

Quantitative Interpretation of Self Potential Anomalies of Some Specific Geometric Bodies

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Summary – Some direct and quantitative methods of SP anomalies caused by some specific geometric bodies have been developed in this paper. The models of current sources which have been considered are i) single pole, ii) a doublet, iii) a pair of single point poles separated by a horizontal distance, iv) single finite line pole, v) single infinite line pole and vi) two similar double infinite vertical line poles separated by a horizontal distance.

Introduction

Interpretations of self potential anomalies are generally qualitative. It is true that if the subsurface causative body is nongeometrical and hence the electrical charge set up nonregular, quantitative interpretation becomes difficult and in some cases even impossible. So some simplified assumptions as regards the shape of geological body causing the SP anomaly therefore obviously become necessary in attempting quantitative interpretations of field data.

The previous works in this line are those of PETROWSKY [1]²⁾, EDGE and LABY [2], HEILAND [3], DE WITTE [4], YUNGUL [5], ROY and CHOWDHURY [6], MEISER [7], PAUL [8], PAUL, DATTA and BANERJEE [9], BANERJEE [10] etc.

It will be relevant to mention here that amongst different quantitative methods of interpretation of SP data, there are some which utilise the whole profile while others utilise some selected points on the profile. It may seem, apparently, that the latter has a distinct disadvantage over the former, but a thorough detailed study will elude this idea. Theoretically, if the causative body is a perfectly geometric one (e.g. sphere, rod, etc.), then the interpretations using either of the methods do not make any difference at all. But, actually, in a field set up where there is not much deviation from the causative body and its geometric model (there are errors of observations too) to make quantitative interpretation effective, we can eliminate those undulations or irregularities by various methods, the easiest of them being smoothing out the curve. Thereafter, we consider the anomaly profile to be due to the assumed geometric model and can interpret it by taking some selected points on the profile. When all the parameters

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²⁾ Numbers in brackets refer to References, page 152.

of the causative body are evaluated, the entire theoretical profile is, then, plotted and compared with the observed profile. If there is fair agreement, the interpretation stands; otherwise it rejects the assumed geometric model for the causative body.

However, before applying any particular method of interpretation, one must have a fair knowledge of the local geology. Let us illustrate it with an example. We know that the nature of SP equianomaly contours are generally, different for different types of electrical charge set up (corresponding to different types of subsurface causative bodies). But one can see from the following study that the SP equianomaly contours are ellipses both for two similar point poles separated by a distance (section iii) and for a single line pole (section iv). In these cases, differentiation should be made on the basis of geological evidences.

Theory

1. Single point pole

It has been pointed out by PAUL, DATTA and BANERJEE [9] in connection with the interpretation of a vertical dipole that 'as the lower end of the dipole extends deeper and deeper, the present method yields results which become more and more uncertain'. In that case, the effect of the lower pole on the anomaly profile being negligible, we can take into consideration only the effect of the upper pole for which the interpretation is given below.

We select our cartesian system of reference such that the surface of observation or datum plane is the plane $z=0$ and Z axis is positive in the vertically downward direction. Let the point pole be situated at the point $(0, 0, H)$.

The equianomaly contours are concentric circles with their centre at the origin. Because of this circular symmetry, it is sufficient to study the profile along any diametral direction, say, along the line $y=0$ on the plane of observation (the plane $z=0$).

If I be the current strength at the source, then the potential at any point $(x, 0, 0)$ is given by

$$V(x, 0, 0) \equiv V(x) = \frac{\rho I}{2\pi} \frac{1}{\sqrt{x^2 + H^2}} \quad (1)$$

where ρ is the resistivity of the surrounding medium.

From relation (1), we can see that $V(x)$ has got one extremum (maximum or minimum depending on positive or negative I) in the finite portion of the curve and it is at $x=0$.

$$V(0) = \frac{\rho I}{2\pi} \frac{1}{H}. \quad (2)$$

Let α and β be the half and quarter width distances of the peak anomaly, that is

$$V(0) = 2 V(\alpha) = 4 V(\beta) \quad (3)$$

where $V(\alpha)$ and $V(\beta)$ are particular cases of (1) as given by

$$V(\alpha) = \frac{\rho I}{2\pi} \frac{1}{\sqrt{\alpha^2 + H^2}}, \quad (4)$$

$$V(\beta) = \frac{\rho I}{2\pi} \frac{1}{\sqrt{\beta^2 + H^2}}. \quad (5)$$

Substituting (2), (4), and (5) in (3), we get

$$H = 0.577 \alpha \quad (6)$$

$$H = 0.258 \beta. \quad (7)$$

The values of α and β which are directly obtained from the profile are substituted in (6) or (7) to give a value for H . Theoretically, the values of H thus calculated should be the same. In practical cases the comparison of the two values as obtained from (6) and (7) will serve as a check on the type of model we have assumed.

2. Vertical and inclined doublets

A doublet is a particular case of a dipole of small dipolar length. So the interpretation of SP anomaly caused by a doublet, either vertical or inclined may be carried out exactly in the same way as suggested by PAUL, DATTA and BANERJEE [9] for the cases of vertical or inclined dipoles. From the set of mathematical curves they have used for solving a pair of transcendental equations, one can see that for values of m slightly greater than 1 (m is the ratio of the depth of the lower pole to the depth of the upper pole, both measured from the free surface i.e. $m > 1$), the resolution of Z (Z is the ratio of the quarter width to the half width of the anomaly profile) is not quite pronounced as seen from Table 1. So a different method of interpretation has

Table 1

m	Z
1.02	1.60858
1.04	1.60866
1.06	1.60881
1.08	1.60900
1.10	1.60925
1.20	1.61114
1.30	1.61391
1.40	1.61736
1.50	1.62134
1.60	1.62572
1.70	1.63040
1.80	1.63532
1.90	1.64044
2.00	1.64565

to be evolved for a dipole of which m is slightly greater than 1 i.e. for a doublet. In that case, the interpretation method consists in evaluating the depth of the centre of the doublet from the free surface and the angle, if there is any, it makes with the vertical.

The interpretation of SP anomaly due to a vertical doublet has been given in HEILAND [3] while in the case of an inclined doublet he has deduced the expression for the potential without offering any method of interpretation.

The nature of SP anomaly contours caused by a vertical or an inclined doublet will be the same as due to a vertical or an inclined dipole respectively, the latter being discussed in detail by PAUL, DATTA and BANERJEE [9].

In the following analysis, a method of interpretation of SP anomaly due to an inclined doublet has been evolved and for completeness, the case of a vertical doublet which has been given in HEILAND is included.

(i) Vertical doublet

Considering the cartesian system of axes as in the previous case, we take a profile along any diametral direction of the circular contours.

If H be the depth of the centre of the doublet from the free surface, then the potential at any point $(x, 0, 0)$ is given by

$$V(x, 0, 0) \equiv V(x) = \frac{K}{(x^2 + H^2)^{3/2}} \quad (8)$$

where K is a constant depending on the geometry and electrical properties of the causative body.

If α and β be the half and quarter width distances i.e. $V(0) = 2 V(\alpha) = 4 V(\beta)$, where $V(0)$ is the maximum or minimum anomaly corresponding to positive and negative current density of the upper pole, then we have

$$H = 1.305 \alpha = 0.811 \beta. \quad (9)$$

(ii) Inclined doublet

In this case there will be two closures in SP equianomaly contours, one corresponding to minimum and the other corresponding to maximum anomaly. We take the profile passing through the two extrema.

If θ be the angle which the doublet makes with the vertical, then the potential at any point $(x, 0, 0)$ is given by

$$V(x, 0, 0) \equiv V(x) = \frac{K}{(x^2 + H^2)^{3/2}} [H \cos \theta + x \sin \theta]. \quad (10)$$

From (10), it follows that $V(x_0) = 0$ when $x_0 = -H \cot \theta$.

Differentiating (10), we can find out the position of minimum and maximum

anomaly. Let these be x_1 and x_2 respectively. Then x_1 and x_2 are given by

$$x_1 = \frac{H}{4} [\sqrt{9 \cot^2 \theta + 8} - 3 \cot \theta] \quad (11)$$

$$x_2 = -\frac{H}{4} [\sqrt{9 \cot^2 \theta + 8} + 3 \cot \theta]. \quad (12)$$

The distances between the zero anomaly point and the two extrema are

$$x_1 - x_0 = \frac{H}{4} [\sqrt{9 \cot^2 \theta + 8} + \cot \theta] = p, \quad \text{say}$$

$$x_2 - x_0 = -\frac{H}{4} [\sqrt{9 \cot^2 \theta + 8} - \cot \theta] = -q, \quad \text{say}.$$

The quantities p and q can readily be obtained from the anomaly profile. Knowing p and q , the unknown parameters H and θ are computed from the following relations:

$$\cot \theta = \frac{1.414(p - q)}{\sqrt{(2p - q)(2q - p)}} \quad (13)$$

$$H = 1.414 \sqrt{(2p - q)(2q - p)}. \quad (14)$$

3. Two similar point poles separated by a distance

With a similar system of reference as in the previous case, let us assume that a pair of point poles both having the same current strength I be situated at $(-d, 0, H)$ and $(d, 0, H)$ respectively where $2d$ is the horizontal separation between the point poles.

In this case the equianomaly contours over the free surface have no longer any circular symmetry but rather look like ellipses. The peak anomaly points are at the centre of those ellipses and a profile in the diametrical direction is symmetric. The major and minor diameters of the ellipses coincide with X and Y axis respectively.

Let us first consider the transverse profile through the origin i.e. the profile in the direction of minor diameter (Y axis); then the SP anomaly at a point $(0, y, 0)$ is given by

$$V(0, y, 0) \equiv V_T(y) = \frac{q I}{2 \pi} \frac{2}{\sqrt{y^2 + H^2 + d^2}}. \quad (15)$$

Relation (15) is analogous to relation (1) so that the transverse profile can be interpreted in the same way as in the case of a single point pole. If α be the half width distances along the transverse profile, then we get a relation similar to (6) which is

$$\sqrt{H^2 + d^2} = \frac{\alpha}{\sqrt{3}} \quad \text{or,} \quad \alpha^2 = 3(H^2 + d^2). \quad (16)$$

Another relation between H and d may be deduced that will enable a unique determination of their values.

We now consider the profile in the direction of major diameter (X axis) i.e. the longitudinal profile through the origin. The SP anomaly at a point $(x, 0, 0)$ is given by

$$V(x, 0, 0) \equiv V_L(x) = \frac{\rho I}{2\pi} \left[\frac{1}{\sqrt{H^2 + (x-d)^2}} + \frac{1}{\sqrt{H^2 + (x+d)^2}} \right]. \tag{17}$$

The SP anomaly at a distance α , the half width distance along the transverse profile is

$$\left. \begin{aligned} V_L(\alpha) &= \frac{\rho I}{2\pi} \left[\frac{1}{\sqrt{H^2 + (\alpha-d)^2}} + \frac{1}{\sqrt{H^2 + (\alpha+d)^2}} \right] \\ &= \frac{\rho I}{2\pi} \left[\frac{1}{\sqrt{\frac{4}{3}\alpha^2 - 2\alpha d}} + \frac{1}{\sqrt{\frac{4}{3}\alpha^2 + 2\alpha d}} \right] \end{aligned} \right\} \tag{18}$$

$$\left. \begin{aligned} \therefore \frac{V_L(\alpha)}{V(0)} &= \frac{1}{4} \left[\frac{1}{\sqrt{1 - 1.5 \frac{d}{\alpha}}} + \frac{1}{\sqrt{1 + 1.5 \frac{d}{\alpha}}} \right] \\ &\equiv F\left(\frac{d}{\alpha}\right), \text{ say.} \end{aligned} \right\} \tag{19}$$

Fig. (1) shows a plot of the curve of p versus $F(p)$ where

$$F(p) = \frac{1}{4} \left[\frac{1}{\sqrt{1 - 1.5 p}} + \frac{1}{\sqrt{1 + 1.5 p}} \right].$$

It may be seen from relation (16) that the maximum value of $p=d/\alpha$ is $1/\sqrt{3}$.

With the above curve, the interpretation is carried out in the following way:

(i) From the transverse profile through the origin i.e. along the minor diameter of the ellipses, the half value distance α is known.

(ii) From the longitudinal profile through the origin i.e. along the major diameter of the ellipses, the SP anomaly at the distance α i.e. $V_L(\alpha)$ is noted. Since $V(0)$ is known, the ratio $V_L(\alpha)/V(0)$ is calculated.

(iii) From the theoretical curve of $F(p)$, the value p_0 of p for which $F(p_0)$ equals the above ratio is found out.

(iv) d and H are computed from the relations $d=p_0 \alpha$ and $H=\sqrt{\frac{1}{3}-p_0^2} \alpha$.

The resolution of $p-F(p)$ curve is not quite sharp at the origin which is evident physically from the fact that two identical point poles either separated by a very small distance or at larger depths behave nearly like a single point pole of different strength.

In our above analysis, there is no imposition on the values of d/H . In fact when $d/H < 1/\sqrt{2}$, the equianomaly contours are ellipses, the longitudinal and transverse

profile have one peak each as mentioned and interpretation may be carried out as described above. But our above analysis still holds good when $d/H > 1/\sqrt{2}$; in this case only the outermost equianomaly contours are ellipses and there are two closures inside. A profile (longitudinal) through the closures will have three extrema viz. if the current strength of the point poles is positive, then there are two maxima and one minimum, the latter one being at the origin. However, a transverse profile through the origin, which can easily be recognised from the contours, shows only one peak.

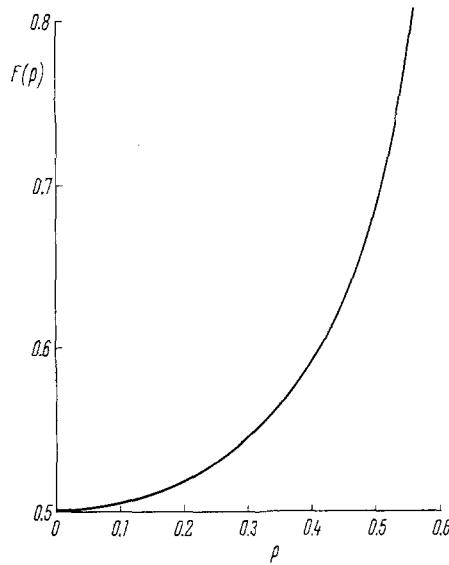


Figure 1

Theoretical curve of $F(\rho)$ to aid interpretation due to a pair of single point poles separated by a distance

4. Single finite linepole

With the cartesian system of reference as in the previous case, we consider the linepole of finite length $2l$ having current density I per unit length extend from $(-l, 0, H)$ to $(l, 0, H)$.

The SP anomaly at any point $(x, y, 0)$ i.e. at any point over the free surface is

$$V(x, y, 0) \equiv V(x, y) = \frac{\rho I}{2\pi} \ln \frac{\sqrt{(x-l)^2 + y^2 + H^2} + \sqrt{(x+l)^2 + y^2 + H^2} + 2l}{\sqrt{(x-l)^2 + y^2 + H^2} + \sqrt{(x+l)^2 + y^2 + H^2} - 2l} \tag{20}$$

From relation (20), we have

$$\frac{e^{(2\pi/\rho I)V(x,y)} + 1}{e^{(2\pi/\rho I)V(x,y)} - 1} = \frac{\sqrt{(x-l)^2 + y^2 + H^2} + \sqrt{(x+l)^2 + y^2 + H^2}}{2l}$$

or,

$$2 l \coth \frac{\pi V(x, y)}{\varrho I} = \sqrt{x^2 + y^2 + l^2 + H^2 - 2 l x} + \sqrt{x^2 + y^2 + l^2 + H^2 + 2 l x}$$

or,

$$2 l^2 \coth^2 \frac{\pi V(x, y)}{\varrho I} - (x^2 + y^2 + l^2 + H^2) = \sqrt{(x^2 + y^2 + l^2 + H^2)^2 - 4 l^2 x^2}$$

or,

$$\frac{x^2}{\cosh^2 \frac{\pi V(x, y)}{\varrho I} \left[l^2 \operatorname{cosech}^2 \frac{\pi V(x, y)}{\varrho I} - H^2 \right]} + \frac{y^2}{l^2 \operatorname{cosech}^2 \frac{\pi V(x, y)}{\varrho I} - H^2} = 1. \quad (21)$$

From (21), it is quite evident that the equipotential contours over the free surface are ellipses with the origin at the common centre of the ellipses, the major and the minor diameters coinciding with the X axis the Y axis respectively.

For any particular $V(x, y) = V$, the ratio of the major axis (a) to the minor axis (b) is

$$\frac{a}{b} = \cosh \frac{\pi V}{\varrho I}. \quad (22)$$

Considering the transverse profile through the origin i.e. along the minor diameter of the ellipses, the SP anomaly at a point is

$$V(0, y, 0) \equiv V(y) = \frac{\varrho I}{2 \pi} \ln \frac{\sqrt{y^2 + l^2 + H^2} + l}{\sqrt{y^2 + l^2 + H^2} - l}. \quad (23)$$

It can, easily, be seen that $y=0$ is the position of the only extremum in the anomaly profile. Let α be the half anomaly value so that $V(0) = 2 V(\alpha)$ where

$$V(0) = \frac{\varrho I}{2 \pi} \ln \frac{\sqrt{l^2 + H^2} + l}{H} \quad (24)$$

$$V(\alpha) = \frac{\varrho I}{2 \pi} \ln \frac{\sqrt{\alpha^2 + l^2 + H^2} + l}{\sqrt{\alpha^2 + l^2 + H^2} - l}. \quad (25)$$

From (24) and (25), we have the following relations

$$\frac{l}{H} = \sinh \frac{\pi V(0)}{\varrho I} \quad (26)$$

$$\alpha^2 = H^2 \left[1 + 2 \cosh \frac{\pi V(0)}{\varrho I} \right]. \quad (27)$$

A direct interpretation of the observed anomaly profile in terms of a single finite line pole is possible with the help of the following steps:

i) From any equianomaly contour, the value of ρI is determined utilising the relation (22).

ii) Substituting the value of ρI in (27), H is easily obtainable while relation (26) helps to find l .

An alternative approach using only the transverse profile through the origin (along the minor diameter of the ellipses) is proposed below:

From (23), we can write

$$\coth \frac{\pi V(y)}{\rho I} = \frac{\sqrt{y^2 + l^2 + H^2}}{l} \tag{28}$$

Simplifying (23), we have

$$y^2 = l^2 \operatorname{cosech}^2 \frac{\pi V(y)}{\rho I} - H^2 \tag{29}$$

Since $y=0$ is the position of extremum in the anomaly profile, H can be eliminated in the following way

$$y^2 = l^2 \left[\operatorname{cosech}^2 \frac{\pi V(y)}{\rho I} - \operatorname{cosech}^2 \frac{\pi V(0)}{\rho I} \right] \tag{30}$$

Let us select a pair of points y_1 and y_2 such that

$$\left. \begin{aligned} V(y_1) &= m_1 V(0), & 0 < m_1 < 1 \\ V(y_2) &= m_2 V(0), & 0 < m_2 < 1. \end{aligned} \right\} \tag{31}$$

From (30) and (31), we get

$$\frac{y_1}{y_2} = \left[\frac{\operatorname{cosech}^2 m_1 p - \operatorname{cosech}^2 p}{\operatorname{cosech}^2 m_2 p - \operatorname{cosech}^2 p} \right]^{1/2} \equiv f(p), \text{ say} \tag{32}$$

where $p = \pi V(0)/\rho I$.

Table 2 furnishes the behaviour of $f(p)$ for different values of p for $m_1=0.25, m_2=0.50; m_1=0.50, m_2=0.75$ and $m_1=0.25, m_2=0.75$. The column for $2l/H (= 2 \sinh p)$ i.e. the ratio of the length of the line pole to its depth from the free surface has also been included in the table.

From the observed profile the value of the left hand side of the expression (32) is determined for any pair of m_1 and m_2 which has been included in the table from which p can be obtained directly. Knowing p, l and H are computed from the relations (26) and (30).

From table 2, it is apparent that the resolution of $f(p)$ versus $2l/H$ is not quite pronounced for small values of $2l/H$ which is to be considered as one of the demerits of the present method.

Table 2

p	$\frac{2I}{H}$	$f(m_1, m_2, p)$	$f(m_1, m_2, p)$	$f(m_1, m_2, p)$
		$m_1 = 0.25$ $m_2 = 0.50$	$m_1 = 0.50$ $m_2 = 0.75$	$m_1 = 0.25$ $m_2 = 0.75$
0.1	0.200334	2.23607	1.96396	4.39156
0.2	0.402672	2.23609	1.96399	4.39166
0.3	0.609041	2.23618	1.96412	4.39213
0.4	0.821505	2.23642	1.96446	4.39335
0.5	1.04219	2.23690	1.96515	4.39585
0.6	1.27331	2.23775	1.96639	4.40042
0.7	1.51717	2.23911	1.96827	4.40718
0.8	1.77621	2.24112	1.97104	4.41734
0.9	2.05303	2.24392	1.97485	4.43141
1.0	2.35040	2.24764	1.97986	4.45003
1.2	3.01892	2.25840	1.99401	4.50327
1.4	3.80860	2.27430	2.01436	4.58127
1.6	4.75114	2.29607	2.04151	4.68744
1.8	5.88435	2.32423	2.07579	4.82461
2.0	7.25372	2.35913	2.11744	4.99524
2.2	8.91421	2.40103	2.16641	5.20160
2.4	10.9325	2.45006	2.22282	5.44604
2.6	13.3895	2.50634	2.28664	5.73110
2.8	16.3838	2.56995	2.35785	6.05958
3.0	20.0358	2.64097	2.43649	6.43470

5. Single infinite line pole

We select our frame of reference such that the datum plane i.e. the free surface is the plane $z=0$ and the Z axis is positive in the vertically downward direction. Let the line pole extend parallel to the Y axis from $-\infty$ to ∞ and $(0, H)$ be the point of intersection of the line pole with the XZ plane.

The equianomaly contours are straight lines parallel to the Y axis being spaced equally at equal intervals. A profile parallel to the X axis shows a symmetric curve with a peak (maximum or minimum according as the current density of the line pole is positive or negative) at the origin.

If I be the current density per unit length of the line pole, then the potential at any point on such a profile is

$$V(x, y, 0) \equiv V(x) = \frac{qI}{2\pi} \ln \frac{x^2 + H^2}{CL^2}. \quad (33)$$

where L and C are unknown quantities, the former having dimension of length while the latter is dimensionless; the quantities are such that they ensure one of the very essential properties of $V(x)$ i.e. $V(x)=0$ as x tends to infinity.

From the well defined part of the curve, we select a pair of points (α, β) such that

$$V(0) - V(\alpha) = 2[V(0) - V(\beta)]. \quad (34)$$

Substituting (33) in (34), we get

$$\frac{\rho I}{2\pi} \ln \frac{H^2}{H^2 + \alpha^2} = \frac{\rho I}{\pi} \ln \frac{H^2}{H^2 + \beta^2}, \quad (35)$$

which when simplified reduces to

$$H = \frac{\beta^2}{\sqrt{\alpha^2 - 2\beta^2}}. \quad (36)$$

Thus H , the depth from the free surface to the line pole can, be calculated from (36) after obtaining the values of α and β from the anomaly profile with the help of (22). A number of pairs of values like (α, β) can be obtained from the well defined part of the profile. If the values of H derived from those sets are compatible (theoretically they should give the same value for H), then the interpretation stands. Otherwise, it rejects the assumed geometry of the causative body. Another precondition for this assumed geometry is $\alpha > \sqrt{2}\beta$.

Since, in this method, we are using the difference of potential values, the constant regional (if there be any) is eliminated. It means that in this case no correction is necessary if the base station is within the zone of influence of the causative body (BANERJEE 1970).

6. Two similar double infinite vertical line poles separated by a distance

We take the coordinate system exactly like the previous section. In this case $(b, 0, H-a)$, $(-b, 0, H-a)$, $(b, 0, H+a)$ and $(-b, 0, H+a)$ are the points of intersection of the line poles (which contain in a vertical plane) with the XZ plane, $H-a$ and $H+a$ denote respectively the depth to the upper and the lower line pole and $2b$ is their horizontal separation.

In our analysis, we will consider the case when b is small compared to other length dimension i.e. H and a ; in other words we will retain upto second power terms of b , the higher powers will be neglected.

The equianomaly contours, just like the previous case, are straight lines parallel to Y axis and a profile parallel to X axis is symmetric with a peak.

If I is the current density per unit length of the line pole – the current line with negative density is situated nearer the free surface, then the potential at any point on such a profile is

$$V(x) = \frac{\rho I}{4\pi} \ln \left[\frac{(x+b)^2 + (H-a)^2}{(x+b)^2 + (H+a)^2} \frac{(x-b)^2 + (H-a)^2}{(x-b)^2 + (H+a)^2} \right]. \quad (37)$$

The right hand side of (37) can be expanded for large value of x and we have the following asymptotic relation of $V(x)$ viz.

$$\lim_{x \rightarrow \infty} x^2 V(x) = -\frac{2}{\pi} \rho I a H. \quad (38)$$

Let $\phi(x)$ be the potential at x due to a single infinite diline pole (BANERJEE, 1970) i.e.

$$V(x) \equiv \phi(x) \quad \text{when} \quad b = 0$$

where

$$\phi(x) = \frac{\rho I}{2\pi} \ln \frac{x^2 + (H - a)^2}{x^2 + (H + a)^2}. \tag{39}$$

We have the following functional relation between $V(x)$ and $\phi(x)$ i.e.

$$V(x) = \frac{1}{2} \phi(x + b) + \frac{1}{2} \phi(x - b). \tag{40}$$

Expanding by Taylor's theorem and retaining terms up to b^2 we have

$$V(x) = \phi(x) + b^2 \phi''(x). \tag{41}$$

Integrating both sides of (41) with respect to x .

$$\left. \begin{aligned} \int_0^\infty V(x) dx &= \int_0^\infty \phi(x) dx + b^2 \phi'(x) \Big|_0^\infty \\ &= \int_0^\infty \phi(x) dx \end{aligned} \right\} \tag{42}$$

since $\phi'(0)$ and $\lim_{x \rightarrow \infty} \phi'(x)$ are both zero.

Now

$$\int \phi(x) dx = \frac{\rho I}{2\pi} \left[\int \ln [x^2 + (H - a)^2] dx - \int \ln [x^2 + (H + a)^2] dx \right].$$

Integrating by parts,

$$\int \phi(x) dx = \frac{\rho I}{2\pi} \left[x \ln \frac{x^2 + (H - a)^2}{x^2 + (H + a)^2} + 2 \left\{ (H - a) \tan^{-1} \frac{x}{H - a} - (H + a) \tan^{-1} \frac{x}{H + a} \right\} \right]$$

Applying L'Hospital's theorem, we can show

$$\lim_{x \rightarrow \infty} x \ln \frac{x^2 + (H - a)^2}{x^2 + (H + a)^2} = 0.$$

From (42),

$$\int_0^\infty V(x) dx = -a \rho I = -A, \quad (\text{say}). \tag{43}$$

To find the position of peak negative anomaly, we see that when $b=0$, $x=0$ is the position of minimum. For non-vanishing b , let $x=\lambda b^2$ be the position of minimum.

Differentiating (37) with respect to x , putting $x=\lambda b^2$ and equating $V'(\lambda b^2)=0$, we get $\lambda=0$. It follows, therefore, that the origin is the position of minimum. This is also physically evident from symmetry.

Let x_i be the position of the point of inflexion of the $\phi(x)$ curve.

$$\phi''(x_i) = 0 \quad \text{so that} \quad V(x_i) = \phi(x_i).$$

Differentiating equation (39) twice and equating it to zero, we obtain, after algebraic reduction, the expression for x_i which is

$$x_i = \frac{-(H^2 + a^2) + 2\sqrt{H^4 - H^2 a^2 + a^4}}{3} \tag{44}$$

from which we get

$$a^2 = \sqrt{H^2 + x_i^2} [\sqrt{H^2 + x_i^2} - 2 x_i]. \tag{45}$$

Now

$$\phi(x_i) = \frac{\rho I}{2\pi} \ln \frac{H^2 + a^2 - 3 a H + \sqrt{H^4 - H^2 a^2 + a^4}}{H^2 + a^2 + 3 a H + \sqrt{H^4 - H^2 a^2 + a^4}}$$

or,

$$\frac{H^2 + a^2 + \sqrt{H^4 - H^2 a^2 + a^4}}{3 a H} = \coth \frac{\pi |\phi(x_i)|}{\rho I}$$

as $\phi(x_i) = -|\phi(x_i)|$, where the modulus denotes the numerical value.

Again, since $\phi(x_i) = V(x_i)$, we derive the expression for a .

$$\left. \begin{aligned} a &= \frac{H^2 + a^2 + \sqrt{H^4 - H^2 a^2 + a^4}}{3 H} \tanh \frac{\pi |V(x_i)|}{\rho I} \\ &= \frac{H^2 + a^2 + \sqrt{H^4 - H^2 a^2 + a^4}}{3 H} \tanh \frac{\pi a |V(x_i)|}{A} \end{aligned} \right\} \tag{46}$$

From (37),

$$V(0) = \frac{\rho I}{2\pi} \ln \frac{b^2 + (H - a)^2}{b^2 + (H + a)^2}$$

or,

$$\left(\frac{H - a}{H + a}\right)^2 \left[1 + \frac{4 a H}{(H^2 - a^2)^2} b^2\right] = e^{-2\pi a |V(0)|/A}$$

correct up to terms containing b^2 .

$$\therefore b^2 = \frac{(H + a)^2}{4 a H} [(H + a)^2 e^{-2\pi a |V(0)|/A} - (H - a)^2]. \tag{47}$$

Method of interpretation

The application of the method of iteration for finding the roots of transcendental equations, which incidentally is not new in the literature of exploration geophysics (PAUL, DATTA and BANERJEE [10], PAUL and BANERJEE [11]) is being utilised in this method of interpretation. The successive steps are as follows:

i) From the observed profile, calculate the area of the anomaly curve under X axis i.e. $\int_0^\infty V(x) dx = -A$ (say).

ii) From the asymptotic part of the curve, find the limiting value of $x^2 V(x)$ for large values of x .

iii) Compute H from the relation

$$H = \frac{\pi |\lim_{x \rightarrow \infty} x^2 V(x)|}{2A} \quad \text{where} \quad |\lim_{x \rightarrow \infty} x^2 V(x)|$$

is the numerical value of $\lim_{x \rightarrow \infty} x^2 V(x)$.

iv) Assume a value for a , determine x_i from (44) and note $V(x_i)$ from the profile.

v) Substitute the values of a , $V(x_i)$ and H in the right hand side of (46) and get the next iterative value of a . Repeat the cycle till $|a_{i+1} - a_i| < \epsilon$ where ϵ is the accuracy desired.

vi) Calculate b from relation (47).

Discussion

It may be mentioned that both the computations for calculating the area of the anomaly profile under X axis and for estimating the value of $\lim_{x \rightarrow \infty} x^2 V(x)$, we require the profile extended to a great length; its tail part is very essential for interpretation by this method.

The area of the profile under X axis can be computed, fairly accurately, for practical purposes, either by a planimeter or by graphical means.

For finding the value of the expression $\lim_{x \rightarrow \infty} x^2 V(x)$, we consider a few points at the farthest end of the curve and calculate the expression at those points and take their average.

The starting value a_1 of a for the iterative process is determined by substituting the position of inflexion point (x_i) of the curve in the relation (45).

This method can also be applied for interpretation due to a thin double infinite line pole i.e. for $b=0$ (BANERJEE, 1970). But in such a case, the technique developed by the author is much simpler than the present method.

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