The Prism Method for Terrain Corrections Using Digital Computers

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Summary – In the prism method of making terrain corrections, the topography is approximated by a model consisting of right rectangular prisms. The vertical component of the gravitational attraction of each prism is calculated and the sum of these components gives the terrain correction.

The prism method as programmed has no computational limitations. It can be used on all sizes of computers; it can be applied to a large area with any fine grid interval; it can be processed in a single run and yet provides complete flexibility for both research and routine computations. This has been achieved by breaking up larger areas into regions which fit into the computer memory. The contributions of these regions are automatically summed up for each station. While processing each region, various controls may be used at each station to exclude the contribution of a distant part of the area, to use approximate expressions farther from the station, to print out details around the station. There is also provision to refine the model by using smaller prisms around each computation point. Thus full use of elevation control can be made to calculate the terrain correction, the accuracy of which depends only on the quality of the input data.

The prism method has been used to calculate terrain corrections for 130 stations in the New Quebec crater area. For five of these stations terrain corrections were also calculated by using Hammer's template. The two independent sets of values differ by less than four per cent.

1. Introduction

The purpose of this paper is to present the prism method of calculating terrain corrections for gravity data. This method has been programmed for a digital computer and it lends itself easily to full automation in the data acquisition stage requiring input over rectangular grids.

Since the gravitational attraction of irregular topography can not be expressed analytically, terrain corrections must be calculated by some method of numerical integration. Several methods employ templates based on cylindrical coordinate systems by which the area around the gravity station is subdivided by concentric circles and radial lines into a number of compartments. The gravitational attraction of the compartments are tabulated as the function of elevation difference between the station and the compartment. The details of this method are discussed by Hayford and Bowie [2] in connection with geodetic computations. To utilize the precision of measurements in gravity prospecting Hammer [3] refined the procedure by subdividing some of the larger compartments and calculating new tables. Some later modifications

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[2]) Numbers in brackets refer to References, page 39.

were made to increase the accuracy by either changing the radii of circles or further subdividing the region near the computation points.

Recently computer oriented systems have been described by BOTT [1], KANE [4] and KARLEMO [5]. In the methods presented by BOTT and KANE the terrain correction for the inner zone (about 2 kilometers square) must be calculated by other means (templates). The computer program calculates the terrain correction from the rest of the area. In the computations approximate expressions are used. These approximations are adequate since the elements are not too close to the computation point. On the other hand KARLEMO includes the near region as well. In his method, based upon a cylindrical coordinate system, the computations are carried out in two stages: the input data are selected along eight symmetrical radial directions from two quadratic nets; the denser net is for the near zone and the coarser one for the more distant region.

2. *The prism method*

The principle of the method described here is very simple and it makes use of a closed expression to calculate the exact value of the vertical component of the gravitational attraction of a right rectangular prism as derived by NAGY [6]. The actual topography is approximated by a model consisting of prisms. The exact terrain correction for the model is then calculated as the sum of the gravitational attractions of all prisms used to represent the topography.

Some terms are defined now. Let R be the region of interest which is subdivided into area elements $dA = dx\,dy$ by superimposing a rectangular grid over it. The vertical coordinates h of each element are provided by the estimates of the representative elevation values. Then the vertical component of the gravitational attraction of a mass element ($dm = \varrho\, dA\, h$) provides the terrain correction at a point P due to that element. This value is a function of the 'coordinates' of the element only. Designating the terrain correction for the mass element as δg_P one can write

$$\delta g_P = \varrho\, f(x, y, h)\, dA\,. \tag{1}$$

The terrain correction for the region, $\varDelta g_R$, may be obtained by summing up the contributions of all elements over the region, that is

$$\varDelta g_R = \sum_R \varrho\, f(x, y, h)\, dA \equiv \sum_R \delta g_P\,. \tag{2}$$

It is interesting to note that in the template procedure the area element changes whereas in the prism method it remains constant. Using the template procedure a change of origin requires a new set of elevation values; on the other hand the prism method, with constant dA over R, requires new information only if a refinement of the model (see Refinement Control) around the station is required. By using an exact expression in the computations, the station is not restricted to be at the center of the area element.

Some details of the prism method are presented now. Let D, the domain of interest, be subdivided into n rectangular areas in any desired manner (see figure 1) so that

$$D = R_1 + R_2 + \cdots + R_n\,. \tag{3}$$

Then the grid spacing for each R_i is defined separately and independently.

We also define a transform function T which, when applied to a given R_i, gives the terrain correction due to that region. If D is subdivided, as indicated in figure 1, then two cases are possible:

a) the computation point P is outside of the region under consideration (e.g. point P and say R_2 in figure 1). This is the ordinary case, no special treatment is necessary;

b) the computation point is within the region (point P and R_3 in figure 1). Since the original grid spacing may not represent the topography adequately around the station, a refinement of the model may become necessary. This refinement is done by using smaller and smaller grids as the computation point is approached. As a consequence of using an exact expression to calculate the gravitational attraction of a prism, no restriction is put on the degree of refinement of the model. The selection of the smaller grids is governed by the topography in the vicinity of the station. The refinement of the model for R_k is symbolically designated as

$$R_k \mid d$$

where d is the degree of refinement. In figure 5 a possible model refinement ($R_{3/2}$) is shown (explained in detail under Refinement Control in the Control Parameters section).

In general one region, R_k, will contain the station for which the above procedure will be applicable.

The terrain correction may now be obtained simply by applying the transform function T to the domain of interest D, that is:

$$\Delta g = T\,D = T(R_1 + R_2 + \cdots + R_k \mid d + \cdots + R_n)$$

or

$$\Delta g = T \sum^{n} R_i\,. \tag{4}$$

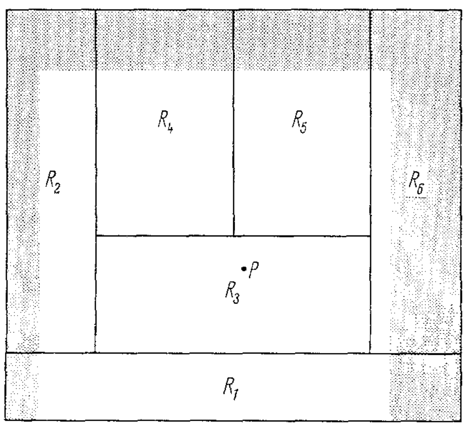


Figure 1
Subdivision of a domain into regions

It remains only to define explicitly the meaning of T. Taking the region R_1 as an example, it is clear from equation (2) that the terrain correction due to this region is

$$\Delta g = \sum_{R_1} f(x, y, h)\, dA_1 \,. \tag{5}$$

that is, T is a function of the coordinates of the element dA_1. This element with height h (elevation difference) defines a right rectangular prism (figure 2). The vertical component of the gravitational attraction of the prism as derived by NAGY (op. cit.) is as follows

$$\delta g_P = G\,\varrho \,\Big|\,\Big|\,\Big|\, x \ln(y + r) + y \ln(x + r) - z \arcsin \frac{z^2 + y^2 + y\,r}{(y + r)\sqrt{y^2 + z^2}} \Big|_{x_1}^{x_2}\Big|_{y_1}^{y_2}\Big|_{z_1}^{z_2} . \tag{6}$$

Substituting the limits and simplifying by letting $z_1 = 0$ and $z_2 = h$, equation (6) takes the form

$$\left.\begin{aligned}
\Delta g = G\,\varrho \Bigg[& x_2 \left\{ \ln \frac{y_2 + \sqrt{x_2^2 + y_2^2}}{y_2 + \sqrt{x_2^2 + y_2^2 + h^2}} - \ln \frac{y_1 + \sqrt{x_2^2 + y_1^2}}{y_1 + \sqrt{x_2^2 + y_1^2 + h^2}} \right\} \\
& - x_1 \left\{ \ln \frac{y_2 + \sqrt{x_1^2 + y_2^2}}{y_2 + \sqrt{x_1^2 + y_2^2 + h^2}} - \ln \frac{y_1 + \sqrt{x_1^2 + y_1^2}}{y_1 + \sqrt{x_1^2 + y_1^2 + h^2}} \right\} \\
& + y_2 \left\{ \ln \frac{x_2 + \sqrt{x_2^2 + y_2^2}}{x_2 + \sqrt{x_2^2 + y_2^2 + h^2}} - \ln \frac{x_1 + \sqrt{x_1^2 + y_2^2}}{x_1 + \sqrt{x_1^2 + y_2^2 + h^2}} \right\} \\
& - y_1 \left\{ \ln \frac{x_2 + \sqrt{x_2^2 + y_1^2}}{x_2 + \sqrt{x_2^2 + y_1^2 + h^2}} - \ln \frac{x_1 + \sqrt{x_1^2 + y_1^2}}{x_1 + \sqrt{x_1^2 + y_1^2 + h^2}} \right\} \\
& + h \Bigg\{ \arcsin \frac{y_2^2 + h^2 + y_2\sqrt{x_2^2 + y_2^2 + h^2}}{\left(y_2 + \sqrt{x_2^2 + y_2^2 + h^2}\right)\sqrt{y_2^2 + h^2}} \\
& - \arcsin \frac{y_2^2 + h^2 + y_2\sqrt{x_1^2 + y_2^2 + h^2}}{\left(y_2 + \sqrt{x_1^2 + y_2^2 + h^2}\right)\sqrt{y_2^2 + h^2}} - \arcsin \frac{y_1^2 + h^2 + y_1\sqrt{x_2^2 + y_1^2 + h^2}}{\left(y_1 + \sqrt{x_2^2 + y_1^2 + h^2}\right)\sqrt{y_1^2 + h^2}} \\
& + \arcsin \frac{y_1^2 + h^2 + y_1\sqrt{x_1^2 + y_1^2 + h^2}}{\left(y_1 + \sqrt{x_1^2 + y_1^2 + h^2}\right)\sqrt{y_1^2 + h^2}} \Bigg\}\Bigg] .
\end{aligned}\right\} \tag{7}$$

This is the transform function T which, when applied to all elements of D, produces the terrain correction. In other words equation (4) is an operator equation: T, the transform function, operates on D which contains n subsets $R_1, R_2, \ldots, R_n$, which all in turn have elements $dA_1, dA_2, \ldots, dA_n$ on which T comes directly into play. Thus equation (4) summarizes in brief the procedure to obtain the terrain correction.

3. Input data

The preparation of input for the computation is simple: the input data for each region (R_i) consist of the elevation matrix, which may contain a single element, and station cards; for each computation point a station card is required which contains station and control parameters. The input data for the area consist of the assembly

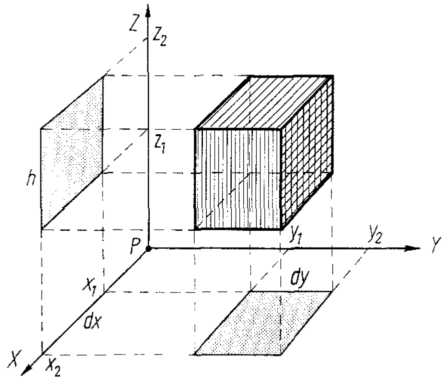


Figure 2
The right rectangular prism

of the input data of all regions put together in any order for the computation. If none of the optional controls are used then the terrain correction using equation (7) is automatically obtained from the domain of interest for each station.

The output for each station comprises the station and control parameters, and the terrain correction.

4. Optimum use of computer memory

So far we have discussed the transform function T of equation (4), and also the possibility of subdividing D into smaller regions has been indicated. Subdivision becomes necessary if D contains more elements than can fit into the computer memory. In this case the regions are selected in such a way that for each R_i the storage requirement should not exceed the available computer memory. Usually the elevation values for a particular region are represented in matrix form in the computer. The size of this array, i.e. the maximum number of rows and columns, must explicitly be given in the *dimension* statement. Let the name of the elevation matrix be A with maximum dimensions M and N such that M times N fits into the computer memory. These values can be chosen freely, but once they are selected they cannot be changed by the program. This means that if $M1$ and $N1$ are the maximum dimensions for R_i, then the data can be processed only if the two conditions $M \geq M1$ and $N \geq N1$ are simultaneously satisfied. In figure 1, each of the regions $R_1 \ldots R_6$ was selected to require the same number of storage locations, say L. If only L locations are available then the computation for this particular subdivision cannot be carried out in a single run; either six runs are required, in which case the *dimension* statement must be changed six times; or the subdivision must be restricted to one single allowable dimension, say that of R_1, and then the area must be subdivided into 'strips' similar to and not larger than R_1. Neither of these possibilities is attractive and both can be overcome by changing the two-dimensional array into a one-dimensional form. This procedure of subscript (or dimension) reduction is quite general but it is shown here only for the above case. Let I and J be the index values of a particular element of A (see figure 3), then the index K of the one-dimensional array can be calculated as

$$K = N * (I - 1) + J .$$

Then if B is the name of the one-dimensional array with a maximum dimension of L, than all matrices representing $R_1, R_2, \ldots, R_6$ in figure 1 can be put into this form, thus achieving single processing without restricting the subdivision. It is noted here that the one-dimensional representation of a two-dimensional array is a built-in feature of the program. The user shall consider his arrays conveniently in their two-dimensional form but without any restriction on the 'form' of the array.

Equivalent Representation

Two dimensional array: $A(M, N)$

M and N are the maximum values for I and J

One particular element: $A(I, J)$

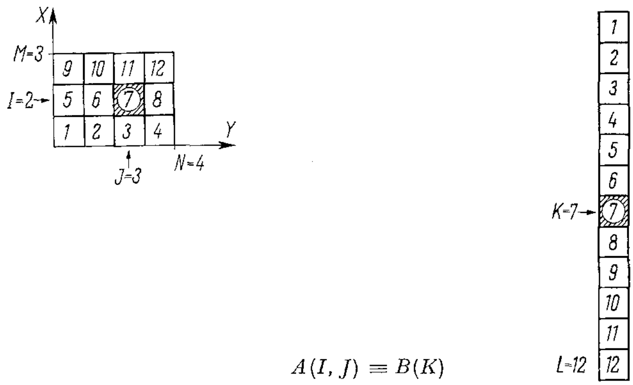
One dimensional array: $B(L)$

L is the maximum value for K such that:

$$L = M * N$$

Corresponding element $B(K)$ where

$$K = N * (I - 1) + J$$



$$A(I, J) \equiv B(K)$$

Figure 3
The relation between two and one dimensional arrays

5. *Control parameters*

Some specific features of the program based upon the method summarized by equation (4) are described below.

Area control: For some stations contributions from only a part of D may be needed for the computations (unshaded part in figure 1). Minimum and maximum for both subscripts representing the elevation matrix can be specified. For example if $M = 20$ and the controls 5 and 15 are specified this means that the contributions from the first 4 rows and the last 5 rows of the elevation matrix will not be included in the computation.

Radius control: In some applications the full accuracy of equation (7) may not be required all over the domain. In this case a circle of radius R with centre at the station may be specified (see figure 4); then outside of this region an approximate transform function may be used.

Slope control: An approximate transform function may be completely adequate, except for one or two compartments with large elevation differences. In this case instead of using equation (7), it is possible to specify a maximum allowable slope, drawn from the station to the compartment (see figure 4). When the radius control is used the slope is evaluated for each compartment: if it exceeds the value specified, then equation (7) is used instead of an approximate expression.

Printing control: One may want to learn some details of the computation in the vicinity of the station. For example if the station is close to water, what are the magnitudes of the contributions of water and land compartments? Or one may wish to study the effect of subdivision of some compartment on the computed value. The printing control may be used for getting such details for 'square' rings around the station (arrays 1×1, 3×3, . . . etc., see figure 4).

Refinement control: This is the most complex of all controls and its full understanding shows the power of the system. First we describe this control in general, then its use is illustrated in an example. As the name implies, this control makes it possible to refine the model in the vicinity of the station. Any irregular shape around the station composed of prisms, may be obtained by a process of two steps: First, a 'regular' structure (1×1, 3×3, 5×5 . . .) around the station is excluded from the computation; second the excluded part is reinserted for processing with some elements further subdivided (refined) and the others unaltered. This will result in an irregular structure with finer subdivision. There is no limitation to the repetition of this procedure which allows any refinement of the model desired.

An example of a refinement is outlined for R_3 of figure 1 (for details see figure 5). First, a 3×3 array (numbered 1 to 9) is excluded from the processing. Parts of this are reinserted for processing without changes (i.e. two input matrices one with

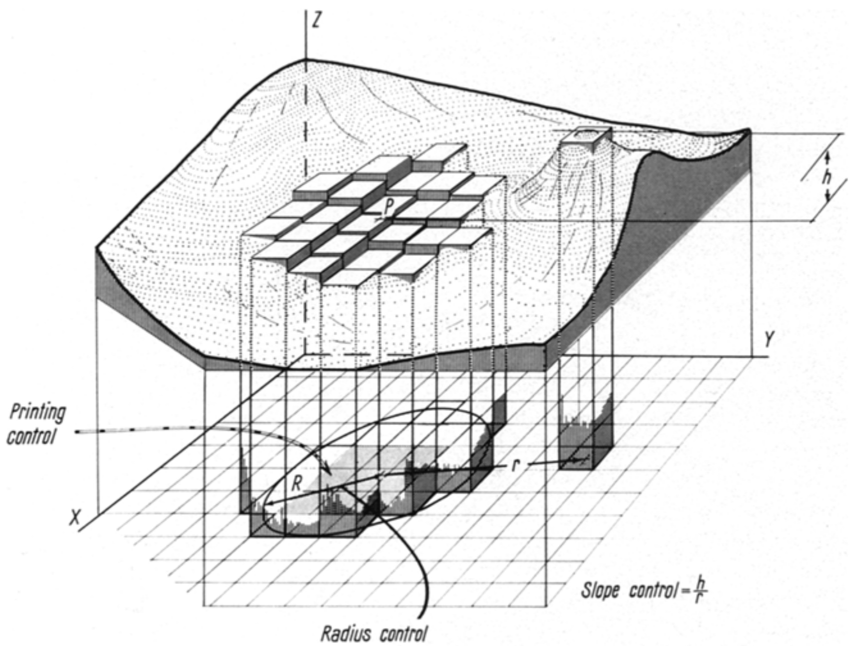


Figure 4
Control parameters

elements 3 and 6 and the other with element 7, 8 and 9). Each of the elements 1, 2, 4 and 5 are subdivided into four parts resulting in an input matrix with 16 elements. Another refinement is shown around the station excluding only the element containing the station (point P) which is further subdivided into 16 parts. In this case two degrees of refinement are carried out for region R_3. If each set of input data is properly prepared, then they can be assembled for computation in any order.

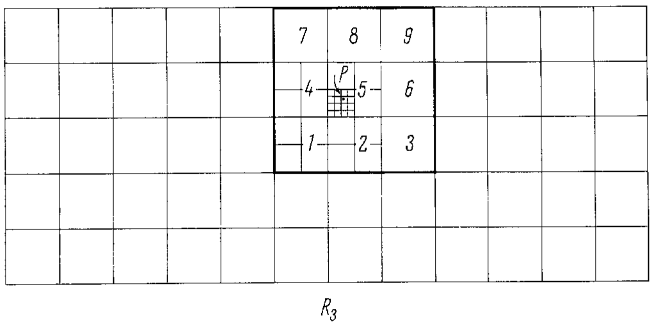


Figure 5
Two degrees of refinement of region R_3

The controls described above are independent of each other in two ways,

i) the use of a specific control is not related to other controls which may or may not be used for the same station;
ii) any control may be changed from station to station in any desired manner.

The method and its implementation for digital computers as described in this paper can satisfy various requirements for both research, where accuracy is the main concern, and routine computations, where efficiency plays a more important role. In order to achieve the latter an optimum compromise in the choice of the control parameters is of great importance. Since no general rule can be set up, a reasonable approach is to use the data to obtain some estimates. One should then select a few representative points for the area and by trial and error find sufficiently good values for the control parameters.

6. *Application of the prism method*

Using the program as outlined, terrain corrections were calculated for a number of stations in the vicinity of the New Quebec Crater. This area is ideally suitable for testing the prism method because of the highly irregular topography of its rim and the great water depth. A new detailed topographic map of the crater area was used to estimate the input values. A ten square kilometer region around the crater was subdivided into 10 000 compartments each 100 metres square. In the program provision has been made to treat one water covered area per region and to calculate terrain correction properly when either a station and/or a compartment falls on water surface. Negative sign for the elevation indicates depth from the water level datum which must be specified on the station card. For comparison, terrain corrections for a few stations were calculated also by using HAMMER's template. The stations for which the terrain corrections were obtained by both procedures are listed in table 1. The selection of a 100 meter square comparement proved to be sufficient for all but one of the five points used to compare the template and prism method. At point 9814 the computer result was 3.14 milligals, differing by about 20% from the template value. Accordingly, an inspection of the neighbourhood of the station indicated that a finer subdivision of two adjacent compartments (25 prisms for each) should improve the result. By this refinement the value, 2.44, for station 9814, as listed in table 1, was obtained.

Table 1

Station No.	Station Coordinates φ	λ	h/ft.	Δg_{mgals} Computer	Template	Percentage Difference
2	61–16–32	73–40–37	– 680	12.16	12.19	0.2
4	61–16–37	73–39–49	– 710	14.19	13.95	1.7
7	61–16–42	73–38–44	– 780	13.07	12.62	3.4
8	61–16–41	73–39–07	– 805	14.35	13.85	3.5
9814	61–16–28	73–41–15	1925	2.44	2.46	0.8

Stations 2, 4, 7 and 8 have been measured on ice at an elevation of 1620 feet. The corresponding values listed under h are depth to the bottom of the lake.

7. *Conclusion*

The prism method for terrain correction and its implementation as presented in this paper has no computational limitations. The basic elements are prisms which form a model of the terrain. For the model exact values of terrain corrections can be calculated without any restriction. But the model is only an approximation of the actual topography. As the model becomes a better and better approximation to the topography, so the calculated terrain correction converges towards its true value.

In conclusion it is emphasized that the critical judgement of the interpreter is of vital importance. He must in each case interpret the results of his computation as a function of the input data: first he must construct a model that provides an optimum approximation of the topography and subsequently, by evaluating the reliability of the elevation matrices, he must estimate the uncertainty of the computed values.

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