Interpretation of Self-Potential Anomalies of Some Simple Geometric Bodies

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Summary – An entirely new procedure for interpreting selfpotential anomalies of spheres, rods and dipping sheets is presented. The anomaly of a sphere is divided into two parts – the anomaly of odd symmetry and the anomaly of even symmetry – from which the depth can be obtained by fitting them with the master curves. The self-potential anomalies of a finite rod are transformed to the anomalies of a veritcal sheet, for which standard curves are presented. The case of a sheet was divided into three parts; (a) finite line of poles; (b) infinite double line of poles and (c) finite double line of poles. For the first case logarithmic curves were prepared and presented; by their comparison with the field profile, different parameters can be obtained. In the second case, a geometrical construction is provided to obtain the various values. In the third case, the anomalies of finite sheet (finite double line of poles) are transformed into those due to an infinite double line of poles for interpretation.

Introduction

Though the applicability of the self-potential method to a variety of problems is well established, the quantitative interpretation of the self-potential anomalies has not been thoroughly developed. The first article dealing with the quantitative interpretation in this field is due to PETROWSKY [6][2]), who has derived expressions for the self-potential anomalies and their space derivatives of a hidden polarised sphere and deduced graphical methods to obtain the depth of the sphere. YUNGUL [14], has also considered the case of a sphere and obtained the depth in terms of some characteristic distances. MEISER [3], has calculated theoretical profiles both for the case of a hidden sphere for different directions of polarisations and for infinite double line of poles with different dip angles. The theoretical profiles were plotted on double logarithmic sheets with appropriate differentiation between (a) the portions of the curve on the positive direction of the origin (right side to the origin, the point on the ground surface immediately above the centre of the body), (b) the portion of the curve on the negative direction (left side to the origin) and (c) any portions having negative values. The field profile is also plotted similarly and matched with one of the theoretical profiles to obtain the various physical parameters. But a great handicap in this method is that *a priori* knowledge of the origin of the body is needed, while as a rule, it is a parameter to be obtained during the interpretation. Furthermore it requires too many

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[2]) Numbers in brackets refer to References, page 77.

charts in the case of dipping doublets or dipping sheets. The plotting of the three different parts of the anomalies on a single paper for all possible cases of a rod or a sheet will be very confusing.

The case of a dipping rod has been attempted only by STERN [13], though a variety of bodies encountered generally in mining Geophysics can be approximated to rods.

Methods dealing with the quantitative interpretation of self-potential anomalies caused by single line of poles and double lines of poles were first published by ROY and CHOUDHURY [11]. In this they treated four separate cases: (a) single infinite line of poles, (b) double infinite line of poles, (c) single finite line of poles and (d) double finite line of poles. In the first three cases, characteristic distances were obtained and their relations to the parameters to be interpreted were established. In the last case model tank experiments were conducted and the anomaly curves for different combinations of dip, length and depth were constructed. The method adopted is that of fitting the field profile into one of the model profiles. These techniques were followed by that of MEISER [3] discussed above, in which both the field and theoretical curves were plotted on double logarithmic paper. Recent publications on the interpretations of self-potential anomalies of infinite sheets include those of M. K. PAUL [4] and P. A. PAUL [5]. In the method due to M. K. PAUL [4] a technique of iteration is employed, which is tedious and suitable only for digital computer and not for routine hand computation. In the method due to P. A. PAUL [5], characteristic distances such as maximum anomaly to minimum anomaly, distances between the points of $\frac{1}{2}[V(\max)+V(\min)]$ and others were calculated and their relation to various parameters presented.

Thus, most of the methods utilise only the characteristic distances while only a few of them, as those of MEISER [3] and ROY and CHOUDHURY [11] (case 4), utilise the fit of the complete field profile with the theoretical curve. But the methods presented by them are neither complete nor unique.

At this stage, it is relevant to mention here that logarithmic curve fitting technique is being applied only in the interpretation of resistivity sounding curves, though HUTCHISON [2], used in 1956 a similar technique in the analysis of magnetic anomalies of dipping dykes. The same logarithmic fitting technique was employed by RAO and RADHAKRISHNA MURTHY [9], for the interpretation of remanent magnetism of dykes and RADHAKRISHNA MURTHY [7, 8], for the interpretation of gravity anomalies of dykes and magnetic anomalies of dipping doublets. But the applicability of the logarithmic curves in the geophysical interpretations other than that of resistivity is not properly understood or appreciated by many. As a rule, most of the anomalies in geophysical interpretation particularly relating to bodies possessing a symmetry about a horizontal or a vertical axis or plane possess either even or odd symmetry or both. When the anomalies are functions of both the anomalies of odd and even symmetry, they can be separated. Hence the technique of logarithmic curve fitting can be applied to the anomalies of all bodies with symmetry about a vertical or a horizontal axis or plane. Furthermore most anomalies of other bodies can be transformed into anomalies of either even symmetry or odd symmetry or a function of both, either by an integral transform (ROY and JAIN [12]) or by obtaining the horizontal derivatives of the anomaly (RADHAKRISHNA MURTHY [7]). Thus it is to be appreciated quite clearly that most of the geophysical anomalies (mainly self-potential, gravity and magnetic) can be interpreted uniquely by logarithmic curve fitting techniques.

The purpose of the present paper is to emphasize the importance of such a technique of interpretation with special reference to the self-potential anomalies of simple geometrical shape. The three types of bodies considered here are: (a) sphere, (b) dipping doublet and (c) sheet. Except the case of a finite sheet, all the problems were found to be amenable to logarithmic curve fitting technique. A suitable interpretation technique for the finite sheets is also presented.

1. Sphere

The self-potential anomaly due to a sphere of depth h and angle of polarisation α is given by HEILAND [1]

$$\left.\begin{aligned} V &= K\left[\frac{h\cos\alpha}{(x^2+h^2)^{3/2}} - \frac{x\sin\alpha}{(x^2+h^2)^{3/2}}\right] \\ &= K(A\cos\alpha - B\sin\alpha) \end{aligned}\right\} \quad (1)$$

where K is a constant term depending upon the dimensions of the sphere and its electrical moment. The self-potential anomaly of a sphere is therefore a weighted sum of two functions A and B – the function of even and odd symmetry respectively.

The interpretation technique to be adopted in this case essentially consists of separating the two anomalies – the anomaly of even symmetry and the anomaly of odd symmetry and matching them with the corresponding master curves shown in

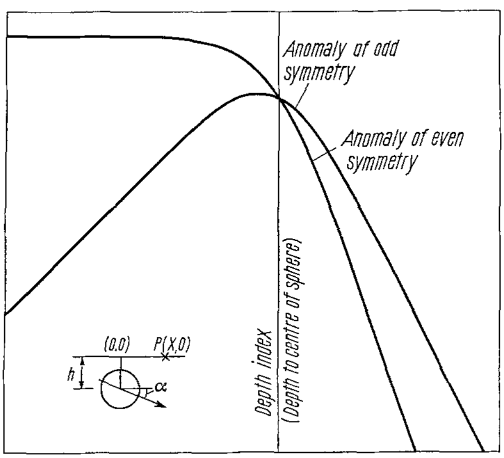

Figure 1

Fig. 1. The technique is quite similar to that originally proposed by HUTCHISON [2] and subsequently used by RAO and RADHAKRISHNA MURTHY [9] and RADHAKRISHNA MURTHY [7]. The origin, i.e. the point immediately above the centre of the sphere is located by iteration. The conditions to be satisfied are that (1) the anomaly of even symmetry is maximum at the origin, decreasing gradually for increasing values of x, (2) the anomaly of odd symmetry is zero at the origin, reaching a maximum and then falling off steeply and (3) both the anomalies should cut at a single point defined by 1.414 of the distance at which the odd function reaches a maximum. The point, that satisfies all these criteria uniquely represents the origin. The anomaly of even symmetry $\frac{1}{2}[(V(x)+V(-x)]$ and the anomaly of odd symmetry $\frac{1}{2}[V(x)-V(-x)]$ are then obtained for increasing values of x from the origin and matched with the theoretical curves. After a perfect match is obtained, the position of the depth index on the abscissa of the field profile defines the depth of the sphere. It shall here be noted that the accuracy of the results obtained is sufficiently improved since same value of depth shall be obtained both from the anomaly of even symmetry and the anomaly of odd symmetry.

It can be noted here that no method was given to obtain α the dip of the polarisation vector. As a matter of fact, it is an unsolved problem why a sphere shall be polarised in any arbitrary direction, far from the vertical. The angle α does not therefore have much significance.

It is relevant to mention here that the curve representing the even symmetry in Fig. 1 can be utilised to obtain the depth of a sphere from its gravity anomaly also.

2. Finite dipping dipole

The self-potential anomaly due to a doublet (Fig. 2A) of length $2\,l$ dipping at an angle of α with the horizontal is given by

$$V(x, y, 0) = \frac{I\,\varrho}{2\,\pi}\left[\frac{1}{[(x + l\cos\alpha)^2 + y^2 + H_1^2]^{1/2}} - \frac{1}{[(x - l\cos\alpha)^2 + y^2 + H_2^2]^{1/2}}\right] \quad (2)$$

where $H_1 = H - l\sin\alpha$ and $H_2 = H + l\sin\alpha$, being the depths to the top and bottom of the dipole and H is the depth to its centre.

The anomalies defined by Eq. (2) do not possess any symmetry. But, the integral of the observed anomalies along X can be written as,

$$\left.\begin{aligned}\int_{-\infty}^{\infty} V(x, y, 0)\,dx &= \frac{I\,\varrho}{2\,\pi}\int_{-\infty}^{\infty}\frac{dx}{[(x + l\cos\alpha)^2 + y^2 + H_1^2]}\\ &\quad - \frac{I\,\varrho}{2\,\pi}\int_{-\infty}^{\infty}\frac{dx}{[(x - l\cos\alpha)^2 + y^2 + H_2^2]^{1/2}} = \frac{I\,\varrho}{2\,\pi}(\log r_1/r_2)\end{aligned}\right\} \quad (3)$$

where

$$r_1^2 = H_1^2 + y^2$$

and

$$r_2^2 = H_2^2 + y^2$$

This implies that the integral of the observed anomalies parallel to the x-axis is equivalent to the anomaly of a vertical line doublet with its lines of poles at the same depths H_1 and H_2. The integration of the observed anomalies can be done by writing

$$\int_{-\infty}^{\infty} V(x, y, 0)\, dx = \sum_{-\infty}^{\infty} V(x, y, 0) \tag{4}$$

where the observations $V(x, y, 0)$ are plotted at the corners of a regular grid system.

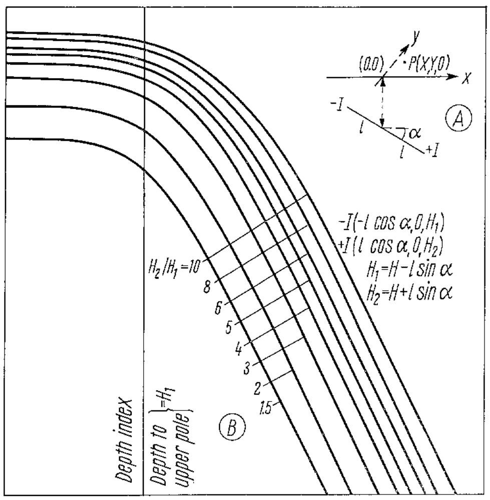

Figure 2

These transformed anomalies, being equivalent to those of a vertical line doublet will have even symmetry about the origin. Fig. 2B represents the anomalies of a vertical line doublet, drawn on a double logarithmic graph paper, in which the depth index at $y = H_1$ is included. The integrated profile is also plotted on a similar graph sheet and matched with one of the theoretical curves in Fig. 2B. The depth to the top pole is defined by the position of the depth index line on the abscissa of the field profile while H_2/H_1 (and hence H_2) is given by the figure on the best fitting curve. The parameters H_1 and H_2 can thus be obtained.

To find out the dip angle of the doublet, one may take into account the profile along the y direction defined by $x = -H \cot\alpha$. Along this line, the potential anomaly can be seen to be

$$V(-H\cot\alpha, y, 0) = \frac{I\varphi}{2\pi}\left[\frac{1}{\sqrt{H_1^2\operatorname{cosec}^2\alpha + y^2}} - \frac{1}{\sqrt{H_2^2\operatorname{cosec}^2\alpha + y^2}}\right] \tag{5}$$

which represents the anomaly due to a vertical doublet with the depth to the poles $H_1 \operatorname{cosec}\alpha$ and $H_2 \operatorname{cosec}\alpha$. The line defined by $x = -H\cot\alpha$ passes through the point of intersection of the doublet with the x-axis (Fig. 3A) and is perpendicular to it.

The point defined by $(-H\cot\alpha, 0, 0)$ is first located approximately and the profile along y through this point drawn and matched with one of the theoretical curves in Fig. 3B. If the point located represents the correct position of $(-H\cot\alpha, 0, 0)$ then the field profile fits closely to the curve having an index H_2/H_1 same as that obtained previously. If it does not fit with that having an index H_2/H_1, the testing point has to be shifted. The exact location of the point $(-H\cot\alpha, 0, 0)$ is thus found out by iteration, the profile along y constructed and matched with the theoretical profile

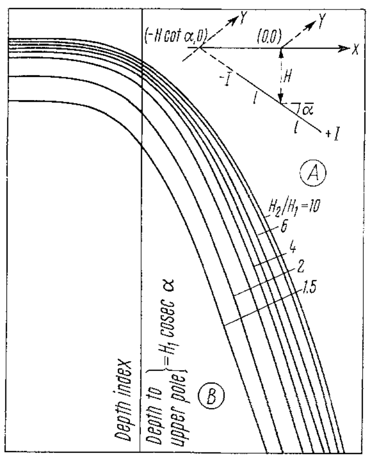

Figure 3

in Fig 3, having an index of H_2/H_1. The position of the depth index is then read out on the abscissa of the field profile to give $H_1 \operatorname{cosec}\alpha$ and hence α can be calculated.

Alternatively the dip angle α can be found out by the integration of the anomalies along y. Integrating the anomalies along y, one gets

$$V(x, y, 0)\,dx = \frac{I\varphi}{2\pi}\log\left[\frac{(x + l\cos\alpha)^2 + H_1^2}{(x - l\cos\alpha)^2 + H_2^2}\right] \tag{6}$$

which is quite similar to the equation of the self-potential anomalies of a infinite double line of poles with a dip α. From the foregoing section, it will be noted that the distance between the position of the maximum anomaly or the position of the minimum anomaly from the point of zero intensity is $\sqrt{H^2 \sec^2\alpha+l^2}$, which is also equal to $\sec\alpha\sqrt{H^2+l^2\cos^2\alpha}$. The parameters H and $l\cos\alpha=\frac{1}{2}(H_2-H_1)$ are obtained previously and hence one can obtain α.

Further the anomalies integrated along the y direction, can also be subjected to a similar treatment as in the case of infinite double line of poles and can be solved for H_1, H_2 and α simultaneously.

3. *Sheet*

The case of a sheet is receiving good attention in recent years. The common notations used in interpreting the self-potential anomalies of a sheet are as follows:

H = depth to the centre of the sheet $=(H_1+H_2)/2$
$H_1 = H-l\sin\alpha$ = Depth to the top pole
$H_2 = H+l\sin\alpha$ = Depth to the bottom pole
α = Dip of the sheet with the horizontal ground surface
$2a$ = Distance between the two lines of poles
$2l$ = Total length extent of the sheet.

The problem of the sheet is divided into three cases – and treated separately. The three cases are: (i) finite line of poles (ii) infinite double line of poles and (iii) finite double line of poles.

(i) *Finite line of poles.* The self-potential anomaly due to a finite line of poles is given by

$$V(x, y, 0) = \frac{I\varphi}{2\pi}\left[\sinh^{-1}\frac{y+l}{\sqrt{x^2+H^2}} - \sinh^{-1}\frac{y-l}{\sqrt{x^2+H^2}}\right] \tag{7}$$

where H is the depth to the line of poles. The above equation represents perfectly a symmetrical function both about the x-axis and y-axis. The self-potential contours are therefore perfectly elliptical, the direction of the major axis representing the direction of the elongation of the line of poles. The centre of any major axis represents the centre of the body. To interpret the length of the line pole and its depth of burial, two sets of curves were prepared and plotted in Fig. 4A and B, representing the self-potential anomalies of line of poles for different combinations of l and H drawn on double logarithmic paper. Fig. 4A represents the anomalies through the origin along the minor axis of the elliptical contours (x-axis) and Fig. 4B represents the anomalies through the origin along their major axis (y-axis). The two field profiles are similarly plotted and matched with the best fitting curve of the corresponding set. The position of the depth index line defines the depth, while the number on the best fitting curve indicates l/H (and hence l). Two values of H and l are thus obtained, one from each chart. The accuracy of the results is increased as the same values of H and l should be obtained from the two charts. If they are widely different, it shall be interpreted

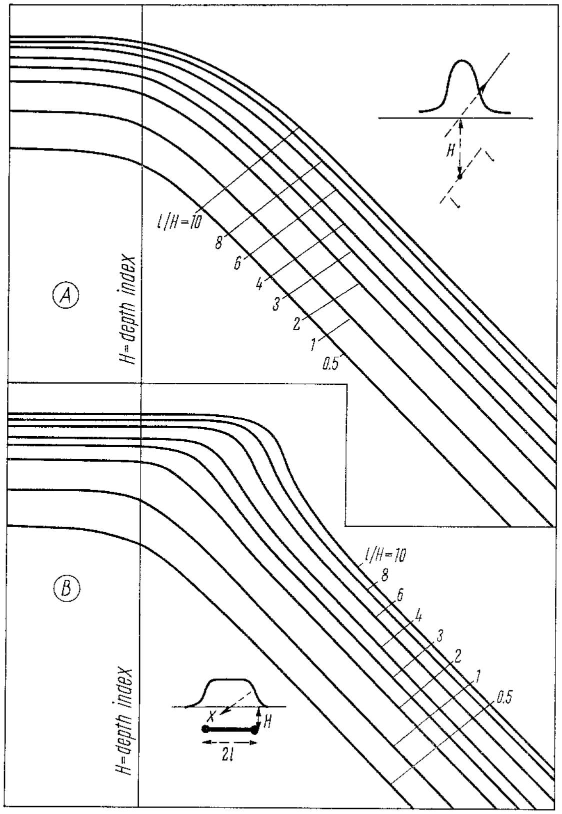

Figure 4

that the body cannot be replaced by a single line of poles, but by a double finite line of poles, one placed quite vertically below the other. In such case, the anomalies are integrated along y, so that they represent the anomalies of infinite vertical line doublets. The parameters namely H_1 and H_2, depths to the top and bottom poles respectively, are then obtained utilising charts in Fig. 2B. The length extent $2\,l$ of each pole is obtained from Fig. 6 to be presented in the foregoing lines.

(ii) *Infinite double line of poles.* The self-potential anomaly due to an infinite double line of poles is given by

$$V(x, 0) = \frac{I\,\varrho}{2\,\pi} \log \frac{(x + a\cos\alpha)^2 + (H - a\sin\alpha)^2}{(x - a\cos\alpha)^2 + (H + a\sin\alpha)^2} \tag{8}$$

where the relative orientations of various parameters are defined in Fig. 5A.

It can be seen from the equation, the zero anomaly occurs at a point defined by

$$x_0 = H\tan\alpha \tag{9}$$

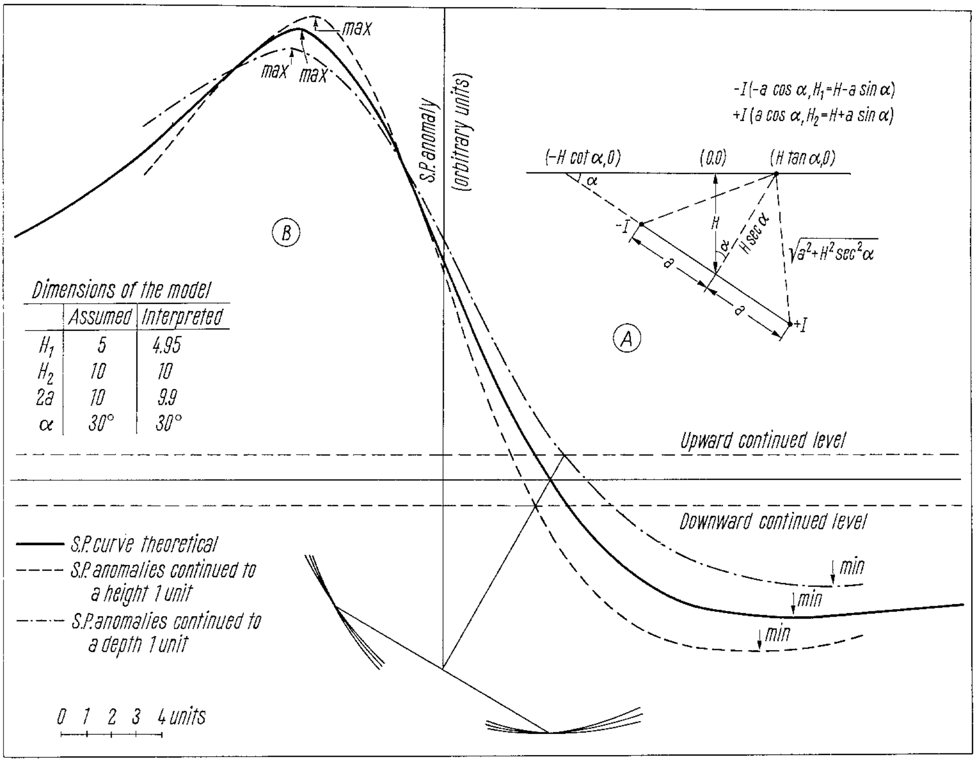

Figure 5

and the maximum and minimum points occur at points defined by

$$x_{\text{max or min}} = H \tan\alpha \pm \sqrt{H^2 + a^2 \sec^2\alpha} \tag{10}$$

so that the point of maximum anomaly and the point of minimum anomaly are at equal distances from the point of zero intensity. The distance between the point of zero intensity and the point of maximum anomaly or the distance from the point of zero anomaly to that of minimum anomaly is $\sqrt{H^2+a^2\sec^2\alpha}$. But from the figure it can be easily seen that the three points, viz. the positions of the two poles and the point of zero intensity, define an isosceles triangle, the lengths of their sides being given by $\sqrt{H^2+a^2\sec^2\alpha}$, $\sqrt{H^2+a^2\sec^2\alpha}$ and $2a$. The length defined by $\sqrt{H^2+a^2\sec^2\alpha}$ therefore defines the distance from either the pole to the point of zero anomaly. This interesting property of the field profile can be utilised to obtain the exact orientation of the line doublet and the steps to be followed are simply as follows:

(a) From the field profile, obtain the points of zero anomaly, maximum anomaly and minimum anomaly. Note that the point of zero anomaly will be midway to the points of maximum and minimum anomalies. Note the distance $\sqrt{H^2+a^2\sec^2\alpha}$ being half the distance between points of maximum and minimum anomalies.

(b) Now draw an arc with the centre at the point of zero anomaly and with the radius $\sqrt{H^2+a^2 \sec^2\alpha}$. Note that the positions of the two poles lie on this arc.

(c) Continue the field data upwards to a unit grid spacing. Repeat steps (a) and (b). Note that the two arcs thus obtained intersect at two points defining the positions of the poles.

(d) Continue the field data downwards also to a unit grid spacing, so that one can have three arcs, and the positions of the two poles can be located with increased accuracy.

The method is illustrated with a numerical example in Fig. 5B. The continuous curve represents the self-potential anomaly due to an infinite line doublet of 10 units breadth (2a) dipping at an angle $(\alpha)=30°$, the depths to the top and bottom poles being 5 and 10 units respectively. Self-potential anomalies due to this model at every point at an interval of 1 unit are plotted. These anomalies are continued both upwards and downwards utilising the formulae given by ROY and BURMAN [10]. and the resulting curves presented. In all the three cases half the distance between the point of maximum anomaly and minimum anomaly is read and an arc is drawn with the centre at the point of zero intensity and with the radius corresponding to half the distance between the points of maximum and minimum anomaly. It can be seen that all the three arcs cut exactly at only two points, which define the positions of the line poles. By joining the two points with a straight line, the exact orientation of the line doublet was obtained and hence all the parameters ($2\,a$, H and α) were deduced. It is interesting to note here that the interpreted values are very close to those assumed indicating the satisfactory application of the proposed method.

(iii) *Finite line doublets or a finite dipping sheet.* The self-potential anomaly due to a finite doublet of line poles is given by

$$\left.\begin{aligned} V(x, y, 0) = \frac{I\,\varrho}{2\,\pi} \log & \frac{(y+l)+\sqrt{A^2+(y+l)^2}}{(y-l)+\sqrt{A^2+(y-l)^2}} \\ - \log & \frac{(y+l)+\sqrt{B^2+(y+l)^2}}{(y-l)+\sqrt{B^2+(y-l)^2}} \end{aligned}\right\} \tag{11}$$

where

$$A^2 = (x + a\cos\alpha)^2 + (H - a\sin\alpha)^2$$

and

$$B^2 = (x - a\cos\alpha)^2 + (H + a\sin\alpha)^2 .$$

As the self-potential anomalies are symmetrical about the x-axis, the line integral of the observed anomalies along the line $x=x_1$ is given by

$$\int_{-\infty}^{\infty} V(x_1, y, 0)\, dy = 2 \int_{0}^{\infty} V(x_1, y, 0)\, dy \tag{12}$$

Noting that

$$\left.\begin{aligned}&\int_0^\infty \log\frac{(y+l)+\sqrt{A^2+(y+l)^2}}{(y-l)+\sqrt{A^2+(y-l)^2}}\,dy\\ &\qquad = [(y+l)\log(y+l+\sqrt{A^2+(y+l)^2})\\ &\qquad - (y-l)\log(y-l+\sqrt{A^2+(y-l)^2})\\ &\qquad - \sqrt{A^2+(y+l)^2}+\sqrt{A^2+(y-l)^2}]_0^\infty\\ &\qquad = -l\log(l+\sqrt{A^2+l^2}) - l\log(-l+\sqrt{A^2+l^2})\\ &\qquad = -l\log A\end{aligned}\right\} \tag{13}$$

and hence

$$\int_0^\infty \log\frac{(y+l)+\sqrt{B^2+(y+l)^2}}{(y-l)+\sqrt{B^2+(y-l)^2}}\,dy = -l\log B \tag{14}$$

the line integral becomes,

$$\int_{-\infty}^{\infty} V(x_1, y, 0) = \frac{I\,\varrho\,l}{\pi}\log\frac{B}{A}. \tag{15}$$

This implies that the line integrals of the observed anomalies due to a finite sheet are equivalent to the anomalies of an infinite line doublet. Thus, while interpreting the self-potential anomalies of a finite sheet, one has to transform the observed anomalies by simple line integration into the anomalies of a infinite line doublet and hence interpret the integrated profile by the steps indicated in case (ii) above. Thus one can

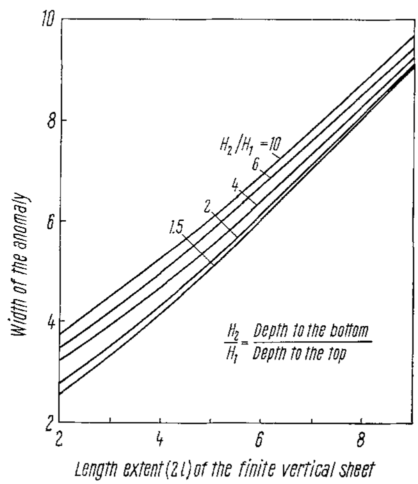

Figure 6

obtain the values α, H and $2a$, leaving only one quantity $2l$, being the length extent of the sheet along y direction to be determined. For this the following procedure is adopted.

Since the exact orientation of the sheet is obtained in a vertical plane, a point defined by the coordinates $(-H\cot\alpha, 0)$ can be located and a profile drawn in the y direction through this point. This profile represents the self-potential anomaly drawn along the y direction of a vertical finite doublet of line of poles. From this profile, the width of the anomaly, being the distance between the two points of half maximum anomaly, is obtained and is utilised to derive $2l$ from Fig. 6, in which the widths of the anomalies for vertical sheets of different lengths and depths are calculated and presented.

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