^2 ,$$

where r is the distance between drop centers ($r = R_1 + R_2 + \Delta$).

For coefficients of capacity c_{ik} connecting the drop charges q_1 and q_2 with their potentials V_1 and V_2 by the formulas

$$q_i = \sum_{k=1}^{2} C_{ik} V_k \quad (i = 1, 2) \tag{1}$$

[3]) By free path length we mean a distance at which drops must disperse in the flow lest their relative velocites in the initial and finite moments should correlate. Value $l_t \approx 1.5 (R_1 + R_2)$ where R_1, R_2—the radius of the larger and smaller drops [12].

the following expressions take place [4]:

$$C_{ii} = R_i(1 - v_i^2) \sum_{m=0}^{\infty} (v_i^2\, u^{2m+1})^m \frac{1 - v_i^2\, u^{4m-1}}{(1 - v_i^2\, u^{2m})\,(1 - u^{2m+1})} \quad (i = 1, 2) , \tag{2}$$

$$C_{12} = C_{21} = -\frac{R_1\, R_2}{r} (1 - u^2) \sum_{m=0}^{\infty} u^{(2m+3)m} \frac{1 - u^{4m+3}}{(1 - u^{2m+2})\,(1 - u^{2m+1})} ,$$

where

$$v_1 = e^{-\mu_1}; \quad v_2 = e^{-|\mu_2|}; \quad u = v_1\, v_2 .$$

Electrostatic force of interaction of two charged spheres — F_e can be represented by an equation of the form (4):

$$F_e = \frac{q_1\, q_2}{r^2} f(r, a, x) , \tag{3}$$

where

$$\left. \begin{aligned} f(r, a, x) &= 1 + l_1(r, a) + x\, l_2(r, a) + \frac{1}{x}\, l_3(r, a) \\ &= -r^2 \frac{ds_{12}}{dr} + x \frac{r^2}{2} \frac{dS_{22}}{dr} + \frac{1}{x} \frac{r^2}{2} \frac{dS_{11}}{dr} . \end{aligned} \right\} \tag{4}$$

Here $a = R_2/R_1 \leq 1$; $x = -q_2/q_1$; S_{ik}-coefficients of induction determined by

$$v_i = \sum_{k=1}^{2} s_{ik}\, q_k \quad (i = 1, 2) . \tag{5}$$

From the equation (1) and (5) we obtain

$$s_{11} = c_{22}\, \delta_c^{-1}; \quad s_{22} = c_{11}\, \delta_c^{-1}; \quad s_{12} = -c_{12}\, \delta_c^{-1} = s_{21}; \quad \delta_c = \begin{vmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{vmatrix} . \tag{6}$$

Computations showed that the series (2) as well as the series for dS_{ik}/dr (by these series (2) and with the help of (6) the derivatives dS_{ik}/dr entering the (4) should be expressed) converge very fast even at the distances between drop surfaces $\Delta \sim 0.01\, R_2$[4]).

This allowed us to program effectively expressions for electrostatic forces on the electronic computer BESM–2. Examples of values of functions l_i computed by the above method are given in table 1.

We consider aerodynamical forces acting on drops in the Stokes approximation because according to our task we shall deal with the drops having $R_1 < 20 \div 30\ \mu$ and at $\Delta < 2 \div 5\, R_1$[5]).

In the Stokes approximation aerodynamical forces f_1 and f_2 acting on drops are linearly related with velocities of drops moving relative to the air. We shall consider

[4]) For example, to compute l_i with an accuracy to 0.1% at $\Delta = 0.01\, R_2$ it is necessary to take all in all 6 to 7 terms of the series (2) and their derivatives.

[5]) The second condition is very essential because Stokes approximation is true not only when the REYNOLD's number $2\, R_1\, v/\nu$ is small but also when $2\, r_r\, v/\nu$ is small [16]. Here v is the drop velocity, ν the kinematic viscosity.

Tabelle 1

Δ/a		20	10	5	2	1	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
$a=1$	l_1	—	0	$132\cdot10^{-6}$	$443\cdot10^{-5}$	$317\cdot10^{-4}$	$130\cdot10^{-3}$	$476\cdot10^{-3}$	$998\cdot10^{-3}$	$189\cdot10^{-2}$	$408\cdot10^{-2}$	$711\cdot10^{-2}$	$123\cdot10^{-1}$	$254\cdot10^{-1}$	$441\cdot10^{-1}$
	$l_2=l_3$	—	$117\cdot10^{-5}$	$601\cdot10^{-5}$	$346\cdot10^{-4}$	$909\cdot10^{-4}$	$187\cdot10^{-3}$	$400\cdot10^{-3}$	$676\cdot10^{-3}$	$113\cdot10^{-3}$	$223\cdot10^{-2}$	$374\cdot10^{-2}$	$633\cdot10^{-2}$	$128\cdot10^{-1}$	$222\cdot10^{-1}$
$a=0.9$	l_1	—	$6\cdot10^{-6}$	$223\cdot10^{-6}$	$524\cdot10^{-5}$	$358\cdot10^{-4}$	$142\cdot10^{-3}$	$505\cdot10^{-3}$	$105\cdot10^{-2}$	$197\cdot10^{-2}$	$425\cdot10^{-2}$	$739\cdot10^{-2}$	$128\cdot10^{-1}$	$264\cdot10^{-1}$	$459\cdot10^{-1}$
	l_2	—	$156\cdot10^{-5}$	$547\cdot10^{-5}$	$444\cdot10^{-4}$	$115\cdot10^{-3}$	$236\cdot10^{-3}$	$505\cdot10^{-3}$	$853\cdot10^{-3}$	$142\cdot10^{-2}$	$280\cdot10^{-2}$	$470\cdot10^{-2}$	$795\cdot10^{-2}$	$161\cdot10^{-1}$	$279\cdot10^{-1}$
	l_3	—	$114\cdot10^{-5}$	$653\cdot10^{-5}$	$317\cdot10^{-4}$	$807\cdot10^{-4}$	$162\cdot10^{-3}$	$342\cdot10^{-3}$	$577\cdot10^{-3}$	$964\cdot10^{-3}$	$190\cdot10^{-2}$	$321\cdot10^{-2}$	$544\cdot10^{-2}$	$110\cdot10^{-1}$	$190\cdot10^{-1}$
$a=0.7$	l_1	—	$11\cdot10^{-6}$	$28\cdot10^{-5}$	$751\cdot10^{-5}$	$460\cdot10^{-4}$	$169\cdot10^{-3}$	$564\cdot10^{-3}$	$115\cdot10^{-2}$	$314\cdot10^{-2}$	$457\cdot10^{-2}$	$793\cdot10^{-2}$	$137\cdot10^{-1}$	$283\cdot10^{-1}$	$492\cdot10^{-1}$
	l_2	—	$310\cdot10^{-5}$	$150\cdot10^{-4}$	$797\cdot10^{-4}$	$201\cdot10^{-3}$	$406\cdot10^{-3}$	$865\cdot10^{-3}$	$146\cdot10^{-2}$	$242\cdot10^{-2}$	$473\cdot10^{-2}$	$791\cdot10^{-2}$	$134\cdot10^{-1}$	$271\cdot10^{-1}$	$468\cdot10^{-1}$
	l_3	—	$105\cdot10^{-5}$	$5\cdot10^{-3}$	$251\cdot10^{-4}$	$591\cdot10^{-4}$	$112\cdot10^{-3}$	$230\cdot10^{-3}$	$386\cdot10^{-3}$	$648\cdot10^{-3}$	$129\cdot10^{-2}$	$217\cdot10^{-2}$	$369\cdot10^{-2}$	$753\cdot10^{-2}$	$131\cdot10^{-1}$
$a=0.5$	l_1	$77\cdot10^{-7}$	$25\cdot10^{-6}$	$506\cdot10^{-6}$	$111\cdot10^{-4}$	$591\cdot10^{-4}$	$197\cdot10^{-3}$	$620\cdot10^{-3}$	$123\cdot10^{-2}$	$227\cdot10^{-2}$	$480\cdot10^{-2}$	$834\cdot10^{-2}$	$144\cdot10^{-1}$	$295\cdot10^{-1}$	$515\cdot10^{-1}$
	l_2	$133\cdot10^{-5}$	$755\cdot10^{-5}$	$344\cdot10^{-4}$	$168\cdot10^{-3}$	$408\cdot10^{-3}$	$811\cdot10^{-3}$	$171\cdot10^{-2}$	$286\cdot10^{-2}$	$472\cdot10^{-2}$	$916\cdot10^{-2}$	$152\cdot10^{-1}$	$257\cdot10^{-1}$	$520\cdot10^{-1}$	$895\cdot10^{-1}$
	l_3	$165\cdot10^{-6}$	$917\cdot10^{-6}$	$40\cdot10^{-4}$	$172\cdot10^{-4}$	$377\cdot10^{-4}$	$647\cdot10^{-4}$	$129\cdot10^{-3}$	$216\cdot10^{-3}$	$364\cdot10^{-3}$	$728\cdot10^{-3}$	$123\cdot10^{-2}$	$210\cdot10^{-2}$	$436\cdot10^{-2}$	$742\cdot10^{-2}$
$a=0.3$	l_1	$2\cdot10^{-6}$	$7\cdot10^{-5}$	$106\cdot10^{-5}$	$167\cdot10^{-4}$	$736\cdot10^{-4}$	$220\cdot10^{-3}$	$650\cdot10^{-3}$	$127\cdot10^{-2}$	$231\cdot10^{-2}$	$484\cdot10^{-2}$	$837\cdot10^{-2}$	$144\cdot10^{-1}$	$296\cdot10^{-1}$	$513\cdot10^{-1}$
	l_2	$529\cdot10^{-5}$	$274\cdot10^{-4}$	$112\cdot10^{-3}$	$487\cdot10^{-3}$	$112\cdot10^{-2}$	$219\cdot10^{-2}$	$455\cdot10^{-2}$	$752\cdot10^{-2}$	$123\cdot10^{-1}$	$236\cdot10^{-1}$	$390\cdot10^{-1}$	$652\cdot10^{-1}$	$131\cdot10^{0}$	$225\cdot10^{0}$
	l_3	$139\cdot10^{-6}$	$679\cdot10^{-6}$	$250\cdot10^{-5}$	$831\cdot10^{-5}$	$151\cdot10^{-4}$	$247\cdot10^{-4}$	$482\cdot10^{-4}$	$814\cdot10^{-4}$	$140\cdot10^{-3}$	$286\cdot10^{-3}$	$483\cdot10^{-3}$	$816\cdot10^{-3}$	$168\cdot10^{-2}$	$228\cdot10^{-2}$

x/a		200	100	50	20	10	5	2	1	0.5	0.2	0.1	0.05	0.02	0.01
$a=0.1$	l_1	—	0	0	$21\cdot10^{-6}$	$276\cdot10^{-6}$	$236\cdot10^{-5}$	$218\cdot10^{-4}$	$786\cdot10^{-4}$	$215\cdot10^{-3}$	$611\cdot10^{-3}$	$118\cdot10^{-2}$	$214\cdot10^{-2}$	$448\cdot10^{-2}$	$770\cdot10^{-2}$
	l_2	—	$148\cdot10^{-5}$	$918\cdot10^{-5}$	$793\cdot10^{-4}$	$320\cdot10^{-3}$	$106\cdot10^{-2}$	$392\cdot10^{-2}$	$868\cdot10^{-2}$	$167\cdot10^{-1}$	$345\cdot10^{-1}$	$565\cdot10^{-1}$	$914\cdot10^{-1}$	$173\cdot10^{0}$	$283\cdot10^{0}$
	l_3	—	$1\cdot10^{-6}$	$88\cdot10^{-7}$	$65\cdot10^{-6}$	$217\cdot10^{-6}$	$49\cdot10^{-5}$	$95\cdot10^{-5}$	$138\cdot10^{-5}$	$218\cdot10^{-5}$	$464\cdot10^{-5}$	$847\cdot10^{-5}$	$15\cdot10^{-3}$	$314\cdot10^{-4}$	$57\cdot10^{-3}$
$a=0.05$	l_1	0	0	$1\cdot10^{-6}$	$4\cdot10^{-5}$	$39\cdot10^{-5}$	$264\cdot10^{-5}$	$212\cdot10^{-4}$	$749\cdot10^{-4}$	$205\cdot10^{-3}$	$584\cdot10^{-3}$	$113\cdot10^{-2}$	$206\cdot10^{-2}$	$430\cdot10^{-2}$	$729\cdot10^{-2}$
	l_2	$150\cdot10^{-5}$	$941\cdot10^{-5}$	$506\cdot10^{-4}$	$352\cdot10^{-3}$	$125\cdot10^{-2}$	$385\cdot10^{-2}$	$140\cdot10^{-1}$	$314\cdot10^{-1}$	$612\cdot10^{-1}$	$127\cdot10^{0}$	$209\cdot10^{0}$	$337\cdot10^{0}$	$635\cdot10^{0}$	$104\cdot10^{1}$
	l_3	0	0	$55\cdot10^{-7}$	$29\cdot10^{-6}$	$67\cdot10^{-6}$	$114\cdot10^{-6}$	$175\cdot10^{-6}$	$238\cdot10^{-6}$	$42\cdot10^{-5}$	$10\cdot10^{-4}$	$2\cdot10^{-3}$	$33\cdot10^{-4}$	$7\cdot10^{-3}$	$127\cdot10^{-4}$
$a=0.02$	l_1	0	0	$3\cdot10^{-6}$	$6\cdot10^{-5}$	$43\cdot10^{-5}$	$255\cdot10^{-5}$	$201\cdot10^{-4}$	$714\cdot10^{-4}$	$197\cdot10^{-3}$	$564\cdot10^{-3}$	$11\cdot10^{-1}$	$199\cdot10^{-2}$	$418\cdot10^{-2}$	$718\cdot10^{-2}$
	l_2	$168\cdot10^{-4}$	$866\cdot10^{-4}$	$374\cdot10^{-3}$	$207\cdot10^{-2}$	$680\cdot10^{-2}$	$209\cdot10^{-1}$	$785\cdot10^{-1}$	$180\cdot10^{0}$	$357\cdot10^{0}$	$747\cdot10^{0}$	$123\cdot10^{1}$	$198\cdot10^{1}$	$374\cdot10^{1}$	$611\cdot10^{1}$
	l_3	0	0	$1\cdot10^{-6}$	$56\cdot10^{-7}$	$88\cdot10^{-7}$	$115\cdot10^{-7}$	$15\cdot10^{-6}$	$1\cdot10^{-5}$	$5\cdot10^{-5}$	$112\cdot10^{-6}$	$29\cdot10^{-5}$	$16\cdot10^{-5}$	$3\cdot10^{-4}$	$7\cdot10^{-4}$

drop movement on the vertical plane. Then in Cartesian coordinates connected with center line (axis 0 t-along the line of center, axis 0 n-normal to it) the relation f_i to v_i can be expressed as

$$\left.\begin{aligned} f_{in} &= (a_{i1,n}\, v_{1n} + a_{i2,n}\, v_{2n})\, 6\,\pi\,\eta\, R_i\,, \\ f_{it} &= (a_{i1,t}\, v_{1t} + a_{i2,t}\, v_{2t})\, 6\,\pi\,\eta\, R_i\,, \end{aligned}\right\} \quad (i = 1, 2)\,, \tag{7}$$

where $a_{ik,n}$ and $a_{ik,t}$-functions of r, R_1, R_2, η-viscosity of the air.

The purpose of a great number of theoretical papers and some experimental works was devoted to find $a_{ik,n}$ and $a_{ik,t}$. [13, 14, 15, 17, 20, 21]. For the case when movement takes place along the line of center $v_{1n} = v_{2n} = 0$) the task has axial symmetry and the exact solution is obtained for it. This task was first solved for the case $v_{1t} = v_{2t}$ by STIMSON and JEFFERY [15].

For arbitrary values of v_{1t} and v_{2t} PSHENAY-SEVERIN obtained $a_{ik,t}$ [17] which can be represented in bispherical coordinates by the following way[6]:

$$\left.\begin{aligned} a_{11,t} = \frac{1}{3}\, sh\,\mu_1 \sum_{m=1}^{\infty} A_m \Big\{ & 2\,(2\,m-1)\,(2\,m+3)\,(\exp[-(2\,m+1)\,\mu_2] \\ & - \exp[-(2\,m+1)\,\mu_1]) - 2(2\,m+1)\,(2\,m+3) \\ & \times sh\left[\left(m+\frac{3}{2}\right)(\mu_1-\mu_2)\right] \exp\left[-\left(m-\frac{1}{2}\right)(\mu_1+\mu_2)\right] \\ & - 2(2\,m+1)\,(2\,m-1) \\ & \times sh\left[\left(m-\frac{1}{2}\right)(\mu_1-\mu_2)\right] \exp\left[-\left(m+\frac{3}{2}\right)(\mu_1+\mu_2)\right]\Big\}\,; \\ a_{12,t} = \frac{1}{3}\, sh\,\mu_1 \sum_{m=1}^{\infty} A_m \Big\{ & 16\,\exp\left[-\left(m+\frac{1}{2}\right)(\mu_1-\mu_2)\right] \\ & \times sh\left[\left(m+\frac{1}{2}\right)(\mu_1-\mu_2)\right] + (2\,m+1)^2\, sh(\mu_1-\mu_2) \\ & \times [(2\,m+3)\exp(\mu_1-\mu_2) + (2\,m-1)\exp(-\mu_1+\mu_2)] \\ & + (2\,m+1)\,(2\,m+3)\,(2\,m-1)\,(sh\,2\,\mu_2 - sh\,2\,\mu_1)\Big\}\,, \end{aligned}\right\} \tag{8}$$

where

$$A_m = \frac{m(m+1)}{(2\,m+3)\,(2\,m-1)} \left\{4\, sh^2\left[\left(m+\frac{1}{2}\right)(\mu_1-\mu_2)\right] - (2\,m+1)^2\, sh^2(\mu_1-\mu_2)\right\}^{-1}.$$

Functions $a_{22,t}$ and $a_{21,t}$ are obtained from $a_{11,t}$ and $a_{12,t}$ respectively by substituting μ_1 for $-\mu_2$ and μ_2 for $-\mu_1$. For case when $v_{in} \neq 0$ a number of authors obtained the following series for functions $a_{ik,t}$ and $a_{ik,n}$ [3, 20]:

$$a_{ik,\,{}^t_n} = \sum_{\mu,\nu=0}^{\infty} P_{\mu\nu} \frac{R_1^{\mu} R_2^{\nu}}{r^{(\mu+\nu)}}\,. \tag{9}$$

[6]) It's a queer thing that just the same solution of the task was published in 3 to 5 years by BRENNER [18] and MAUDE [19].

In particular, HOCKING [3] obtained expressions for $a_{ik,n}$ and $a_{ik,t}$ accurate to the terms $0(r^{-7})$ (9). We compared results by HOCKING with the exact formulas by PSHENAY-SEVERIN.

For this purpose coefficients $a_{ik,t}$ were computed from the formulas (8) and from HOCKING's series. These computations showed that at small $\Delta (\Delta < 0.5\, R_2)$. HOCKING's expressions led to essential errors in determination of $a_{ik,t}$ (figure 2 and table 2). This is connected with slow convergence of series (9). At the same time we noticed that when determining v_{in} and v_{it} from the system (7)

$$\left.\begin{aligned} v_{in} &= \left(b_{i1,n} \frac{f_{1n}}{6\pi\eta R_1} + b_{i2,n} \frac{f_{2n}}{6\pi\eta R_2}\right), \\ v_{it} &= \left(b_{i1,t} \frac{f_{1t}}{6\pi\eta R_1} + b_{i2,t} \frac{f_{2t}}{6\pi\eta R_2}\right), \end{aligned}\right\} \quad (i = 1, 2)\,, \tag{10}$$

functions $b_{ik,n}$ and $b_{ik,t}$ can be obtained for which series of (9) type converge significantly sooner than for $a_{ik,n}$ and $a_{ik,t}$. This allowed us within the limits of the same approximation that HOCKING obtained $(0(r^{-7})$ to find expansion in series of (9) type for $b_{ik,n}$ and $b_{ik,t}$:

$$\left.\begin{aligned} -b_{11,n} &= 1 - \frac{3}{4}\frac{R_1 R_2^3}{r^4} - \frac{17}{16}\frac{R_1 R_2^5}{r^6} + 0\,(r^{-8})\,, \\ -b_{12,n} &= \frac{3}{4}\frac{R_2}{r} + \frac{R_2(R_1^2 + R_2^2)}{4\,r^3} + \frac{3\,R_1^3 R_2^4}{8\,r^7} + 0\,(r^{-9})\,, \\ -b_{11,t} &= 1 - \frac{15}{4}\frac{R_1 R_2^3}{r^4} - 2\,\frac{R_1 R_2^5}{r^6} + \frac{15}{2}\frac{R_1^3 R_2^3}{r^6} + 0\,(r^{-8})\,, \\ -b_{12,t} &= \frac{3}{2}\frac{R_2}{r} - \frac{R_2(R_1^2 + R\ )}{2\,r^3} + \frac{75}{4}\frac{R_1^3 R_2^3}{r^7} + 0\,(r^{-9})\,. \end{aligned}\right\} \tag{11}$$

Functions $b_{22,{}^{t}_{n}}$ and $b_{21,{}^{t}_{n}}$ are obtained from $b_{11,{}^{t}_{n}}$ and $b_{12,{}^{t}_{n}}$ respectively by transposition of R_1 and R_2.

Hence we obtained expressions for $a_{ik,t}$[7]).

$$\left.\begin{aligned} a_{11,t} &= b_{22,t}\,\delta_t^{-1}; \qquad a_{12,t} = -\,b_{12,t}\,\delta_t^{-1}; \\ a_{21,t} &= -\,b_{21,t}\,\delta_t^{-1}; \quad a_{22,t} = b_{11,t}\,\delta_t^{-1}\,. \end{aligned}\right\} \quad \delta_t = \begin{vmatrix} b_{11,t} & b_{12,t} \\ b_{21,t} & b_{22,t} \end{vmatrix}. \tag{12}$$

Functions $a_{ik,t}$ obtained from formulas (12) approach to the exact solution of (8) at small values of the distance Δ better than that obtained by HOCKING. This is due to the fact that in formulas (12) we compute determinants δ_t and δ_n substituting there values of $b_{ik,{}^{t}_{n}}$ from (11) while HOCKING when computing these determinants limited himself to the terms $0\,(r^{-7})$. Calculations showed that the obtained approximation proved to be much better for drops of comparable sizes $(R_2/R_1 > 0.5)$. This illustrates in figures 2, table 2. The values of $\lambda = (a_{11,t} + a_{12,t})$ are given in table 2, for the case when two identical drops move along the center line at one and the same velocity v_t. The values λ characterizes the relation of the force acting on one of these drops to the

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[7]) Functions $a_{ik,n}$ are expressed by $b_{ik,n}$ from the formulas similar to (12).

force acting on any of these drops (when it is considered isolated) moving at the same velocity. As it is seen from table 2, at $\Delta < 0.1\ R$ the force estimated from HOCKING's formulas is 1.5 to 2.5 times less than the exact value while the value obtained from (12) differs from the exact one by 4%.

Table 2

Δ/R	0	0.1	0.2	0.5	0.8	1.0
λ (exact)	0.645	0.655	0.660	0.675	0.692	0.705
λ (HOCKING)	0.257	0.440	0.525	0.636	0.675	0.695
λ (formulas (12))	0.617	0.630	0.642	0.670	0.688	0.705

As it was demanded by the task stated in the beginning of the paper we considered a system of dimensionless equations of movement for drops with electrostatic, aerodynamical forces and gravity act upon (the coordinate system is shown in figure 3):

$$I \frac{dw_1}{dt} = 1 - \frac{z\, l_1 - x\, t_1}{r} + a^3\, \alpha\, \frac{z\, f(r)}{r^3}\,; \tag{13a}$$

$$I \frac{du_1}{dt} = - \frac{x\, l_1 + z\, t_1}{r} + a^3\, \alpha\, \frac{x\, f(r)}{r^3}\,; \tag{13b}$$

$$I \frac{dw_2}{dt} = 1 - \frac{z\, l_2 - x\, t_2}{a^2\, r} - \alpha\, \frac{z\, f(r)}{r^3}\,; \tag{13c}$$

$$I \frac{du_2}{dt} = - \frac{x\, l_2 + z\, t_2}{a^2\, r} - \alpha\, \frac{x\, f(r)}{r^3}\,; \tag{13d}$$

$$\frac{dx}{dt} = u_2 - u_1\,; \tag{13e}$$

$$\frac{dz}{dt} = w_2 - w_1\,; \tag{13f}$$

where

$$z = z_2 - z_1; \quad x = x_2 - x_1; \quad r^2 = x^2 + z^2; \quad u_i = \frac{dx_i}{dt}; \quad w_i = \frac{dz_i}{dt};$$

x_1, z_1, x_2, z_2 Cartesian coordinates of drop centers ($0\,z$ is vertical), $a = R_2/R_1 \le 1$: $I = 4\, \varrho^2\, R_1^3\, g/81\, \eta$ is an analogue of the Stokes number[8]). $\alpha = -\, q_1\, q_2/6\, \pi\, \eta\, R_1^3\, a^3\, v_{1s}$ is a parameter characterising the relation of electrostatic forces to aerodynamical ones, ϱ is the drops density, g the gravity acceleration, $v_{1s} = 2\, \varrho\, R_1^2\, g/9\, \eta$ the sedimentation velocity of the larger drop.

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[8]) It is easily noticed that $I = \tau_1\, v_{1s}/R_1$ where $\tau_1 = 2\, \varrho\, R_1^2/9\, \eta$ relaxation time for the larger drop. The Stokes number is $\kappa = \tau_2 (v_{1s} - v_{2s})/R_1$, where $\tau_2 = 2\, \varrho\, R_2^2/9\, \eta$-relaxation time for the smaller drop, $v_{2s} = 2\, \varrho\, R_2^2\, g/9\, \eta = v_{1s} \cdot a^2$-sedimentation velocity of the smaller drop. As $\tau_2 = a^2\, \tau_1$, Stokes number $\kappa = I\, a^2 (1 - a^2)$.

Functions

$$l_i = - u_{t_1} a_{i1,t} - u_{t_2} a_{i2,t} ; \tag{14a}$$

$$t_i = - u_{n_1} a_{i1,n} - u_{n_2} a_{i2,n} ; \tag{14b}$$

$$u_{t_i} = \frac{1}{r} (u_i x + w_i z) ; \tag{14c}$$

$$u_{n_i} = \frac{1}{r} (u_i z - w_i x) ; \quad (i = 1, 2) , \tag{14d}$$

where $a_{ik,t}$ and $a_{ik,n}$ are determined by (12) and (11).

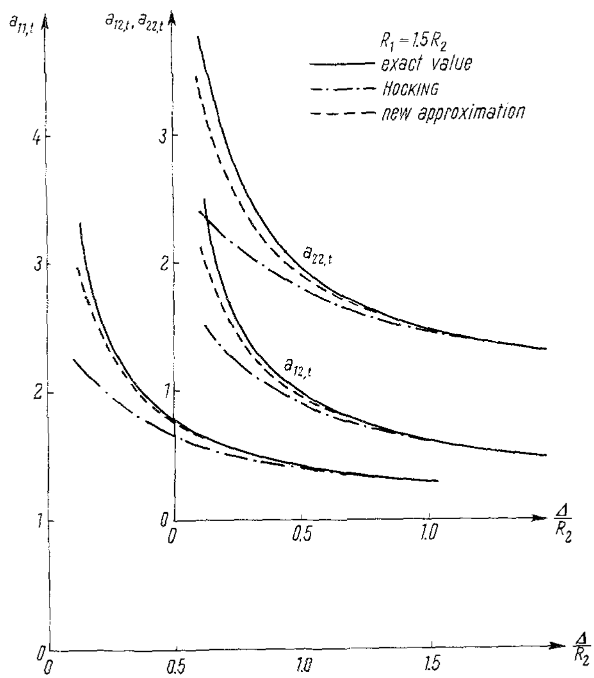


Figure 2
Dependence of coefficients $a_{ik,t}$ on the distance Δ between drop surfaces

This system of equations (13) was programmed for the electronic computer BESM-2 and was solved under the following initial conditions:

$$\text{at} \quad t = 0; \quad x = x_0; \quad z = z_0; \quad w_{10} = 1; \quad u_{10} = 0; \quad w_{20} = a^2; \quad u_{20} = 0 . \tag{15}$$

The grazing trajectory of gravity center of the smaller drop tangent the sphere 0 with its center in the larger drop center and with its radius $r = R_1 + R_2$ (in dimensionless values $r = 1 + a$) was determined for the given values of 1, a, z_0. For this purpose the trajectory of drops was determined at some arbitrary value of the initial impact parameter $x_0 = x_{0,0}$. If this trajectory of the smaller drop went past the sphere 0, the impact parameter two times decreased. If the next trajectory went past the same sphere, the impact parameter two times decreased again. And this was repeated until the trajectory intersected the sphere. Analogous to this if the initially taken trajectory of the smaller drop intersected the sphere, the impact parameter two times increased.

If the next trajectory intersected the same sphere, the impact parameter also two times increased. This was repeated until the trajectory went past the sphere 0. After several of such trajectory computations we obtained two values of the impact parameter ($x_0 = x_{0,i}$ and $x_0 = x_{0,i-1}$).

At one of these values the trajectory intersected the sphere and at the other it went past the sphere. After it a trajectory was calculated with $x_0 = x_{0,i+1} = 1/2$ $(x_{0,i-1} + x_{0,i})$. And then out of these 3 trajectories a pair was chosen one of which intersected the sphere and the other went past it. Such an operation continued until $x_{0,i+1} - x_{0,i}$ did not become less than some value given beforehand by the necessary accuracy of collection efficiency calculations.

Having found in such a way the impact parameter for the grazing trajectory $- x_{0,gr}$ we determine the collection efficiency E of smaller drops by greater ones according to the formula

$$E = x_{0,gr}^2 . \tag{16}$$

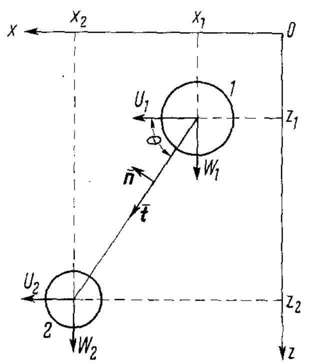


Figure 3
Geometry of the considered problem

The collection efficiency was calculated for three cases. When calculating trajectories for the first case we assumed the impact parameter z_0 of drops very high ($z_0 = 50$) and determined the collection efficiency of gravitational coagulation E_g which was calculated by a number of authors including HOCKING. To determine E_{tg} of turbulent gravitational coagulation on the base of the two-layer model described in the beginning of the paper it was assumed

$$z_0 = l_t + R_1 + R_2 = 2.5\,(R_1 + R_2) \tag{17a}$$

or in dimensionless values

$$z_0 = 2.5\,(1 + a) . \tag{17b}$$

On the base of the same two layer model the collection efficiency E_{teg} of turbulent electrostatic gravitational coagulation was determined. For its calculation electrostatic interaction of opposite charged drops took into account.

Calculated values of collection efficiency are listed in table 3 and figure 3. Collection efficiency values calculated by HOCKING and LANGMUIR are also given for comparison[9]).

Calculations were made at the following values of parameters: $\varrho = 1\ g \cdot cm^{-3}$; $g = 980\ cm \cdot sec^{-2}$; $\eta = 1.8 \cdot 10^{-4} P$ ($R_1 = 25$ and $20\ \mu$: $I = 23.3$ and 11.9, respectively). To calculate the mean value of the parameter $\alpha = \bar{\alpha}$, characterizing electrostatic interaction of particles the values of q_1 and q_2 charges were assumed equal to the mean value of cloud drop charges determined by [4]:

$$|q| = 10^{-4} R \quad (\text{in} \quad CGSE) .$$

Table 3

	$R_1 = 25\ \mu;\ I = 23.3$					$R_1 = 20\ \mu;\ I = 11.9$		
a/E	E_{tg}	E_g ($z_0 = 50$)	E_g HOCKING	E_g LANGMUIR	a/E	E_{tg}	E_g ($z_0 = 50$)	E_{teg} ($\alpha = \bar{\alpha}$)
0.25	$< 10^{-3}$	$< 10^{-3}$	10^{-3}	0.05	0.45	$< 10^{-3}$	$< 10^{-4}$	0.02
0.30	0.27	0.033	0.059	0.16	0.50	0.089	$< 10^{-4}$	0.10
0.40	0.79	0.29	0.33	0.34	0.60	0.24	$\sim 10^{-4}$	0.25
0.55	1.29	0.61	0.66	0.47	0.70	0.10	$< 10^{-4}$	0.12
0.70	1.49	0.69	0.80	0.51	0.75	$< 10^{-3}$	$< 10^{-4}$	0.02
0.85	0.79	0.19	0.44	0.46				
0.88	0.26	$< 10^{-4}$	0.26	0.41				
0.90	$< 10^{-4}$			0.38				

From table 3 and figure 3 the following conclusions can be drawn:

a) Refinement of expressions for hydrodynamical forces carried out in this paper is essential at small values of $d(R < 20\ \mu)$ and it slightly changes the values of the collection officiency in comparison to these calculated by HOCKING if E_g is higher than 0.3 to 0.4.

b) The collection efficiency E_{tg} for turbulent gravitational coagulation is significantly higher than E_g. But values of critical parameters $a = R_2/R_1$ beyond the limits of which ($a > a_{max}$ and $a < a_{min}$) coagulation is impossible ($E = 0$) are not changed by turbulence. It means that critical sizes of cloud drops obtained taking turbulence into account are so that drop growth by coagulation must occur only for drops with the diameter $d > 40\ \mu$. Thus, the account of turbulent does not take off the problems which were put in the beginning of the paper.

c) Due regard for electrostatic forces does not change the situation. Electric charges mean in magnitude in fact do not effect[10]). Charges 3 to 10 times higher the

[9]) Several calculations made by us from HOCKING's formulas gave values of E_g which agree reasonably well with E_g values reported by HOCKING.

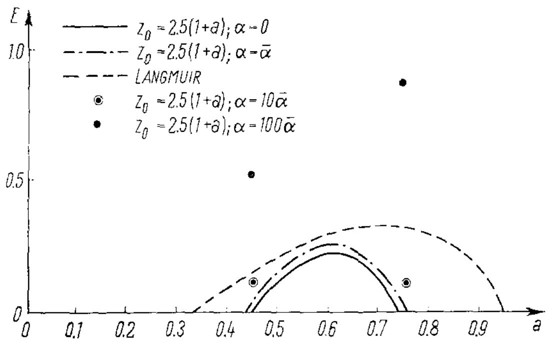
LANGMUIR's calculations are given by us as functions of and a due to the fact that the Stokes number $\kappa = I\ a^2(1 - a^2)$.

[10]) The case when a is very close to 1, demands additional consideration. But even now it is clear that it cannot change the given conclusion.

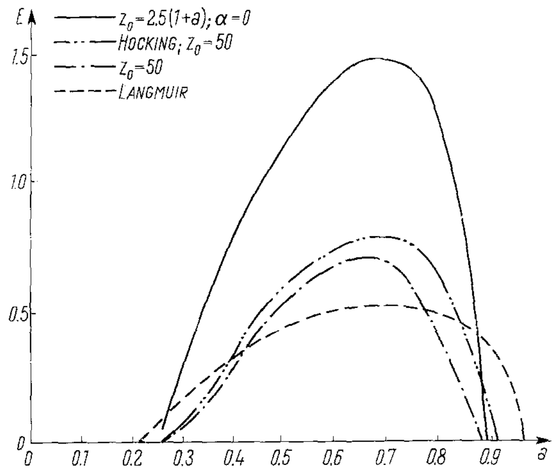
mean ones ($\alpha = 10\,\bar{\alpha}$ and $\alpha = 100\,\bar{\alpha}$) are necessary for significant increase of E to take place (E_{teg} is considerably larger than E_{tg}). However, the appearence of such charges in non-thunderstorm clouds is very unlikely[11]).

Summarizing all this the following can be said. In the considered model at the assumed boundary conditions turbulence provided approach of drops to each other at small distances with relatively high velocities.

Therefore results obtained with the help of computations based upon this model allow to conclude that turbulent diffusion even together with gravitational and electrostatic coagulation cannot provide growth with $R < 18 - 20\ \mu$ by coagulation.



a) $R_1 = 20\ \mu$



b) $R_1 = 25\ \mu$

Figure 4
Dependence of collection efficiency on the ratio a of drop radii

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[11]) In thunderstorm cloud drops are so large that various kinds of coagulation and purely gravitational one among them are possible in them.

We think that formation of rather wide spectrum of cloud drops and appearance of large cloud drops (with $R > 25$ to 30 μ) are connected with condensation processes occurring under of fluctuation of main parameters: flow velocity, temperature and supersaturation. This demands special investigations.

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