of the Stanton and Gibbs numbers; Pe, fluctuation value of the Peclet number, characterizing the ratio of the mean and fluctuation components of crystal growth rate; p, Fourier transform variable; Q, mass flux; r, r_*, radii of crystal and critical nucleus; t, time; u, dimensionless supersaturation; V, function introduced in (2); St, Stanton number; β, mean rate of crystal growth; $\gamma(r)$, rate of crystal extraction; γ_0, Γ_0, Γ_*, coefficients pertaining to $\gamma(r)$; ρ, density of crystal; ω, frequency of oscillation. The subscript s denotes quantities corresponding to the steady-state regime of crystallization; the superscript ° denotes values on the surface of neutral stability; the index a denotes that a quantity pertains to the natural frequency of oscillation; an asterisk denotes averaging over the ensemble; brackets denote averaging over time.

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OPTICAL AND RADIANT CHARACTERISTICS OF TUNGSTEN AT HIGH TEMPERATURES

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It is proposed that thermal radiation spectra of solid materials at high temperatures be treated by the method of moments.

Extensive experimental data have been accumulated on radiant and optical characteristics of solid materials at high temperatures [1-3]. However, the problem of the theoretical treatment of radiant spectra and the related determination of the frequency—temperature dependencies of optical constants is still open today. In the present study, a method is offered that allows one to recreate the radiant spectra from the integral characteristics of the system. The given method has been tested on tungsten. From the generalized Wien displacement law, we determine the temperature for the material under investigation. The calculated values reproduce well the measurement results.

In [4], it is proposed that the absorption spectra be treated by the method of moments. It has been shown that if the experimental outline is a smooth function then it is well reproduced by the asymptotic Edgeworth series. In the present work, a similar method was

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TABLE 1. Temperature Dependence of the Initial and Central Moments of the Radiant Spectral Density for Tungsten

T, K	m_0, 10^{-3} J·sec/(m^3·deg^4)	m_1, 10^{11} J/(m^3·deg^5)	$\bar{m}_2$, 10^{24} J/(sec·m^3·deg^6)	$\bar{m}_3$, 10^{38} J/(sec^2·m^3·deg^7)	$\bar{m}_4$, 10^{52} J/(sec^3·m^3·deg^8)
1200	0,1691	0,2359	0,5289	0,2811	0,6589
1400	0,4114	0,6426	1,7	0,9811	2,768
1600	0,8596	1,48	4,5	2,867	9,39
1800	1,607	3,013	10,37	7,405	27,38
2000	2,759	5,59	21,59	17,34	71,04
2200	4,449	9,691	42,26	40,77	188,9
2400	6,764	15,71	75,06	75,41	364
2600	9,879	24,39	128,6	142,4	735,2

assumed based on the description of the optical and radiant characteristics of the thermal radiation of solid materials.

It has been known [5] that knowledge of a finite number of moments of the spectral density of radiation allows one to approximate it in the form of the finite sum

$$I(\nu, T) = \frac{m_0}{\sigma}\left[\varphi(\xi) - \frac{\gamma_1}{3!}\varphi^{(3)}(\xi) + \frac{\gamma_2}{4!}\varphi^{(4)}(\xi) + \frac{10\gamma_1^2}{6!}\varphi^{(6)}(\xi)\right], \quad (1)$$

where $m_n = \int \nu^n I(\nu, T)\, d\nu$; $\bar{m}_n = \int (\nu - \bar{\nu}) I(\nu, T)\, d\nu$ are the initial and central moments of the distribution function $I(\nu, T)$, $n = 0, 1, 2, 3, 4, \ldots$ is the order of the moments; $\xi = (\nu - \bar{\nu})/\sigma$; $\bar{\nu} = m_1/m_0$; $\sigma^2 = \bar{m}_2/m_0$; $\gamma_1 = \bar{m}_3/(\bar{m}_2)^{3/2}\sqrt{m_0}$; $\gamma_2 = \bar{m}_4/(\bar{m}_2)^2 m_0 - 3$; $\varphi(\xi) = 1/\sqrt{2\pi}\exp(-0.5\xi^2)$.

The first moment is equal to the area of the distribution, the ratio m_1/m_0 is equal to the center of gravity $\bar{\nu}$. The second central moment is related to the width of the distribution, and the third, to its asymmetry. The third and the fourth terms of (1) refine the form of the function $I(\nu, T)$ in the tails of the distribution and near its maximum. The negative sign of the third central moment shows that the decline of the curve $I(\nu, T)$ to the right from the maximum is considerably sharper than the decline to the left of the maximum ($\bar{\nu} < \nu_{max}$). Strictly speaking, the function $I(\nu, T)$ is characterized completely only by all the moments m_n or $\bar{m}_n$. However, for a single-valued description of all the properties of the outline of the thermal radiation spectrum it is sufficient to have 4-5 first moments [5]. The values of the moments are calculated on the basis of the experimental curve $I(\nu, T)$. The integration is performed along the entire outline of the radiant spectrum.

It is characteristic that for thermal radiation, good agreement with experimental data is obtained for high temperatures or low frequencies, such that $1.1 \leqslant x \leqslant 7.2$, where $x = h\nu/kT$. Such an estimate can be readily obtained in the simplest case of the spectrum of an ideal black body, the frequency moments of which are easily calculated and are equal to: $m_0 = 7.64 \cdot 10^{-16}\ T^4$ J·sec/(m^3·deg^4), $m_1 = 6.02 \cdot 10^{-5}\ T^5$ J/(m^3·deg^5), $\bar{m}_2 = 1.35 \cdot 10^6\ T^6$ J/(sec·m^3·deg^6), $\bar{m}_3 = 5.61 \cdot 10^{16}\ T^7$ J/(sec^2·m^3·deg^7), $\bar{m}_4 = 1.07 \cdot 10^{28}\ T^8$ J/(sec^3·m^3·deg^8).

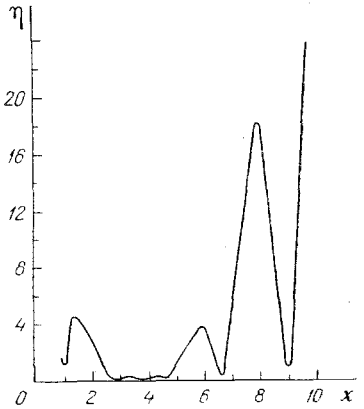

Fig. 1. Frequency–temperature dependence of the relative error of approximation of the radiant spectral density for tungsten by function (1). η, %; $x = h\nu/kT$.

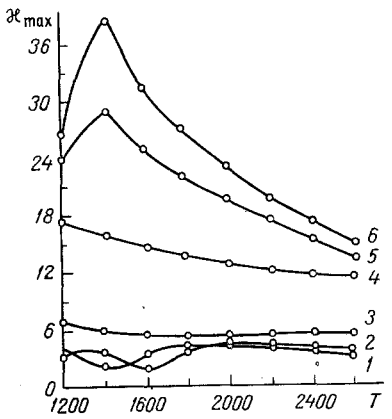

Fig. 2

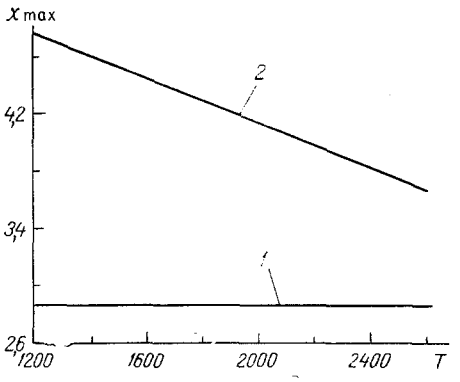

Fig. 3

Fig. 2. Temperature dependence of $\varkappa_{max}$ for six wavelengths: 1) λ = 0.75 μm; 2) 0.9; 3) 1.5; 4) 3; 5) 4.2; 6) 5 μm; dots) experimental data, curves) calculated values. T, K.

Fig. 3. Investigation of the Wien displacement law for tungsten: 1) ideal black body; 2) tungsten.

For the case of an ideal black body, function (1) with the known moments approximates well Planck's function [6] in the region $0.8 \leqslant x \leqslant 9$.

The position of the maximum of the radiant spectral density can be represented in the form

$$\nu_{max} = \frac{m_1}{m_0}\left(1 + \xi_1 \frac{m_0}{m_1}\sigma\right), \tag{2}$$

where ξ_1 is a real root of the equation

$$\xi^7 + \left(3\frac{\gamma_2}{\gamma_1^2} - 21\right)\xi^5 + \frac{12}{\gamma_1}\xi^4 + \left(105 - \frac{30\gamma_2}{\gamma_1^2}\right)\xi^3 - \frac{72}{\gamma_1}\xi^2 + \left(\frac{72 + 45\gamma_2}{\gamma_1^2} - 105\right)\xi + \frac{36}{\gamma_1} = 0.$$

For an ideal black body, $\xi_1 = -0.466$ and Eq. (2) with the known moments assumes the form $h\nu_{max}/kT = 2.887$. Equation (2) differs insignificantly from the well-known expression for the Wien displacement law: $h\nu_{max}/kT = 2.822$. The error in the determination of the temperature in this case does not exceed 2%.

Below we will be concerned with the investigation of the optical and radiant characteristics of tungsten. In [7], a procedure is offered for estimating optical constants directly from the spectral radiating power in which an upper estimate for the absorption coefficient $\varkappa$ for the given radiating power ε_ν is obtained in the form

$$\varkappa_{max} = \sqrt{[(2 - \varepsilon_\nu)/\varepsilon_\nu]^2 - 1}. \tag{3}$$

Next it is pointed out that when the temperature is increased the difference between $\varkappa_{max}$ and the experimental value $\varkappa$ for tungsten decreases.

In this work, for the description of a high-temperature dependence $\varkappa_{max}$, asymptotic series (1) is used, which is related to the radiating power entering into (3) via the expression $\varepsilon_\nu = I(\nu, T)/I^0(\nu, T)$. According to the experimental data [1], the frequency moments of the radiant spectral density for tungsten have been calculated by the method offered in [8] and are given in Table 1. From the given values of the moments the radiant spectral densities (1) were recreated for the eight investigated temperatures. A numerical analysis has shown that the calculated values reproduce well the experimental data in the region $1{,}1 \leqslant x \leqslant 7{,}2$. In Fig. 1, a direct estimate of the relative error of approximation is given; the accuracy in the investigated region does not fall below 5%. By knowing the calculated values of ε_ν, the values of $\varkappa_{max}$ were determined from (3). In Fig. 2, the temperature de dependence of the absorption coefficient is given for six wavelengths. The calculated values give good quantitative reproduction of the experimental data at high temperatures; the latter, in turn, indicates that the approximating equation (1) was a good choice.

The temperature dependence of the coordinate of the radiation maximum $h\nu_{max}/kT$ obtained from Table 1 and Eq. (2) is represented in Fig. 3. As is seen from Fig. 3, x_{max} is not constant (as for the ideal black body) but is linear and is approximated by the equation

$$x_{max} = h\nu_{max}/kT = a + bT, \tag{4}$$

where a = 5.58; b = $-7.09 \cdot 10^{-4}$ deg^{-1}. From Eq. (4), the determination of the tungsten temperature follows in the form

$$T = -a/2b\,[1 - \sqrt{1 - 19{,}2 \cdot 10^{-10} b\nu_{max}/a^2}]. \tag{5}$$

It follows from (5) that for a single-valued determination of the temperature from the experimental data, only precise measurements of the maximum of the radiant spectral density are necessary. The accuracy in the determination of the temperature in this case is no worse than 1%. We note that Eq. (5) for tungsten apparently can be extrapolated to the regions of high and low temperatures. In order to prove this statement, additional experimental data are required.

One of the important characteristics of radiation of real bodies is the X-point. The coordinate of the X-point is determined by Eq. (6):

$$\varepsilon(\lambda, T_1) = \varepsilon(\lambda, T_2) = \ldots = \varepsilon(\lambda, T_8) \tag{6}$$

(λ is the wavelength of the radiation corresponding to the X-point). The numerical solution of Eq. (6) for the eight investigated temperatures gives the value of the coordinate of the X-point λ = 1.276 μm. The calculated values are in good agreement with the experimental data [1].

In conclusion we note that the proposed asymptotic Edgeworth series (1) reproduces well the experimental values in the intermediate region $1.1 \leqslant x \leqslant 7.2$ only. In order to describe the frequency–temperature behavior of the spectrum in the regions x < 1.1 and x > 7.2 correctly, additional studies are required.

NOTATION

ν, frequency of radiation; ξ, x, dimensionless parameters; k, Boltzmann constant; T, temperature; h, Planck constant; $\varkappa$, absorption coefficient; ε_ν, monochromatic radiating power; $I^0(\nu, T)$, spectral density of radiation of an ideal black body; λ, wavelength of the radiation; η, relative error of approximation; m_n and $\bar{m}_n$, initial and central moments of the distribution function; σ, γ_1, and γ_2, dispersion, coefficient of asymmetry, coefficient of the excess of distribution; $\varphi(\xi)$, density function of the normal distribution.

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