ISAAC LEVI

ITERATION OF CONDITIONALS AND THE RAMSEY TEST*

Peter Gärdenfors (1986) has recently posed a dilemma for advocates of belief revision models for conditionals. Proponents of such models have often held that conditionals are best understood as having acceptability conditions characterized by the so-called "Ramsey test" which Gärdenfors formulates as follows:

(R) Accept a proposition of the form "if A then C" in a state of belief K if and only if the minimal change of K needed to accept A also requires accepting C.

In addition, advocates of belief revision models have adopted views which satisfy the following "preservation criterion":

(P) If a proposition B is accepted in a given state of belief K and A is consistent with the beliefs in K, then B is still accepted in the minimal change of K needed to accept A.

Gärdenfors goes on to claim that "on pain of triviality, the Ramsey test and the preservation criterion are inconsistent with each other". The dilemma facing advocates of belief revision models of conditionals is how to choose between (R) and (P).

Gärdenfors does acknowledge that the inconsistency relies on other assumptions. Some of them seem innocuous enough, but one of them is far from innocuous:

(O) Belief sets include propositions containing the conditional connective $>$ as elements.

In my view, those who advocate belief revision models of conditionals ought to be happy to give up (O). In my own discussion of this view I have always insisted on it. Only someone, like Gärdenfors himself, who seeks to mimic the formal features of accounts of conditionals of the sorts advocated by Stalnaker and Lewis within the framework of a belief revision approach should be discomfited by Gärdenfors' result.

In my opinion, it is futile to try to bridge the gap between those who think that conditionals are objects of beliefs which bear truth values

Synthese **76** (1988) 49–81.

and are subject to appraisal with respect to epistemic possibility and probability, and those who regard conditionals to be appraisals of serious or epistemic possibility relative to transformations of the current corpus of knowledge or belief set K. Gärdenfors' contribution is to offer us a rather powerful argument for the futility of the effort.

Gärdenfors himself sees matters differently. He thinks that abandoning (O) threatens us with the incapacity to supply acceptability conditions for iterated conditionals. Until the threat is addressed, he believes the predicament facing advocates of belief revision models of conditionals entails abandoning either the Ramsey test or the preservation condition.

I have always been mystified why so many serious authors have thought that the problem of iterated conditionals is so important. Be that as it may, my aim in this essay will be to establish the following:

(a) The threat to the preservation condition can be derived from epistemic conditions for the acceptability of judgements of possibility and impossibility without appealing to the Ramsey test, provided one admits judgements of possibility and impossibility into the corpus or belief set which is subject to revision according to condition (P).

(b) The epistemic conditions for the acceptability of judgements of possibility and impossibility will be shown to be consequences of a version of the Ramsey test. Combining the Ramsey test with condition (O) will insure that judgements of possibility and impossibility are admitted into the corpus of belief. Hence, the Ramsey test will be shown to conflict with (P) and (O) by a line of reasoning different from Gärdenfors' argument. Since the pivotal consideration in the argument concerns the acceptability of judgements of possibility and impossibility rather than conditionals, abandoning the Ramsey test for conditionals will not avoid the threat to the preservation condition and (O). We would need instead to abandon the acceptability conditions for judgements of possibility and impossibility. Because I take this alternative to be unacceptable and because of virtues of the preservation condition, condition (O) will be rendered suspect. Abandoning (O) will then relieve the pressure on the Ramsey test.

(c) I shall then offer an account of the acceptability of some types of sentences which exhibit the grammatical features of iterated conditionals without requiring that conditionals be objects of belief eligible for membership in belief sets or corpora of knowledge as condition (O) requires.

This should sustain my contention that the worries registered by Gärdenfors (and by L. J. Cohen before him in 1981) concerning my dismissal of iterated conditionals are excessive. Gärdenfors' recognition of the conflict between (O), (R) and (P) has brought home to him the problems facing advocates of belief revision models who wish to regard conditionals (and modal assessments) as objects of belief. I cannot demonstrate to him that he should abandon the modal realism implicit in this view. I can try to show, however, that as far as the question of iteration is concerned, the resulting reification of epistemic notions is a gratuitous, *verdoppelte Metaphysik*.

2.

To formulate the issues in a precise manner comparable to Gärdenfors' own formulation, I shall need to consider three formal languages L, L^* and L^{**}. L will be a language containing truth functional connectives and an underlying truth functional logic. One could allow for resources for quantification and take the logic to be first order predicate logic. Gärdenfors follows the former course, but it will not matter for our purposes one way or the other. It is crucial, however, that L contain no modal connectives. Given L, we have a "consequence" relation of deducibility $\vdash$ according to the classical truth functional logic, or quantificational theory as the case may be. L^* will be a language obtained from L by adding a single modal connective $\Diamond$ for "it is possible that". L^{**} will be language obtained from L by adding the binary conditional connective $>$.

Gärdenfors and I agree that a corpus expressible in a given language is closed under the consequence relation $\vdash$. With this understanding, we may classify all revisions of corpus into four types:

(a) K' is an expansion of K by adding h if and only if K' is obtained by adding some sentence h (in the given language) to K and forming the closure under consequence. In this case, $K' = K/h$.

(b) K' is a contraction of K iff $K = K'/h$.

(c) K' is a replacement of K if and only if there is an h in K such that $\sim h$ is in K'.

(d) K' is a residual shift of K if and only if K' is neither an expansion, contraction or replacement of K.

Of course, this classification applies to shifts expressible in L^* and in L^{**} as well as to those expressible in L.

I maintain that all legitimate revisions of a corpus ought to be analysable as sequences of legitimate expansions and contractions (Levi 1974, 1975, 1980, 1983). That is to say, there are no justifiable replacements or residual shifts unless they are justifiable as sequences of contractions and expansions.

According to the directive of the so called "Ramsey test" introduced by Stalnaker (1968) and elaborated by Harper (1974, 1975, 1976) and by Gärdenfors himself, the acceptability of a conditional $h > g$ depends upon whether a "minimal revision" of the current corpus so that h becomes accepted results in a corpus which also contains g.

If all legitimate revisions of a corpus should be sequences of justifiable contractions and expansions, and if minimal revisions are at least sometimes justifiable, it must be possible to analyse such justifiable minimal revisions into sequences of contractions and expansions. Indeed, if we could proceed in this way, it seems sensible to suppose that minimal revisions are of one of the following sorts:

(1) if h is in K, a minimal revision K_h of K via adding h is identical with K.

(2) if neither h nor $\sim h$ are in K, $K_h = K/h$ – i.e., the expansion of K.

(3) if h is not in K but $\sim h$ is, K_h is the outcome of first contracting by deleting $\sim h$ with a minimum loss of informational value and then expanding the result by adding h. Let $K \dot{-} h$ be a contraction of h by deleting h incurring minimum loss of informational value. (It is possible to introduce a rule for ties to select one of these minimal contractions when there are more than one (Levi 1980, p. 62).) Then $K_h = (K \dot{-} \sim h)/h$

(4) if h is not in K but $\sim h$ is (as in case 3), K_h is the outcome of first expanding by adding h to K, thereby obtaining an inconsistent corpus and then contracting the inconsistent corpus to a consistent deductively closed corpus while removing $\sim h$ (Levi 1980, pp. 40, 59–60). Then $K_h = (K/h) \dot{-} \sim h$. The notion of a minimal contraction of an inconsistent corpus needs special attention. I contend that when the minimal contraction yields a replacement of type (4), $(K/h) \dot{-} \sim h$ should be the same as $(K \dot{-} \sim h)/h$ so that

type (4) replacements can be reduced to type (3) replacements.

I have never used the notion of a minimal revision of belief in my work but have instead worked exclusively with contractions and expansions. However, my account of the acceptability of conditionals can be formulated in terms of a notion of minimal revision satisfying (1), (2) and (3). I shall call such revisions L-minimal.

Let us suppose that $\sim h$ is not in the corpus K and let the minimal revision K_h of K be L-minimal. If h is in K, $K_h = K$ as (1) requires. If h is not in K, $K_h = K/h$ as (2) requires. No matter which of these two cases is operative, for every g in K, g is in K_h. That is to say, the requirements of Gärdenfors' preservation condition are met by all L-minimal revisions.

There are many different types of revision rationalisable as sequences of justifiable expansions and contractions which are not L-minimal. For example, a contraction of K which minimizes loss of information while removing $\sim h$ from K is not L-minimal because the removal of $\sim h$ is not followed by the addition of h. Or one might remove $h \supset \sim g$ from K with minimum loss of information and then add $h \supset g$. This too is not an L-minimal revision, but it is a revision which is rationalizable as a sequence of contractions and expansions.

I have not insisted on the requirement that legitimate revisions be rationalized as sequences of justifiable expansions and contractions because of my account of conditionals. To the contrary, the account of conditionals is a byproduct of a larger epistemological project. In particular, my commitment to the view that all minimal revisions used in determining the acceptability of conditionals via the Ramsey test be L-minimal and, hence, satisfy (P) derives from considerations of epistemological principle which cover a broader range of changes of cognitive state than L-minimal revisions. I shall try now to summarize some of these considerations.

3.

According to Levi (1974, 1976, 1979a, 1979b, 1980, 1983), a corpus of knowledge (belief set) is a resource for deliberation and is to be analysed in terms of its function as such a resource rather than in terms of its "pedigree" – i.e., in terms of the way it is justified or its causal origins. And the chief function I have attributed to a corpus of knowledge is to serve as a standard for serious possibility. If h is in the corpus, its negation is not a serious possibility. If h is not in the corpus, its negation is a serious possibility.

One of the ramifications of this view is that from the agent's point of view at the time that a given corpus is endorsed by him, all items in the corpus are true. Hence, if the agent is concerned to avoid error in revising his corpus, he should avoid deliberately adding to his corpus propositions which, according to that corpus, are false. To do so is to deliberately import error. This prohibition against the deliberate importation of error does not preclude risking the importation of error in an expansion as long as there is a prospect of avoiding error. Nor does it preclude removing items from a corpus via contraction; for to remove a proposition concerning whose truth one is certain from the initial point of view is not, from that point of view, to import error. Rather it is to lose error-free information. I have contended elsewhere that there are sometimes good reasons for suffering such losses. We need not consider the matter here. The point I mean to emphasize is that if we are concerned to avoid the deliberate importation of error into a corpus where error is judged on the assumption that all items in the initial corpus are true, no revision can be legitimate unless it is a contraction or an expansion.

The position I am taking stands in opposition to views like those of Kuhn and Feyerabend who insist that there are legitimate replacements which cannot be rationalized as sequences of legitimate contractions and expansions because contraction to a "neutral basis" is somehow incoherent. I am, of course, fully aware that my view is not universally accepted. My current concern, however, is not to convince everyone of my epistemological outlook but to explain how that outlook motivates my insistence that all legitimate minimal revisions be L-minimal and, hence, obey the preservation condition (P). As far as I have gone, the argument is that the *deliberate* importation of error into a corpus should be illegitimate. This rules out deliberate expansion into inconsistency and deliberate replacement of items in the initial corpus by items inconsistent with them. It also rules out replacements involving deliberate expansion into contradiction followed by contraction as in replacements of type (4). It does not rule out deliberate contraction followed by deliberate expansion where the net effect is a replacement of type (3). That is to say, it does not rule out any L-minimal revisions.

Although I am prepared to entertain the possibility of legitimate replacements of type (4), I would do so only in those cases where the initial expansion into inconsistency is a "routine" expansion in which reports made in response to sensory stimulation or made by witnesses are added to the initial corpus according to some program to which the agent is committed by nature, nature of deliberation beforehand (Levi 1980, pp. 35–41). Unlike deliberate inferential expansion, routine expansion may legitimately result in expansion into contradiction and require a subsequent contraction. Although I do not think this likely, it could happen that the net product of an expansion of this type and the subsequent contraction is a replacement of type (4) (Levi 1980, pp. 62–63).

But even if we entertain such a possibility, revisions of type (4) are not fully deliberate. I contend that an account of the acceptability conditions for conditionals in terms of minimal revisions ought to be modeled on legitimate changes in belief which can be seen as deliberate. In making conditional judgements of possibility and impossibility the agent is invited to modify his views, for the sake of the argument, in a deliberate fashion. If the revision required is a replacement, it should be, therefore, of type (3). Setting this point to one side, however, legitimate, nondeliberate replacements of type (4) ought to be formally equivalent to deliberate replacements of type (3). If one makes observations, for example, which contradict some settled assumption $\sim h$ and one trusts one's observational routines while abandoning $\sim h$, the net result ought to be the same as what would have happened if one had opened one's mind up to the possibility that h is true before observation by a minimal contraction removing $\sim h$ and then had let the observations decide. Thus, I contend that all minimal revisions relevant to the analysis of acceptability of conditionals should be L-minimal.

4.

Appraisals of serious possibility take as objects only truth value bearing propositions. Similarly, the only items eligible for membership in a corpus of knowledge are sentences expressing such truth value bearing propositions. If statements like "it is possible that h" are neither true nor false, if follows that such sentences could not appear in a standard for serious possibility. Hence, the language in which a corpus is expressed cannot be like L^*. Indeed, for much the same reasons, it cannot be L^{**}. We need to restrict the corpus to sentences expressible in L.

This does not mean, however, that we cannot recognize a derived corpus expressible in L^* of those sentences expressible in L^* whose acceptability in L^* is grounded on the adoption of the corpus K in L. I have never intended to prohibit anyone from uttering sentences like "it is possible that h". Nor do I deny that they have a purpose. They express the propositional attitude of an agent who regards the truth of h as a serious possibility and, hence, who does not accept $\sim h$ in his corpus expressible in L. Of course, "it is possible that h" is not a biographical remark.[1] It does not assert that h is a serious possibility according to agent X at t. If it did, it would have a truth value after all. On the other hand, when uttered by X, it may express the attitude whose presence is announced by the biographical remark. This expression is neither truth nor false.

We can, if we like, consider the set of sentences which includes all sentences in K as well as all those expressions of appraisals of serious possibility and impossibility relative to K of the elliptical and, hence, non-truth-value-bearing sort and include them together in Poss(K) expressible in L^*. And we can, if we like, impose conditions on the derivation of Poss(K) from K which insure that Poss(K) is closed under consequence according to a suitable modal logic. Thus, formally at any rate, Poss(K) can be endowed with all the properties of a corpus expressible in L^*. However, it must be remembered that Poss(K) is derived from K by certain principles (about which more will be said shortly). Revisions of Poss(K) are the product of revisions of K. Thus, if K_h is an L-minimal revision of K, the shift is mirrored in L^* by a shift from Poss(K) to Poss(K_h). However, the latter shift need not and, in general, will not satisfy the requirements for an L-minimal revision of corpus. Nor, on the view which I advocate, is it required that such revisions of corpora expressible in L^* via the Poss-function be L-minimal revisions.

Given that K is the standard for serious possibility, we can see that K determines Poss(K) according to the following principle:

(Poss) (a) if h is in K, h is in Poss(K).
(b) if h is in K, $\sim\Diamond\sim h$ is in Poss(K).
(c) if h is not in K, $\Diamond\sim h$ is in Poss(K).

(d) if h is deducible (according to truth functional or first order logic) from Poss(K), h is in Poss(K).
(e) Only those sentences in L^* whose membership in Poss(K) is derivable from clauses (a)–(d) are in Poss(K).

(Poss) allows us to define a set of sentences Poss(K) expressible in L^* as a function of a deductively closed set K of sentences expressible in L. It reflects the role of K as a standard for serious possibility. However, we can apply the schema characterizing (Poss) to deductively closed sets of sentences expressible in L^* to obtain another set of sentences expressible in L^*. Beginning with a deductively closed corpus K^* in L^*, consider the infinite sequence K^*, Poss(K^*), Poss(Poss(K^*), Poss(Poss(Poss(K^*))), ... Form the union Poss*(K^*) of all sets in the sequence. Because the corpus K^* which initiated the sequence will not, in general, qualify as a standard for serious possibility, neither the sequence of applications of Poss nor the limiting output Poss*(K^*) seem to be more than formal epiphenomena.

Nonetheless, just as Poss(K) is uniquely determined by K, Poss*(K^*) is uniquely determined by K^* for any corpus expressible in L^* closed under truth functional or first order deduction. In particular we may substitute Poss(K) for K^* to obtain Poss*(Poss(K)). This corpus in L^* is uniquely determined by the corpus K of nonmodal sentences in L. It contains K and Poss(K). Finally, if we consider the logical truths of L^* to be those sentences which are in every corpus Poss*(Poss(K)) determined by some consistent and deductively closed corpus K of L, these logical truths are the logical truths of an S5 modal logic. Except when asked to supply an account of iterated modal sentences, I see little advantages in focusing on Poss*(Poss(K)). Poss(K) will do. In either case, the corpus expressible in L^* is determined by the contents of K.

Consider now a corpus K which contains neither h nor $\sim h$. According to (Poss), Poss(K) contains both $\Diamond h$ and $\Diamond \sim h$.

K/h is identical with the L-minimal revision K_h by adding h to K.

Poss(K/h) cannot be an expansion of Poss(K). Poss(K) contains $\Diamond \sim h$. Poss(K/h) contains $\sim\Diamond \sim h$ and does not contain $\Diamond \sim h$. Hence, Poss(K/h) cannot represent an L-minimal shift from Poss(K). Even though neither h nor $\sim h$ is in Poss(K), the result of adding h to Poss(K) in a manner which satisfies (Poss) must lead to the removal of $\Diamond \sim h$.

But perhaps one might wish to regard this shift as minimal in a sense different from L-minimality. No matter how we explicate this sense of minimality, as long as condition (P) is required to apply to corpora expressible in L^* and such corpora are restricted to sets derived from corpora expressible in L using the Poss operation, condition (P) asserts that if a proposition B is in Poss(K) and A is consistent with Poss(K), B is contained in the minimal revision of Poss(K) needed to accept A. But we need to explain what it means to say that A is consistent with Poss(K). This could mean that the result Poss(K)/A of adding A to Poss(K) and forming the deductive closure is consistent (which is so if and only if A is consistent with the corpus K expressible in L). According to this reading, it is obvious that the preservation condition (P) will be violated. For the minimal revision of Poss(K) obtained by adding A will be Poss(K/A). Since $\diamond \sim A$ is in Poss(K) but not in Poss(K/A), condition (P) will be violated.

There is, however, another reading of condition (P). We might say that A is consistent with Poss(K) if and only if Poss(Poss(K/A)) is consistent. On this reading, no A not already in Poss(K) is consistent with Poss(K). Hence, condition (P) is vacuously satisfied.

Thus, if potential corpora expressible in L^* are restricted to corpora closed under Poss (as they must be if they are to reflect the notion that the corpus expressible in K represents the verdicts concerning possibility delivered by a standard for serious possibility), (P) cannot be nonvacuously satisfied.

None of this poses a problem for those who, like myself, wish to regard the corpus expressible in L as a standard for serious possibility and recognize as legitimate only those revisions of corpora expressible in L which satisfy the preservation condition.

The conflict with (P) just constructed makes no reference to the Ramsey test for the acceptability of conditionals. Nothing is said about conditionals at all. Instead, we have substituted a principle (Poss) specifying "acceptability conditions" for judgements of serious possibility. Given that (Poss) functions here in a manner analogous to the Ramsey test in the case of conditionals, applying (P) to revisions of corpora expressible in L^* corresponds to endorsing (P) corresponds to Gärdenfors' assumption (O) for conditionals. By refusing to endorse this analogue for (O) and resting content with imposing condition (P) only on the corpus expressible in nonmodal L, the tension between (Poss) and (P) described above is avoided.

Nothing in this argument depends upon whether the corpus expressible in L^* allows for the iteration of $\Diamond$. Since Poss*(Poss(K)) contains Poss(K), condition (P) cannot be applied consistently and nonvacuously to revisions of corpora in L^* which are derivable from changes of corpora in L via the function Poss*(Poss(K)). But we do not need to consider Poss*(Poss(K)) to obtain this unattractive result. As long as the preservation condition (P) is applied to revisions of corpora in L^* derivable from revisions of corpora in L via (Poss), there will be trouble. The application of (P) should be restricted to revisions of corpora expressible in L – at least if we are to think of a corpus of knowledge as a standard for serious possibility

The shift from Poss(K) to poss(K/h) qualifies as a replacement in my technical sense because $\Diamond \sim h$ is removed and $\sim\Diamond \sim h$ is substituted in its stead. If these modal sentences are regarded as truth-value-bearing *in a sense in which avoidance of error (falsity) is a concern of inquiry*, then from the point of view of the agent endorsing Poss(K), $\Diamond \sim h$ is true and $\sim\Diamond \sim h$ is false. To deliberately replace the former by the latter is, from that point of view, to deliberately replace truth with error.

According to the position I have taken, such modal judgements lack truth values in any sense in which avoidance of error is a value which ought to be promoted in revising beliefs. Hence, the shift from Poss(K) to Poss(K/h) does not involve the deliberate substitution of error for truth.

Of course, I am supposing that the sentences in L do represent truth value bearing propositions in a sense in which avoidance of error matters. Thus, although I do object to deliberate substitution of error for truth via the replacement of one corpus expressible in L by another, I do not object to replacement of corpora expressible in L^*. This explains why I am concerned to observe (P) for revisions of corpora expressible in L but do not require it for corpora expressible in L^*.

If one objects that on this view one cannot iterate the possibility operator since it is presumably supposed to apply to truth-value-bearing claims, I agree in the following sense: the only sentences whose serious possibility are appraised by the corpus are sentences in L and only such sentences are eligible for membership in the corpus.

This does not mean that there is any obstacle to allowing for forms of expressions either in some natural language or in a regimented language such as L^* involving the iteration of modal connectives. The principle (Poss*) does just that in a perfectly harmless way. However, no modal sentence in L^* (or a paraphrase in natural language) is to be evaluated with respect to truth value in the context of a concern to avoid error in belief revision.

Thus, the sort of modal realism I oppose is one which insists that modal judgements belong in the "corpus" or "belief set" which is subject to revision and where it not only makes sense to be concerned to avoid error in revision of that corpus but one should do so. If one embraces such modal realism, it should be the case that the corpus K^* expressible in L^* is legitimately revisable if and only if it is legitimately revisable as a sequence of contractions and expansions. Hence, all minimal revisions in L^* would have to be L-minimal and in conformity with preservation principle (P). As a consequence, the corpus expressible in L^* could not be derivable from the corpus expressible in L via (Poss) or even (Poss) supplemented by (Poss*).

But since the corpus K^* expressible in L^* is now considered the object of revision, it becomes the standard for serious possibility. Hence, there must be a notion of possibliity distinct from the realistically construed notion of possibility which appears in statements in L^*. We must be able to ask whether $\Diamond h$ is true or false, and if we are in doubt, both alternatives are serious possibilities. The set of sentences Poss(K^*) expressing judgements of serious possibility concerning sentences in L^* would itself be expressed in another modal language distinct from L^* though including it. The connective for serious possibility would be different from the connective $\Diamond$ now used for the realistically construed version of de dicto possibility, and the corpus K^* could not be identified with Poss(K).

Thus, so it seems to me, a modal realist will not avoid the need to consider judgements of serious possibility of the sort I am considering. He should insist, however, that there are two types of de dicto possibility: an objective and a subjective variety – just as many others have maintained that there are two kinds of de dicto probability: an objective probability and a subjective or epistemic one. And, as is to be expected from such a realist, the objective variety of possibility is taken to be irreducible to the epistemic variety by appeal to (Poss).

Although I am committed to the view that there are objective statistical probabilities, I deny that in any interesting and useful sense these are de dicto (Levi 1978, 1980). But one cannot insist that there will never be a useful theory in which de dicto objective probabilities figure in an important way. I have not seen any such theory as yet. The position I take vis-à-vis de dicto probability applies just as well to de dicto possibility. No one has offered any good reason for introducing a notion of objective de dicto possibility. The proposals which abound and which are realistically construed are defended by appealing to the fact that we do after all make modal judgements. This point, of course, is not under dispute. But it also ought not to be under dispute that we make judgements of serious possibility – i.e., judgements as to what is and is not ruled out by the current doctrine. What needs to be argued is whether we also make judgements of realistically construed de dicto possibility.

5.

How does all of this bear on the use of the Ramsey test as a criterion for the acceptability of conditional sentences?

According to the version of the belief revision account of conditionals I favor, acceptability conditions for sentences in L^{**} of the form $h > g$ and $\sim(h > g)$ where h and g are in L are formalized as follows:

(RL) (a) K is a subset of RL(K).
(b) $h > g$ is in RL(K) if $\sim\Diamond\sim g$ is in Poss(K_h) (i.e., $g \in K_h$) where K_h is L-minimal.
(c) $\sim(h > g)$ is in RL(K) if $\Diamond\sim g$ is in Poss(K_h) (i.e., g is not in K_h) where K_h is L-minimal.
(d) RL(K) is closed under truth functional (or first order) deduction.
(e) f is in RL(K) only if its membership is derivable via (a)–(d).

The key idea behind condition (RL) is this: a conditional of the (regimented) type $h > g$ is a judgement concerning the serious possiblity of g relative to a *transformation* of the current corpus or belief set K expressible in L and not relative to the current corpus itself. The transformation T(K) of K is the L-minimal revision of K which is subject to the sole constraint that h be a member of T(K).[2]

Observe that $\sim(h > g)$ is also judgement of serious possibility relative to T(K) which is a minimal transformation of K where h is made a member of T(K) which is a minimal transformation of K where h is made a member of T(K). The difference between the two judgements $(h > g)$ and $\sim(h > g)$ is that the former renders the verdict that the falsity of g is not a serious possibility relative to T(K) and the latter declares that $\sim g$ is a serious possibility relative to T(K).

Consequently, conditional sentences ought not to be construed as truth-value-bearing any more than judgements of serious possibility ought to be. They are expressions of our evaluations of truth-value-bearing hypotheses with respect to serious possibility relative to transformations of the current corpus (Levi 1977, 1980).

Of course, as far as linguistic representation is concerned, we can provide for a language L^{**} in which such conditional judgements are expressed and consider the corpus RL(K) of sentences in L^{**} which are derived from the corpus K in L by taking K and applying the Ramsey test (RL).

When h is in K, $h > g$ is in RL(K) if and only if $\sim\Diamond\sim g$ is in Poss(K). Similarly, when h is in K, $\sim(h > g)$ is in RL(K) if and only if $\Diamond\sim g$ is in Poss(K). If, therefore, we take $\Diamond$ to be a defined connective in L^{**}, we can regard Poss(K) as translatable into a subset of RL(K).

RL(K), like Poss(K), contains no iterations of the connective $\Diamond$. Nor does it contain sentences in which there are iterations of $>$ or in which occurrences of one of these two connectives are nested in an occurrence of the other. We may allow for iterations of $\Diamond$ by extending RL(K) to Poss*(RL(K)) just as previously we extended Poss(K) to Poss*(Poss(K)). The other types of iteration are still excluded in Poss*(RL(K)). We shall discuss this matter later on.

Consider now a corpus K which contains neither h nor $\sim h$ and look at the expansion of K by adding h. K/h is K_h if minimal revisions are L-minimal.

Observe, however, that $\Diamond\sim h$ is a member of RL(K) but is not a member of RL(K/h) = RL(K_h). Indeed, $\sim\Diamond\sim h$ is a member of the latter set. Hence, the relation between RL(K) and RL(K/h) is one of replacement and not of expansion. Moreover, it is easily seen that there is no way in which one can decompose the shift from RL(K) to RL(K/h) into a sequence of contractions and expansions expressible in L^{**} where each corpus in the sequence is derivable from a corresponding consistent corpus in L via (RL). Unlike the shift from K to K/h, the shift from RL(K) to RL(K/h) cannot be considered to be an L-minimal revision. The same example shows that the shift from RL(K) to RL(K/h) violates the preservation condition (P). Of course, the shift from K to K/h is L-minimal and obeys (P) even though (RL) is obeyed in deriving the corpus expressible in L^{**}.

Thus, we have reached the dilemma posed by Gärdenfors but by a different route. We have a "Ramsey test condition" (RL) for deriving a corpus expressible in L^{**} from a corpus expressible in nonmodal L. To endorse condition (RL) as a requirement of belief revision is to insist, in effect, that any belief state must be such that the corpus expressible in L^{**} is related to the corpus expressible in L according to (RL).

We have a preservation condition (P) on minimal revisions. When the condition is required to apply to minimal revisions of corpus expressible in L^{**}, we have the conjunction of (P) and (O) as stated by Gärdenfors. When the condition is required to apply to minimal revisions expressible in L, we have (P) and the denial of (O). The condition (RL) conflicts with the demand that minimal revisions of corpora expressible in L^{**} satisfy (P) – i.e., (RL) conflicts with the conjunction of (P) and (O).

6.

My condition (RL) is not quite the same as Gärdenfors' condition. His version of the Ramsey test condition runs as follows:

(RG) Let K^{**} be a set of sentences in L^{**} closed under truth functional (first order) consequence.
(a) K^{**} is a subset of RG(K^{**}).
(b) $h > g$ is in RG(K^{**}) if and only if g is in RG$(K^{**})_h$.
(c) RG(K^{**}) is closed under deduction.
(d) Nothing is in RG(K^{**}) unless its membership is derivable via clauses (a)–(c).

There are three respects in which (RG) differs from (RL):

(1) There is nothing in (RG) to prevent h and g in $h > g$ from being sentences in L^{**} (or L^{*}) which are not also in L. (RL) precludes conditionals of that type from being members of RL(K).

(2) (RG) provides for the acceptance of $h > g$ but not for the acceptance of $\sim(h > g)$.

(3) (RG) specifies membership of a conditional in RG(K^{**}) if and

only if a certain sentence is an element of the minimal revision of RG(K^{**}) – rather than the minimal revision of the subset K of RG(K^{**}) (and, hence, of K^{**}) of nonmodal and nonconditional sentences in L.

Difference (3) is not as considerable as it might appear. Given RG(K^{**}), let K be the subset of RG(K^{**}) consisting of the elements of that set which are in the nonmodal L. RG(K) is the set in L^{**} produced by applying (RG) to K rather than K^{**}. RG′(K) is the result of applying (RL) with clause (c) deleted to K. The difference between RG(K) and RG′(K) is that $h > g$ can be in RG(K) even if h or g is in L^{**} but not in L. For the condition of acceptability in RG(K) refers to minimal revision of RG(K) by adding h to it. $h > g$ can be in RG′(K), by way of contrast, only if h and g are in L, for the condition of acceptability refers to minimal revision of K and not of RG′(K).

Even though RG′(K) disallows iteration of the conditional connective, the monotonicity condition (M) – which asserts that if h is not in K, K_h is contained in K'_h if K is a subset of K' – holds for revisions of the corpus RG′(K) expressible in L^{**}. On this score, there is no difference between RG(K^{**}), RG(K) or RG′(K). The crux of Gärdenfors' proof of an inconsistency between preservation and the Ramsey test is an argument showing that monotonicity as applied to corpora in L^{**} conflicts with (P) as applied to revisions of corpora in L^{**}. It does not matter whether the corpora in L^{**} are of the type RG(K^{**}), RG(K) or RG′(K).

To see this for RG′, let RG′(K) be contained in RG′(K'). Suppose that the minimal revision of K contains g. Then RG′(K_h) contains g. If we are adopting (RG′), it is plausible to suppose that minimal revisions of corpora in L^{**} satisfy the condition that $\{RG'(K)\}_h = RG'(K_h)$. This is because h is in L and the subset of $\{RG'(K)\}_h$ in L ought to be identical with K_h. Since however, $\{RG'(K)\}_h$ is determined by the application of RG′ to a corpus in L, it must be the result of applying RG′ to K_h. Hence, $\{RG'(K)\}_h = RG'(K_h)$.

It follows from this that $\{RG'(K)\}_h$ contains g. Hence, $h > g$ is a member of RG′(K) and also RG′(K'). But this means that g is a member of K'_h and of $RG'(K'_h) = \{RG'(K')\}_h$. Thus, (RG′) implies satisfaction of the monotonicity condition by minimal revisions of corpora RG′(K) expressible in L^{**} even though there is no iteration of the conditional connective.

Because (RG′) is derived from (RL) by deleting clause (c), it is obvious that this result could be obtained from (RL) – provided we focus on minimal revisions of RL(K) rather than K and define $\{RL(K)\}_h$ to be identical with RL(K_h).

Thus, Gärdenfors' monotonicity requirement (M) is met by corpora derived from K via (RL). This is the requirement Gärdenfors shows to conflict with (P) and (O). For the purposes of his argument, the possibility of formulating acceptability conditions for iterated conditionals with the aid of (RG) does not appear to be relevant to the status of the monotonicity condition as applied to minimal revisions of corpora in L^{**}. Thus, we may ignore difference (1) as well as difference (3) at least insofar as the dilemma posed by Gärdenfors is concerned. To be sure, insofar as the question of iteration remains a matter of concern, difference (1) is very important indeed. My version of the Ramsey test cannot be invoked to specify acceptability conditions for iterated conditionals where Gärdenfors' principle can. I rest content, for the present, with pointing out that the conflict with preservation obtains *when preservation is applied to revisions of corpora in* L^{**} whether (RL) or (RG) is deployed.

Difference (2) is more substantial. The difference between RL(K) and RG(K) is that RL(K) contains sentences of the type $\sim(h > g)$. RG(K) does not contain rules for incorporating such sentences. Gärdenfors' achievement is to rely on the fact that (RG) suffices to imply the monotonicity principle and that this conflicts with (P) and (O). By way of contrast, the argument I offer here depends on acceptability conditions for negations of conditionals as well as for conditionals themselves. In that sense, my approach relies on stronger assumptions than does Gärdenfors'. On the other hand, my argument does not really depend on the full force of (RL) but only on the special case where (RL) is equivalent to (Poss) for deriving the corpus Poss(K). That is to say, my argument appeals solely to that feature of the Ramsey test condition which relies on the function of a corpus K expressible in L as a standard for serious possibility. In this context, controversies concerning negations of conditionals need not preoccupy us.

Thus, it seems to me that one cannot question the Ramsey test condition account of conditionals without questioning the function of a corpus of knowledge as a standard for serious possibility, for to retain a version of (Poss) and abandon the rest of (RL) will still breed conflict with (P) when that is applied as a condition on revisions of corpora expressible in (L^{**}). But to abandon the conception of knowledge as a standard for serious possibility is to deprive us of any clear conception of the function of knowledge as a resource in deliberation.

For this reason, it seems to me that the "dilemma" identified by Gärdenfors cannot be resolved by abandoning the Ramsey test condition. As long as we retain that part of the Ramsey test condition formulated by (Poss), the dilemma posed by Gärdenfors remains, and to abandon (Poss) is to abandon the idea that a corpus should serve as a standard for serious possibility.

If we retain the Ramsey test condition and are prepared to endorse it in the full form (RL), conflict can be avoided only by abandoning the preservation condition as a condition on revisions of corpora expressible in L^{**} but not as a condition on corpora expressible in L. That is to say, we ought to abandon the conjunction of (P) and (O).

If this view is endorsed, revisions of belief sets or corpora of knowledge are to be viewed as revisions of corpora expressible in the nonmodal language L. There is nothing to prevent us from identifying for each corpus expressible in L an associated corpus expressible in L^{**}. But on the view being proposed here, the revisions of corpora expressible in L^{**} are parasitic on the revisions of corpora expressible in L. It is only the truth values and the informational values of the contents of such corpora which matter when we seek to improve our knowledge by modifying it.

7.

I have just argued that proposed resolutions of the conflict between the Ramsey test and the application of the preservation condition to revisions of corpora expressible in L^{**} ought to hold fast to the idea that a body of knowledge or set of beliefs should serve as the standard for serious possibility, while insisting that the minimal revisions of corpora relative to which the Ramsey test is to be applied ought to be L-minimal and, hence, should satisfy the preservation condition. For this reason, it seems to me that (1) the Ramsey test condition should be restricted in application to revisions of corpus expressible in L and (2) the preservation condition should be restricted to the same revisions of corpus expressible in L.

Gärdenfors does not want to proceed in this way. He explicitly embraces condition (O) which implies that the corpus which is to be subjected to minimal revision in applying the Ramsey test is to be a corpus in L^{**}. He defends this posture on the grounds that failure to provide acceptability conditions via the Ramsey test for iterated conditionals represents a defeat for the belief-revision account of conditionals.

Gärdenfors also holds fast to another ambition. He not only insists on a belief-revision account of iterated conditionals grounded on the Ramsey test but also seeks to reconstruct on this basis an approach to conditionals of the sort developed by Stalnaker and Lewis.

In the next section, I shall show that one can develop a belief-revision approach to the acceptability of iterated conditionals provided one does not insist that the acceptability of iterated conditionals be determined by a Ramsey test. Before turning to this matter, however, I would like to examine Gärdenfors' search for a belief revision model for the Lewis theory of conditionals.

Gärdenfors (1978 and 1981) did develop a reconstruction of the Lewis–Stalnaker view in terms of a Ramsey test criterion for the acceptability of conditionals. As he points out in his 1986 paper, the Ramsey test he employed in the 1978 paper is (in essence) (RG) which, so he emphasizes, entails the monotonicity condition but does not mandate preservation as a condition on minimal revisions of corpora expressible in L^{**}.

Crucial to Gärdenfors' (1978) reconstruction of the Lewis system is a condition (C10b). According to this condition, if $\sim(h > \sim g)$ is in K^{**}, K_h^{**}/g is a subset of K_h^{**} & g. In the special case where h is in K^{**}, this condition stipulates that if $\Diamond g$ is in K^{**}, K^{**}/g is a subset of $K^{**}g$. (C10b) avoids implying the preservation condition because Gärdenfors has avoided commitment to the requirement that if $\sim g$ is not in K_h^{**}, $\sim(h > \sim g)$ is in K^{**}. Hence, if $\sim g$ is not in K^{**}, Gärdenfors' acceptability conditions leave unsettled whether $\Diamond g$ is acceptable in K^{**}.[3]

But surely there is an important sense in which one does want to say that g is possible if it is compatible with what one knows. When g is consistent with what is known one may coherently assign a positive probability to g and define conditional probabilities (in the sense of a standard probability measure – not a Popper measure) on the condition g. And if one insists, nonetheless, that it remains unsettled in general whether one may "accept" g as possible when g is consistent with what one knows, the conception of possibility involved must be different from an epistemic conception congenial to advocates of belief revision approaches to conditionals.

The common feature of the account of conditionals I advocate and the one presented by Gärdenfors is the requirement captured by (RL) (b) and (RG) (b) respectively. This requirement insures commitment to monotonicity which Gärdenfors deploys in establishing the conflict between the Ramsey test and the condition of preservation on changes in corpora expressible in L^{**}.

Having recognized the conflict, Gärdenfors sees himself facing a choice between giving up the Ramsey test and giving up preservation. He thinks there is something to be lost by both moves. Giving up the Ramsey test means abandoning his epistemic reconstruction of the Lewis theory of conditionals. Giving up on preservation means substantial modification of the approach to criteria for rational belief revision on which he collaborated with Alchourron and Makinson (Gärdenfors 1982, 1984 and Alchourron, Gärdenfors and Makinson 1985). Although there are some points of detail concerning which I would differ with these authors, the general approach to belief revision they endorse conforms well with the ideas I favor. The difference between Gärdenfors and me is that he seems to think that there is something to be lost by commitment to this approach and abandonment of the effort to reconstruct the Stalnaker–Lewis vision in terms of models of belief revision. I see no loss.

My contention is that Gärdenfors' vision of the alternatives is hampered by his failure to address the question of the role of the body of beliefs or corpus of knowledge in deliberation and inquiry. If it is right to suppose that a corpus serves to define the space of possibilities over which subjective probabilities are defined as I maintain, we should incorporate an account of acceptability conditions for "it is not possible that h" as well as "it is possible that h". Similarly, we should identify acceptability conditions for "if h is true, g might be true" as well as "if h is true, g would be true".

It is ironic that Gärdenfors complains that I have failed to provide acceptability conditions for interated conditionals when he had failed to do so for the negation of conditionals. And the irony deepens when it becomes apparent that his effort to obtain a Lewis system depends upon his refusing to do so.

The moral of the story would seem to be that efforts to reconstruct a theory of conditionals along the lines of Stalnaker and Lewis in terms of belief revisions ought to be abandoned. Such theories cannot be reconstructed along such lines. If they make sense at all, they make sense within a framework which takes realism about possible worlds seriously. I for one cannot find it in my heart to embrace such metaphysics gratuitously. Gärdenfors exhibits a similar penchant but, at the same time, displays a devotion to the Stalnaker–Lewis ideas. The need to accommodate Stalnaker–Lewis is so great that he seems prepared to give up the core of the belief revision approach – to wit, that bodies of knowledge define the spaces of serious possibility. At this point, we should remember one of the fundamental theorems about possibility – you can't have your cake and eat it!

8.

Even though I insist that revisions of knowledge of belief are in the first instance revisions of corpora expressible in non modal language, this does not mean that modal judgements are meaningless. Nor, for that matter, does it preclude some forms of iterated modal statements. Thus, I have allowed for Poss(K) to be enriched by iterated statements through (Poss*). (Poss*) can be applied to RL(K) in the same spirit.

Given a corpus K in L, therefore, we can use (Poss*) to derive a corpus Poss(K) which allows iterated modal sentences. Indeed, it should be apparent that $\sim\Diamond\sim h$ will be in K^* if and only if $\sim\Diamond\Diamond\sim h$ (i.e., $\Box\Box h$) is in K^* and $\Diamond\sim h$ will be in K^* if and only if $\sim\Diamond\sim\Diamond\sim h$ (i.e., $\Box\Diamond\sim h$) is in K^*. So K^* will be closed under the consequence relation appropriate to the S5 modal system.

Thus, insisting that all revisions are grounded on revisions of corpora expressible in L does not preclude identifying a corpus of judgements of possibility (the corpus Poss(K)) where iterated judgements of modality are allowed. And the corpus Poss(K) can be incorporated into RL(K) as indicated.

Even so, as long as all revisions of corpora are in the first instance revisions of corpora expressible in L, all L-minimal revisions of the sort required for the Ramsey test conditional are revisions where nonmodal sentences are added to the corpus K in L. Moreover, the hypotheses which are evaluated with respect to serious possibility relative to a L-minimal revision K_h are also nonmodal judgements expressible in L. Consequently, it appears that Gärdenfors' complaint that the account of conditionals I favor does not allow for iterated conditionals is correct in the following sense: No iterated conditionals are acceptable in the corpus RL(K) (or the corpus Poss*(RL(K))) via the Ramsey test. I deny, however, that this is worrisome. I shall now indicate how the anxiety about iteration can be mitigated without in any way abandoning my scepticism about iteration.

What is the main feature of the position I am taking about belief revision? It is that all belief revisions are in the first instance revisions of corpora expressible in nonmodal language L. Furthermore, all legitimate revisions at this level are to be seen as sequences of justifiable contractions and expansions. This implies that condition (RL) cannot be used to generate acceptance conditions for iterated conditions. To try to do so is to consider L-minimal revisions of corpora expressible in L^{**} rather than those expressible in L.

Keep in mind, however, that even noniterated conditionals derived from (RL) are essentially appraisals of serious possibility relative to transformations of the corpus K. The transformations we have focused on are L-minimal revisions in which the condition specified in the if-clause of the conditional becomes a member of the transformed corpus. But there are revisions of K which are not L-minimal revisions of this kind.

Perhaps the most important of these is revision of K via contraction where $\sim h$ is removed from a corpus containing it. Now suppose we are concerned with evaluation of g with respect to serious possibility relative to the contraction of K by removing $\sim h$ – let us call it $K \dot{-} h$. Before contraction, h is not a serious possibility. Afterwards it is. We cannot use the Ramsey condition (RL) where the if clause does not contain a modal condition to make such a conditional judgement of serious possiblity. We can, however, use sentences like "if h were possible, g would be true" or "if h were possible, g might be true".

Those who seek to apply the Ramsey test to such cases construe them as belief revision conditionals where the corpus Poss(K) (Poss*(Poss(K)), RL(K), Poss*(RL(K))) containing $\sim\Diamond h$ is minimally revised so that $\Diamond h$ is added. However, to focus on this revision is to suggest that our account of belief revision should address changes in the corpus expressible in L^* or, perhaps, even L^{**}. That, however, is not so. We can construe "if h were possible, g would be true" as an appraisal of the serious possibility of nonmodal g relative to a corpus expressible in L which is a contraction of K where $\sim h$ is deleted with a minimal loss of information. The hypothetical declares that g is in the resultant contraction.

As this illustration indicates, there is nothing to prevent us from allowing modal expressions appearing in the if-clauses in this way. We can even allow such expressions to appear in the conditionals represented in the regimented language L^{**}. However, the acceptability conditions for such conditionals depend entirely on modal appraisals relative to revisions of corpora expressible entirely in nonmodal terms. It is plain that the acceptability conditions do not conform to the Ramsey test conditions as specified by Gärdenfors or by myself. The revision of K relative to which judgements of serious possibility are made is not L-minimal. Perhaps we shall want to introduce a generalized sense of minimal revision according to which a contraction is a minimal revision, but it is not a revision resulting in the addition of anything to the corpus relative to which judgements of serious possibility are to be made.

This does not appear to me to entail any abandonment of principle. The nerve of the belief revision modal of conditionals as I understand it is that conditionals are appraisals of serious possibility relative to transformations of the current corpus where the if-clause gives instructions as to the kind of transformation to be made. The Ramsey test can be used to determine the kind of transformation when the if clause contains a nonmodal condition. (Even this is not strictly correct. As V. Dudman has exhaustively established, tense plays a crucial role in determining the kind of transformation which is appropriate.) And what I am now suggesting is that there are other occasions where the use of modality in the if clause can contribute to specifying the transformation. Of course, the acceptability of such conditionals is not defined by (RL) but by supplementary rules. We could introduce a new rule:

(Contract)

(a) Poss*(RL(K)) is a subset of Contract(K).
(b) If g is in $K \dot{-} \sim h$, $\Diamond h > g$ is in Contract(K).
(c) If g is not in $K \dot{-} \sim h$, $\sim(\Diamond h > g)$ is in Contract(K).
(d) If g is a deductive consequence of Contract(K), g is in Contract(K).

(e) Only sentences in L^{**} whose membership in Contract(K) is derivable from (a)–(d) are in Contract(K).

Given Contract(K), we can then construct Poss*(Contract(K)). All conditionals in Contract(K) can be viewed as judgements of possibility and impossibility relative to corpora which are transformations of K. Many of them are L-minimal. Others are not because they are contractions. Hence, they are not conditionals whose acceptability is decided by a Ramsey test in the strict sense. These exceptional conditionals are of the form $\Diamond h > g$. If one thinks of the acceptability of such a conditional as depending on applying a Ramsey test, the tension between the Ramsey criterion and the preservation requirement emerges. But if one thinks of the acceptability of the conditional as depending on a different test appealing to transformations of K different from L-minimal ones but still rationalizable as sequences of contractions and expansions and in conformity with (P), there is no trouble at all.

Iterative forms may be used to characterize more sophisticated transformations of corpora expressible in L without admitting that the transformations operate on corpora expressible in some modal language.

To illustrate: consider the transformation of K obtained by first shifting to K_h and then shifting to $(K_h)g$ before evaluating proposition f in L with respect to serious possibility. If f is in the transformed corpus $(K_h)g$, the agent might be said to "accept" the iterated conditional $h > (g > f)$.

Of course, this iterated conditional is not in RL(K) as I have defined it. But it can be added to RL(K) by appealing to the additional rule of derivation I have just specified just as RL(K) can be enriched by rules for adding sentences like $\Diamond h > g$.

The precise details of the rules supplementing the Ramsey test criterion for the acceptability of conditionals expressible in natural languages will, no doubt, be heavily controlled by constraints due to the grammatical features of the particular languages under investigation – as Dudman's work on conditionals impressively establishes (Dudman 1983, 1984). One problem which arises when one has iterated transformations of corpus is the following: if one L-minimally revises K by adding h and then L-minimally revises the result by adding g, it can happen that the result no longer contains h.

On the other hand, we may want to stipulate that in this sequence of iterations, h is not to be removed when g is added. The sequence of minimal revisions is to be subjected to this constraint. (The idea of imposing such a constraint is not unintelligible. If h and g are incompatible so that h must go, the test for the conditional confronts a degenerate case which is to be decided as convenience requires.) The question is whether the acceptability of an iterated conditional like $h>(f>g)$ is subject to such a conservation constraint on iterated transformations. I take it that the appropriate response is that it is possible to identify at least two types of iterated conditionals. However, one might wonder which type best articulates the messages conveyed in English or some other natural language by iterated conditions of the form under investigation.

I do not know the answer to this question. As far as the belief revision account of conditionals I favor is concerned, the answer makes no fundamental difference to the approach but only to details about specific natural languages. Conditionals express assessments of serious possibility relative to transformations of the corpus of non-modal beliefs representable in regimented L. How the transformations are specified is a detailed question of grammar and semantics which need not be answered fully in order to sustain the coherence and plausibility of the belief revision view. However, if the conservation constraint on iterated transformations is right, the conditional $h>(g>f)$ has the same acceptability conditions as $h \& g>f$ as Vann McGee has recently and plausibly suggested (McGee 1985).

On this view, it is worth noting that $h>(g>f)$ may be acceptable along with h itself and yet $g>f$ need not be acceptable. Once more, McGee has offered some useful illustrations.

What about claims of the form $(h>g)>(i>j)$? Examples of this type worry Gärdenfors. I am not entirely convinced that bona fide examples of such iterated conditionals are used in serious discourse or inquiry. I shall explain why shortly. However, I do not mean to rely on such scepticism concerning this form of iterated conditional and shall sketch an approach to such conditionals which should be adequate when and if they may prove of use.

To obtain a clue how one ought to proceed, consider the way in which the construal of $\Diamond h>g$, and $h>(g>f)$ was approached. In the case of $\Diamond h>g$, the idea is to contract K and assess serious possibility relative to the contracted corpus. This construal suggests that the transformation of K recommended by the if-clause advises a modification of K such that relative to the modified corpus $T(K)$, h is a serious possibility. To achieve that end, one needs to remove $\sim h$ from K if it is in K and otherwise to do nothing. If one contracts, of course, it is recommended that the contraction minimize loss of informational value.

Suppose now that we contemplate the conditional $\Diamond h > (g > f)$. Once again we are advised to contract by removing $\sim h$ if need be and modify the resultant $T(K)$ by an L-minimal revision eventuating in the addition of g.

This last conditional offers us a precedent for approaching $(h > g) > (i > j)$. The if-clause contains $(h > g)$. This clause instructs us to transform K so that relative to the transformation $T'(K)$, $h > g$ is a member of $RL(T'(K))$, just as when $\Diamond h$ is in the if-clause, we are advised to deploy a transformation $T(K)$ such that $\Diamond h$ is a member of $Poss(T(K))$.

Once we have undertaken a transformation $T'(K)$ of K expressible in the nonmodal language L satisfying this condition, we are invited to ascertain whether $i > j$ is also a member of $RL(T'(K))$. Equivalently, we are asked to perform an L-minimal revision of $T'(K)$ so that i is added and to judge the possibility of j relative to the resulting corpus.

There is, therefore, an analogy between the treatment of $\Diamond h > \langle g > f \rangle$ and the treatment of $(h > g) > (i > j)$. But there is a disanalogy. In the former case, it seems quite clear what kind of transformation of K is deployed in order to secure that h is a serious possibility relative to $T'(K)$. Either h is already a serious possibility according to K in which case K already $= T(K)$, or h is not a serious possibility according to K in which case $T(K)$ is a contraction of K minimizing loss of information while deleting $\sim h$. The problem in the case of $(h > g) > (i > j)$ is that it is as yet unclear what modification of the corpus K in the nonmodal language L is being recommended which will meet the specification that $h > g$ will be in $RL(T'(K))$.

I am not sure that there is a unique answer to that question. As I have already intimated, there may be no need for an answer anyhow, for iterated conditionals of the type under consideration may rarely if ever find a useful application.

I do not mean to deny that judgements which appear to be of that form are sometimes used. Consider the example Gärdenfors cites from van Fraassen:

If this vase breaks if dropped on the floor, then it breaks if thrown against the wall.

To deny that claims such as this are made would be foolish. But the claim made by van Fraasen's English sentence is not easily interpretable as an iterated conditional instantiating the form $(h > g) > (i > j)$. To see this focus on the if-clause. "This vase breaks if dropped on the floor" or the alternative "If this vase is dropped on the floor it breaks". These sentences do not appear to be construable as either hypotheticals or conditionals according to Dudman's insightful taxonomy classifying if-sentences into hypotheticals, conditionals and generalizations. But as I explain in Note 2, the English sentences to be paraphrased into the regimented form $h > g$ according to the belief-revision account I favor are equivalent to if-sentences under either the hypothetical or conditional construal.

"This vase breaks if dropped on the floor" is a conditional belonging to the kind Dudman calls "generalizations". Dudman is quite clear that these generalizations are not "exponible Frege's way" (Dudman 1984, p. 149). Dudman does not produce a systematic account of how such "generalizations" are to be understood but I think it is plausible to suggest that such "generalizations" are tantamount to attributions of dispositions to objects. The vase is alleged to have the disposition to break upon being dropped. The interesting insight afforded by Dudman is that grammatical clues distinguish if-sentences serving as devices for predicating dispositions of systems or of systems of some type from if-sentences under the hypothetical or conditional construals. The putative example of an iterated conditional cited by Gärdenfors seems to me to be a hypothetical if-sentence in which the if-clause contains an if-sentence under the generality or dispositional construal as does the then-clause.

According to the position I have taken elsewhere (e.g., in Levi 1977 and 1980), disposition sentences are not if-sentences construable according to the belief revision model. In this sense, I deny that they are conditionals representable by the form $h > g$. Consequently, if the van Fraassen–Gärdenfors example of an iterated conditional is really to be construed as a conditional where both the if-clause and the then-clause contain attributions of dispositions, the example fails to instantiate the form $(h > g) > (i > j)$ after all.

To attribute the disposition to break upon being dropped to the vase is to assert that the vase satisfies a given condition or has some property. The assertion has a truth value. It is eligible for membership in a potential corpus and it may be evaluated with respect to serious possibility and credal probability. To be sure, disposition predicates are place holders in stopgap explanations pending research which may yield better integration of the predicates within explanatory schemes or their replacement by explanatorily more satisfactory descriptions. (Levi and Morgenbesser 1964, Levi 1967). But precisely because they serve this placeholder function, they are treated as predicates true or false of objects or systems. They are to be distinguished, therefore, from conditionals of the type $h > g$ which, according to the account I am advocating, express assessments of serious possibility relative to transformation of the current standard for serious possibility. Conditions of this latter sort, unlike disposition statements, lack truth values and are not appropriate objects of propositional attitudes such as credal probability judgements.

To be sure, a predicate like "is disposed to break upon being dropped on the floor" (which I will abbreviate by "is a D(B/DF)" is related to conditionals. The reason is easy to see. Such disposition predicates are introduced in order to formulate stopgap laws such as "Every D(B/DF) dropped on a floor breaks". Such a stopgap law is, of course, one clause in a Carnapian reduction sentence. But given a corpus containing the stopgap law and "v is a D(B/DF)", the conditional "if the vase is dropped on the floor, it will break" is acceptable in the corpus RL(K). Thus, even though disposition statements are truth-value-bearing and, hence, eligible for membership in the standard for serious possibility while conditionals are not, acceptance of disposition statements in a corpus can support (render acceptable) conditionals (Levi 1977 and 1980, Chap. 11).

According to my reading of the van Fraassen example, it does not instantiate an iterated conditional at all. Instead it instantiates the form $x > y$. It may be represented by "v is a D(B/DF) > "v is a D(B/DW)" or in a closer facsimile to English, "if this vase is disposed to break on being dropped on the floor, it is disposed to break on being thrown against the wall".

Although it seems to me that this reading of the van Fraassen example, these two if-sentences are conditionals in Dudman's sense and although I doubt whether there is a widespread English usage which drives us to acknowledge the importance of iterated conditionals of the form under consideration, I will not insist on this position.

There is another one available. Let "$h > g$" be construed as "if this vase is dropped on the floor, it will break" and "$i > j$" as "if this vase is thrown against the wall, it will break". Unlike the van Fraassen example, these two if-sentences are conditionals in Dudman's sense and, hence, suitable for regimentation along the lines indicated. The iterated conditional $(h > g) > (i > j)$ can be construed along the lines suggested initially before consideration of dispositions was introduced. The instruction is to transform K by shifting to $T'(K)$ where $h > g$ is in $RL(T'(K))$.

Previously, we were not in a position to make a definite suggestion as to the character of the transformation $T'(K)$. Now, however, we can make one. (Perhaps there are others.) $T'(K)$ is the L-minimal revision of K which contains "v is a D(B/DF)". Having made this transformation, we can then follow the procedures described previously for determining whether $i > j$ is also in $RL(T'(K))$.

It should be apparent that the acceptability of $(h > g) > (i > j)$ will correlate with the acceptability of "v is a D(B/F) $>$ v is a D(B/DW)" so that disputes as to whether to favor one reading of van Fraassen's example or the other may well seem pointless. Nonetheless, both approaches rely on deploying disposition predicates understood along the lines I have suggested which are distinguished from the conditionals they support.

The upshot is that if one seeks to avoid the use of disposition predicates on the grounds that they are meaningless in the same spirit that de Finetti eschews objective statistical probabilities or chances, there will be no general account of iterated conditionals of the type $(h > g) > (i > j)$ along the lines I favor. This result ought not to push one to the conclusion that conditionals should be regarded as truth-value-bearing propositions eligible for membership in a standard for serious possibility. At least, anyone who, like de Finetti, is suspicious of the metaphysical character of chances and dispositions ought to be leery of the modal realism such a view entails. I am inclined to think de Finetti's positivism excessive even though I mean also to keep modal and stochastic realism at arms length. I am prepared to allow for dispositions, abilities and chances while eschewing the quest for a semantics other than what can be learned from the way in which disposition, ability and chance predicates are integrated within the theories in which they are employed. My aim here is not, however, to justify the middle course I wish to follow but to explain the extent to which iterated conditionals can be accounted for according to the view of conditionals I favor.

Of course, this account of iterated conditionals is not fully systematic in the sense that I have failed to supply a single acceptability condition applicable to both noniterated and iterated conditionals. To this charge I cheerfully plead guilty. On the other hand, the approach I favor does systematically hew to the idea that conditionals are appraisals of serious possibility relative to transformations of a body of nonmodal beliefs. In this sense, I continue to reject iterated conditionals. I deny that the vagaries of natural language or the exigencies of science demand otherwise.

NOTES

* Thanks are due to Andre Fuhrmann, Peter Gärdenfors, James Higgenbotham, Peter Laveves and David Makinson for helpful comments and identification of egregious errors.

[1] "It is possible that *h* according to agent *X* at time *t*" is a biographical remark bearing a truth value. My practice (see Levi 1980) has been to represent the information conveyed by such a sentence by a sentence in a regimented metalanguage L^1 for language *L*. I have no vested interest, however, in representing the agent's beliefs about his own beliefs by means of a metacorpus rather than in a corpus in an enriched object language. L^1 should not be confused with L^* which includes sentences of the type "It is possible that *h*" without the relativity to persons, corpora or times. According to my antirealist construal, such unrelativized modal sentences lack truth values. They express but do not describe *X*'s propositional attitudes at time *t*. Because they lack truth values, they cannot be included in the set of sentences representing *X*'s beliefs at a given time not even when it includes sentences describing *X*'s beliefs at various times. According to realist construals of such unrelativized modal sentences, such sentences can be included in the set of sentences representing *X*'s belief state so that a corpus representable in L^* rather than *L* can be the object of belief revision. Even so, for the realist and the antirealist alike, "it is possible that *h*" is not an elliptical rephrasal of "it is possible that *h* according to *X* at *t*". In that sense, it cannot be said to "change its meaning" as *X*'s corpus changes.

[2] V. Dudman (1983, 1984) has offered a taxonomy of if-sentences in English at variance with a widely endorsed classification into indicatives and subjunctive. Dudman favors a tripartite division which makes provision for if-sentences used to formulate generalizations as a special type. Setting this type to one side, he contrasts hypotheticals with conditionals in a manner at variance with the indicative-subjunctive contrast. As a classification of English sentences, Dudman's approach and account is very attractive and I am inclined to endorse it. However, my use of the $>$ connective is intended to represent a connective in a regimented language L^{**} built on the regimented language *L*. I wish to emphasize that how sentences in L^{**} map into sentences in English involves subtleties of grammar concerning which one should be instructed by masters of

English grammar of the calibre of Dudman. However, I do wish to make one comment here. Setting aside degenerate cases where the h in $h > g$ is already in K, we can distinguish cases where $\sim h$ is not in K and cases where it is. The former are cases where $h > g$ is an open conditional and the latter are cases where it is counterfactual. As I understand it, Dudman's English hypotheticals always express open conditionals whereas his conditionals can be either open or counterfactual. The Ramsey test as I construe it specifies assertibility conditions for both kinds of conditionals. It should be apparent that my sense of "conditional" which applies to sentences in the regimented language and their natural language correlates is different from Dudman's. Dudman's hypotheticals express conditionals (open conditionals) in my sense. His conditionals also express conditionals (sometimes open, sometimes counterfactual) in my sense. Finally, the sentences h and g in $h > g$ are intended to be truth-value-bearing when h and g are in L as I assume in (R). Dudman insists that the "if-clause" in his conditionals is an "undeclarative" which cannot be used on its own to make a truth value bearing assertion. Thus, "Grannie misses the bus tonight" in "If Grannie misses the bus tonight, she will not get any dinner" is an undeclarative. It would not be good English to use it to make an assertion. Still it does express a "proposition" which is expressed also by the declarative "Grannie will miss the bus tonight". In my regimented L^{**} where the niceties of tense in English and how they operate in the encoding of Dudmanian conditionals are glossed over, we use sentences in L. Nonetheless, I agree with Dudman that conditionals (in my general sense as well as in his sense) are modal judgements where the if-clause serves as an adverbial modification. Without such a clause, we have modal judgement relative to the current doctrine. The if-clause qualifies the modal judgement by relativizing it to a transformation of the current doctrine.

The Ramsey test, as I construe it, takes the modal judgement to be relative to the L-minimal revision of K required to add the condition specified in the if-clause to the corpus. However, there may be additional constraints on the revision in addition to the stipulation that the condition specified by the if-clause be added to the corpus. If that condition is incompatible with K, to incorporate it K will first have to be contracted. According to the view I favor, the contraction should minimize loss of information. However, there may be additional constraints on the contraction strategy imposed by the context. Dudman's account of the tense of conditionals suggests that the contraction strategy leave unsullied any part of the corpus focused on events prior to or simultaneous with a "crossover point" specified by the form of the very in the if-clause. Strictly speaking the minimal revision need not be L-minimal because the contraction step will not minimize loss of information but only loss of information subject to the Dudman constraint. I have never intended to deny the presence of contextual constraints on the contraction step but until reading Dudman's work had not appreciated the extent to which systematic treatments of such constraints and their operation could be given.

Even though the existence of contextual constraints means that the minimal revisions in the Ramsey test cannot strictly speaking be L-minimal, they can be required to be L-minimal given the contextual constraints. It will turn out that such contextualization of L-minimality has negligible relevance to the issues pertaining to the consistency of the Ramsey test, preservation and Gärdenfors condition (O). The reason is that L-minimal revisions of K by adding h when h is consistent with K are supposed to be expansions in any event and this suffices to imply preservation.

[3] In his 1978 paper, Gärdenfors mentions a condition (C9) which states that when $\sim h$ is not in K, $K/h = K_h$. Whether this condition is imposed on corpora in L, L^* or L^{**}, it does, in effect, imply the preservation condition on revisions of corpora expressible in the appropriate language. That is to say the claim that when $\sim h$ is not in K, the expansion by adding h is identical with the minimal revision by adding h implies that the expansion, under those conditions, is included in the minimal revision. But this is the so-called "inclusion property" which Gärdenfors acknowledges to be equivalent to the preservation condition given that we require h to be in K_h (Gärdenfors 1986). In the light of his 1986 paper, it seems that he does not regard himself to have been endorsing (C9) in 1978, where Gärdenfors explicitly asserts that (C10) is stronger than (C9).

These remarks may puzzle the reader as they have puzzled me, but a closer reading suggests that Gärdenfors' claim can be made consistent even if it is misleading.

(C10a) asserts that K_h & g is a subset of K_h/g.
(C10b) asserts that if $\sim(h > \sim g)$ is in K, K_h/g is a subset of K_h & g.

In the special case where h is in K, (C10a) asserts that K_g is a subset of K/g and (C10b) asserts that if $\Diamond g$ is in K, K/g is a subset of Kg.

Because (RG) contains no clause like clause (c) of (RL) specifying conditions for the acceptability of $\sim(h > \sim g)$ and, hence, for the acceptability of $\Diamond g$, it does not follow that if $\sim g$ is not in K, $\Diamond g$ is in K. Hence, (C10b) does not imply that if $\sim g$ is not in K, K/g is contained in Kg. (C10a) clearly does not do so. Hence, (C10) cannot suffice to derive (C9).

Since Gärdenfors' proofs of the claim that he can model Lewis' theory of conditionals on his belief revision system with the aid of the Ramsey test make no use of (C9) but do rely on (C10), it appears that Gärdenfors can make good his claim to have skirted a commitment to the preservation condition.

Thus, although the innuendo that (C10) is stronger than (C9) is false, it plays no role in Gärdenfors' discussion.

REFERENCES

Alchourrón, C. E., P. Gärdenfors and D. Makinson: 1985, 'On the Logic of Theory Change: Partial Meet Functions for Contraction and Revision', *Journal for Symbolic Logic* **50**, 510–530.

Cohen, L. J.: 1981, 'Review of Levi', *Mind*, Vol., 297–300.

Dudman, V. H.: 1983, 'Tense and Time in English Verb Clusters of the Primary Pattern', *Australian Journal of Linguistics* **3**, 25–44.

Dudman, V. H.: 1984, 'Conditional Interpretations of If-Sentences', *Australian Journal of Linguistics* **4**, 143–204.

Dudman, V. H.: 1984b, 'Parsing 'If'-Sentences', *Analysis* **44**, 145–153.

Gärdenfors, P.: 1978, 'Conditionals and Changes in Belief', in I. Niiniluoto and R. Tuomela (eds.) *The Logic and Epistemology of Scientific Change*, North Holland.

Gärdenfors, P.: 1981, 'An Epistemic Approach to Conditionals', *American Philosophical Quarterly* **18**, 203–11.

Gärdenfors, P.: 1982, 'Rules for Rational Belief Change', *Philosophical Studies* published by the Philosophical Society and the Department of Philosophy, Uppsala University, no. 34.

Gärdenfors, P.: 1984, 'Epistemic Importance and Minimal Changes of Belief', *Australasian Journal of Philosophy* **62**, 137–57.

Gärdenfors, P.: 1986, 'Belief Revisions and The Ramsey Test for Conditionals', *Philosophical Review* **96**, 81–93.

Harper, W. L.: 1974, *Counterfactuals and Representations of Rational Belief*, Doctoral Dissertation, University of Rochester, Rochester, NY.

Harper, W. L.: 1975, 'Rational Belief Change, Popper Functions and Counterfactuals', reprinted in W. L. Harper and C. A. Hooker, *Foundations of Probability Theory, Statistical Inference and Statistical Theories of Science*, vol. **1**, Reidel, pp. 73–113.

Harper, W. L.: 1976, 'Ramsey Test Conditionals and Iterated Belief Change', in W. L. Harper and C. A. Hooker, *Foundations of Probability Theory, Statistical Inference and Statistical Theories of Science*, vol. **1**, Reidel, pp. 117–136.

Levi, I.: 1967, *Gambling with Truth*, Knopf reprinted in paper by MIT in 1973.

Levi, I.: 1974, 'On Indeterminate Probabilities', *Journal of Philosophy* **71**, 391–418.

Levi, I.: 1976, 'Acceptance Revisited', in R. Bogdan (ed.), *Local Induction*, Reidel, Dodrecht, pp. 1–71.

Levi, I.: 1977, 'Subjunctives, Dispositions and Chances', reprinted in Levi: 1984, pp. 165–191.

Levi, I.: 1979a, 'Serious Possibility', reprinted in Levi 1984, pp. 147–61.

Levi, I.: 1979b, 'Abduction and Demands for Information', reprinted in Levi 1984, pp. 87–106.

Levi, I.: 1980, *The Enterprise of Knowledge*, MIT Press, Cambridge, Mass.

Levi, I.: 1983, 'Truth, Fallibility and the Growth of Knowledge', reprinted in Levi: 1984, pp. 109–27.

Levi, I.: 1984, *Decisions and Revisions*, Cambridge Univ. Press, Cambridge.

Levi, I. and S. Morgenbesser: 1964, 'Belief and Disposition', *American Philosophical Quarterly* **1**, 221–32.

Lewis, D. K.: 1973, *Counterfactuals*, Blackwell, Oxford.

McGee, Vann: 1985, 'A Counterexample to Modus Ponens', *Journal of Philosophy* **82**, 462–71.

Stalnaker, R.: 1968, 'A Theory of Conditionals', *Studies in Logical Theory* **#2**, Blackwell, Oxford.

Department of Philosophy
Columbia University
New York, NY 10027
U.S.A.