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# THE WAR PROPENSITY OF INTERNATIONAL SYSTEMS*

ABSTRACT. The conjecture that international system structure determines war propensity has met with mixed results in past theory in political science. This question is reexamined within the context of a dynamic model of inter-nation hostile behavior. System structure is defined in terms of the degrees of grievance, fear, etc., among nations and also in terms of the qualitative patterns of hostile behavior that are possible. Propensity for war is measured in terms of the likelihood of progress to war within a given class of hostile behaviors. Then the dynamic model is used to analyze in detail and interpret the relationship between system structure and war propensity.

What factors determine the likelihood of war within a system of nations? This important and long-standing question has been studied at various levels. At the systemic level of analysis researchers have sought answers through relationships between a variety of variables. The independent variables that have been examined include (1) alliance configuration, degree of polarity (Singer et al. 1968, Wallace 1973, Bueno de Mesquita 1978), (2) power distribution (Singer et al. 1972, Bremer 1980), and (3) relative status of nations within the international system (Wallace 1971, 1972). Dependent variables, taken largely from the Correlations of War project, include the frequency, duration, and death toll of wars. Underlying each of these choices is the argument that certain structural attributes of an international system, such as its degree of polarization, make war, however measured, more (or less) likely. In brief, *the structural attributes of a system determine its behavioral attributes.*

The results to date have been both intriguing and puzzling. They suggest that answers to the central question are heavily influenced by how variables are measured (e.g., the definition and indexing of polarization) and depend critically on the time period from which the data were obtained. One reaction to these mixed findings might be to try additional measures and data sets. We feel, however, that the ambiguities they present are rooted more deeply in the definition of structure and in the nature of the structure-behavior relationships being studied. Consequently, a different approach to the problem is suggested.

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This paper will reinterpret the structure-behavior question within the context of a model of inter-nation hostile behavior and provide a new view of the notion of "war propensity". The model we consider has been developed and analyzed in a series of previous studies (Muncaster et al. 1983, 1988) and has been termed the Pure Hostility Model. It is a dynamic model of hostile activity that provides a mapping from perceptions such as grievance and fear to external patterns of hostile behavior. In the next section we describe the model and summarize some of its relevant properties. This will provide the basis for the subsequent section in which precise definitions of *system structure* and the *potential for war* are given. Thereafter we show, purely by analysis of the model, how system structure and war propensity are related.

It is useful to contrast the relationships posited between system structure and war propensity in the Pure Hostility Model with those that are seen in the earlier empirical studies. These studies exhibit three noteworthy characteristics. First, the attributes of the international system – its polarity or its power distribution – are obtained through an *aggregation* of observable individual attributes – the number of alliance bonds of each nation or the power capabilities of each nation. Second, these aggregated characteristics are *attributes* or qualities of nations, in contrast, for example, to the behaviors of nations. Thus systemic measures are obtained by considering the amount of power of each nation rather than the way in which nations behave towards one another. Third, a system's propensity for war in these studies is measured with reference to the amount of violence that occurs between the nations during a proposed time span. *War* is defined in terms of a minimum number of individuals killed among a set of two or more interacting entities that must pass certain nation-state qualifications. An international system's war propensity is then measured in terms of the frequency with which wars occur, the length of time nations spend in wars, or the number of people killed in wars, over a given time span. A system is said to be more war prone the greater the amount of violence that takes place among the nations composing that system.

In the Pure Hostility Model we also consider a set or system of nations. However, the attributes or characteristics of the system are not obtained by aggregating certain qualities over all nations. Instead, it is assumed that the qualities of the system at any one point in time cannot be directly observed, but are partially manifested through the behaviors of any one of the nations. Second, the quality or characteristic of the system under consideration is the behavior of the nations, not attributes like power. Third, a system's propensity for war may be defined in terms of the likelihood that a war will break out between the set of nations under examination. We will not be concerned with the size of the war either in terms of its duration, the number of nations involved, or the number killed. The focus will be on the likelihood that a war will occur given that the set of nations is manifesting certain kinds of behavior.

## THE PURE HOSTILITY MODEL

To this point we have used the vocabulary most common among researchers investigating the question of the propensity for war. It is important in what follows, however, to clarify several key concepts. We do not talk about a system of nations, preferring instead to use the term *set of nations* to designate those interacting nations of interest. This is necessary because we wish to use the term *system* to refer not only to the nations but also to their behaviors. Such an approach is consistent, at least implicitly, with common usage. When researchers consider an *international system*, they implicitly focus not only on a set of nations but on relationships among those nations. These relationships might specify an inter-nation hostility system, or an inter-nation trade system, or an inter-nation cooperative system. Thus we see the term *international system* as a broad but somewhat ill-defined concept that refers to a set of nations and any specified type of relationship that might bind them together.

One could consider many *systems* defined over a set of nations, depending upon the type of behavior of interest. The focus here is on a hostility system, i.e., the hostile behaviors of the interacting nations. The principal variable in the hostility system is $H(t)$, the level of hostility between the members of the set of nations at time $t$. Conceptually $H(t)$ can be viewed as the total, i.e., aggregated, amount of hostility over all the nations at time $t$. However, it differs from the aggregate measures of previous studies in that it is never completely observable at any given time point $t$. While nations always feel hostile towards one another, they may not overtly express hostility in a visible, measurable form at any given time $t$. We assume that $H(t) > 0$

for any given set of nations, i.e., there is some residue of hostility, latent or manifest, within any set of nations. Although the total hostility $H(t)$ within the defined set of nations may not be directly observable, whenever one nation behaves in a hostile manner towards another the level of hostility of this act is a partial indicator of the level of system hostility. When no nation manifests overt hostility, hostility exists but is not observable; when nations actually display hostility, the hostile acts are surrogate measures of the underlying hostility system. The individual hostile acts of nations are thus not aggregated unless they occur simultaneously.

We view *escalation* $E(t)$ as a measure of the changing levels of hostility in the system and so let $E(t) = \dot{H}(t)$ (overdots denote time derivatives). Equivalently the levels of $E(t)$ measure the speed with which the system is changing or the momentum of the hostility system at a given time $t$. The Pure Hostility Model is based upon the assumption that the escalation of a system remains constant unless forced to change by pressures within the system. Thus

$$\dot{E}(t) = f_1(t) + f_2(t) + \cdots + f_N(t),$$

where $f_1, f_2, \ldots, f_N$ measure the levels of various forces or pressures (a more extensive discussion of this assumption has been given by Muncaster et al. 1983).

The model furthermore postulates which forces are most prominent in a hostility system and gives their analytical form. Previous research on hostile interactions and the approach to war suggests four factors that should affect changes in the momentum of a hostility system:

(1) previous levels of hostility (short term memory),
(2) past relationships between the nations (longer term memory),
(3) a fear of war, and
(4) a pull to war.

If each of these forces were represented as an *exogenous* variable, say $A(t)$, $B(t)$, $C(t)$, and $D(t)$, respectively, we would obtain a model of the form

$$\dot{E}(t) = aA(t) + bB(t) + cC(t) + dD(t),$$

where $a$, $b$, $c$, and $d$ are parameters. Such an approach would reflect the tradition in previous empirical studies. In the Pure Hostility

Model, however, we prefer to represent each of the forces *endogenously*: each force is modeled in terms of the way in which it is manifested through the levels of hostility $H(t)$ and escalation $E(t)$:

(1) short term memory: $f_1(t) = aE(t)$,

(2) long term memory: $f_2(t) = \frac{b}{H(t)}$,

(3) fear: $f_3(t) = -\frac{c}{e + (H^* - H(t))^2}$,

(4) pull to war: $f_4(t) = \frac{p}{H^{**} - H(t)}$.

**Short term memory.** $f_1$ represents the hostility system's most recent memory, the parameter $a$ being a measure of the system's *excitability*. If $a > 0$ then past hostility reinforces and enhances subsequent hostility. If $a < 0$ then past hostility dampens or cools the hostility process. If $a = 0$, the system is said to have no recent memory.

**Long term memory.** $f_2$ represents the pressures due to past relationships between the nations, i.e., long term memory. It is assumed that the effects of past relationships only become operative when the level of systemic hostility is very low. This assumption is reflected in the form of $f_2$ in the following ways. If $H(t)$ is large the focus of attention is on current problems, and then long term past relationships fade into the background. Analytically, if $H(t)$ is very large then $b/H(t)$ is small, and thus long term memory of past relationships play only a small role. On the other hand, if the level of system hostility is low, nations have the time and energy to focus on prior relationships. Thus, when $H(t)$ gets very small, $b/H(t)$ becomes very large. The parameter $b$ appearing in $f_2$ governs two important aspects of the relationship between long term memory and the level of hostility. Its magnitude describes the significance of the relationship for the hostility system; its sign describes the "type" of relationship. If $b > 0$, past relationships increase the hostility between the nations. We see this as the manifestation of past grievances – old, long term hostilities between the nations. The larger $b$ is, the more important these past grievances. When $b < 0$, past relationships decrease the hostility between the nations. This we view as reflecting long term feelings of friendship. Similarly, the smaller $b$ is (since $b < 0$), the more important are past friendship ties for the current hostility process.

**Fear.** $f_3$ and $f_4$ function in a way analogous to $f_2$, but they reflect very different kinds of forces. In both $f_3$ and $f_4$, when the denominator becomes very large, the force has almost no impact, and when the denominator becomes very small, the force has an overwhelming impact. In both cases the denominator becomes very small as $H(t)$ approaches a special value. In $f_3$ the special value is $H^*$, the *fear level*. This is the level of hostility at which the nations become most frightened of the possibility of a war. As can be seen in $f_3$, as $H(t)$ approaches $H^*$ this force has an overwhelming negative impact on the hostility system, greatly decreasing the levels of hostility. The parameter $e$ is a positive measure of the *pervasiveness* of fear in the system. For small values of $e$ the fear is manifested only in a small range of hostility levels about $H^*$. For large values of $e$ the fear is felt more broadly, from low through high levels of hostility, though always reaching its maximum at the threshold $H^*$. Since $e > 0$, the hostility system can pass its fear threshold, either upward in an escalating situation or downward in de-escalation. The placement of $H^*$ and the choice of the value of $e$ can discriminate between different hostility systems. The smaller $H^*$ is, the sooner (in terms of levels of hostility) the hostility system becomes fearful; the larger $e$ the more easily it becomes fearful. For a given value of $c$, one might argue that the smaller $H^*$ is, the less likely a hostility system will go to war. If $c$ is large, for example, it would intuitively seem that war would be less likely when the fear level $H^*$ is set at a very low level than if it were set at a very high level. As will be shown below, our intuition is only partially correct.

**Pull to war.** In $f_4$ the critical value in the denominator is $H^{**}$. As $H(t)$ approaches this value the factor $f_4$ has an overwhelming impact on the hostility system. Unlike $f_3$, however, this force is positive, thus increasing the levels of hostility. It can be shown[1] that $H^{**}$ is that level of hostility at which the escalation of the hostility system goes to infinity, and that this blow-up in escalation must occur at a finite time which we denote as $t_{war}$. We therefore define *war* within the model as infinite escalation at a finite level of hostility, and $H^{**}$ is that level of hostility at which war occurs. In the Pure Hostility Model the value of $H^{**}$ is exogenous to the system;[2] i.e., it is arbitrarily set independent of other effects in the model. As $H^{**}$ is increased, the level of hostility necessary for war is increased. One would assume that the higher $H^{**}$

the less likely it is that a war will occur. We will examine this proposition more closely below.

In both $f_3$ and $f_4$ the parameters $c$ and $p$ measure the importance or impact of the particular force; the larger the parameter the greater the significance of that force. Force $f_3$ however is always negative, decreasing levels of hostility, while force $f_4$ is always positive, increasing levels of hostility. In contrast the parameters $a$ and $b$, as noted earlier, could be either positive or negative, lending a somewhat different interpretation to the corresponding force in each case. In a similar vein we can extend the interpretations of the forces $f_3$ and $f_4$. These two forces are essentially opposite in structure: if we change the sign of $f_3$ it operates much like $f_4$ and simply becomes another pull to war. Similarly, with a change of sign $f_4$ becomes another fear. This suggests that we could have a model with two fears, two pulls to war, or perhaps more interesting, a model with a pull to war which occurs at a level of hostility below the level of fear. These possibilities have been introduced and examined in detail by Zinnes et al. (1988). For the present analysis, however, we restrict attention to the cases for which $c > 0$, $p > 0$, and $H^* < H^{**}$.

Combining the four forces and recalling that $E$ is $\dot{H}$, we can write the model as the non-linear differential equation

$$(5) \qquad \ddot{H} = a\dot{H} + \frac{b}{H} - \frac{c}{e + (H^* - H)^2} + \frac{p}{H^{**} - H}.$$

We would ideally like to solve this equation for $H$ as a function of $t$, but the non-linearities and discontinuities make this impossible. Fortunately we can obtain all the information we need about the hostility system modeled in (5) through phase portrait methodology. Certain aspects of this methodology have been described by Zinnes et al. (1984), and most of the phase portrait analyses for the model given by (5) have already been provided (Muncaster et al. 1983, 1988; Zinnes et al. 1988). We will not repeat those discussions here. However, certain aspects of phase portrait methodology must be described in order to make the analyses which follow meaningful and to clarify our use of such terms as *system structure* and the *propensity for war*.

## CONCEPTS: SYSTEM STRUCTURE, PROPENSITY FOR WAR

A phase portrait is a set of curves drawn in a plane whose axes represent, for our purposes, the level of hostility $H$ (the $x$-axis) and the level of escalation $E$ (the $y$-axis). Any point in this plane gives, for a particular time point $t$, both the level $H(t)$ of hostility and the changes $E(t)$ in the level of hostility. A curve is drawn by connecting, in a time ordered sequence, the successive points $(H(t), E(t))$. The resulting curve indicates how the two variables of the model change through time in relation to one another. Since the analysis of equation (5) involves integration and hence arbitrary constants representing initial conditions, we must consider not a single curve but rather a set of curves. For every constant of integration we obtain a new "parallel" curve. Since constants of integration are arbitrary, a phase portrait essentially consists of an infinite number of "comparable" curves.

A typical phase portrait for the case $a = 0$ and $b > 0$ is shown in Figure 1. Several "sample" trajectories are shown for different initial conditions. The arrows on the trajectories indicate the time determined sequence of the process. On curves in the quadrant where $H(t) > 0$ and $E(t) > 0$, the process moves from left to right; on curves

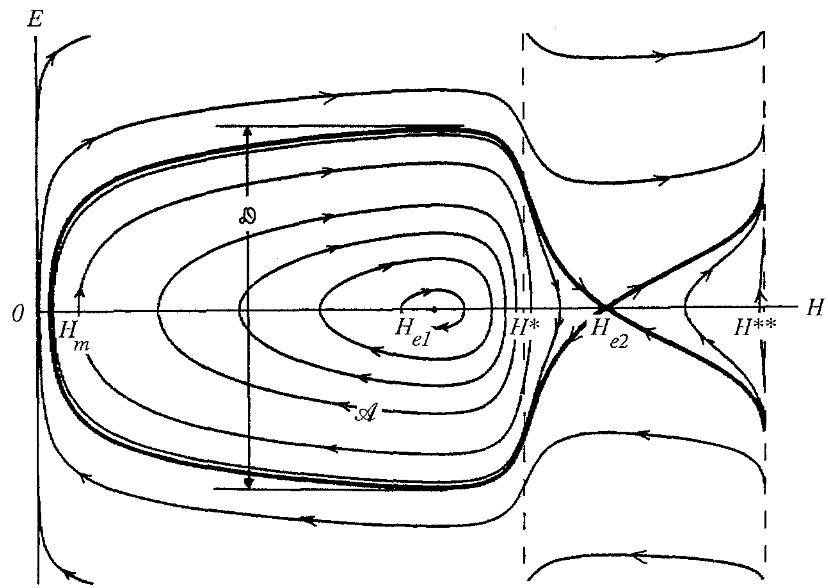


Fig. 1.

in the quadrant where $H(t) > 0$ and $E(t) < 0$, movement is from right to left. It must be emphasized that Figure 1 is a typical phase portrait given the parameter constraints of $a = 0$ and $b > 0$. Each combination of values of the four parameters which satisfy these constraints produces a slightly different phase portrait. Figures 2 and 3 are two other possible phase portraits which meet the same basic constraints. These portraits might be seen as "distorted" versions of Figure 1.

All three of the preceding portraits share certain characteristics. In each there is an enclosed region, denoted by $\mathscr{A}$. We call this a region of *protracted conflict* since levels of hostility and escalation wax and wane here but never eventuate in war. Outside of this region all trajectories go to war. The dark curve enclosing $\mathscr{A}$ is a boundary; the curves in the phase portrait cannot cross it. Thus trajectories within the boundary never go to war, but all trajectories outside the boundary inevitably go to war. These features are common to a whole class of phase portraits. Indeed, all phase portraits that satisfy the conditions that $a = 0$ and $b > 0$ have these same qualitative features, that is, a

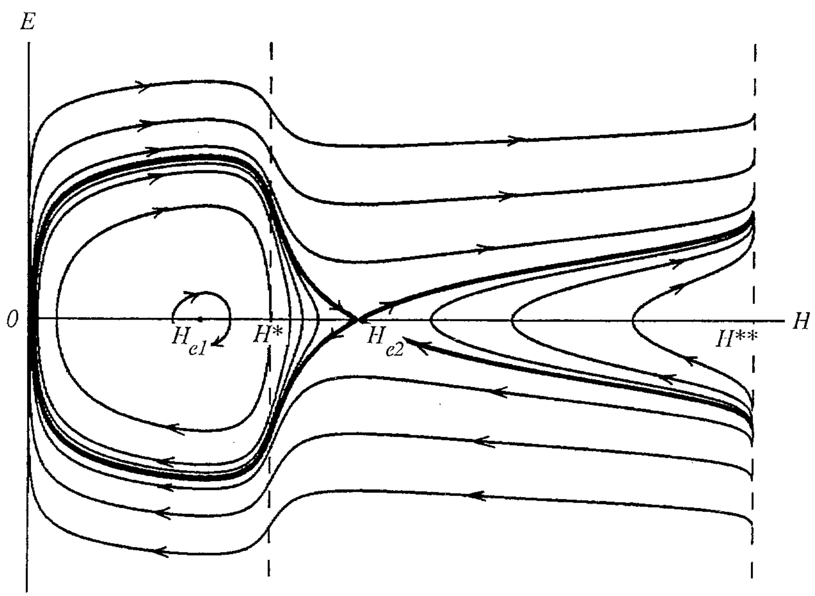


Fig. 2.

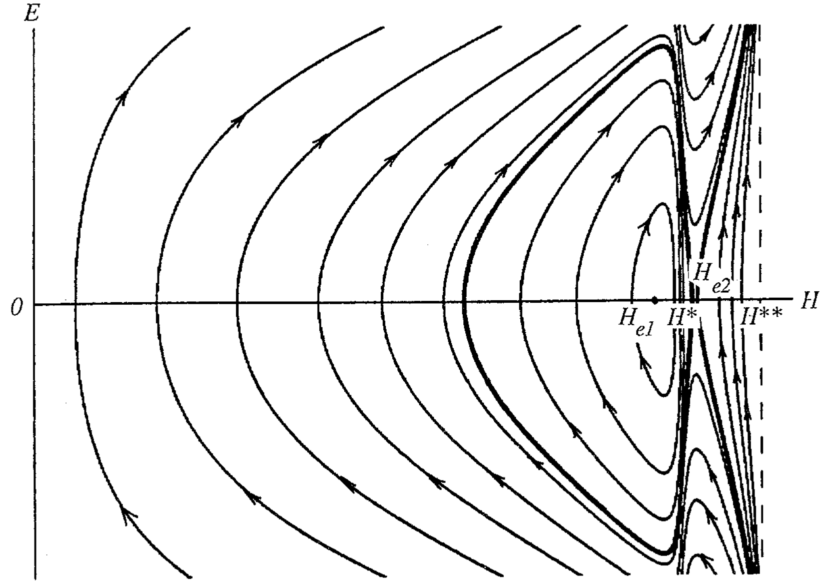


Fig. 3.

bounded protracted conflict region and an unbounded outer region that inevitably goes to war.

For our purposes *system structure* refers to a specific combination of parameter values. Consequently, every combination of parameters produces a different structure for the hostility system. Varying any one or more parameters produces a variation in the structure of the hostility system. There are, however, two different ways in which system structure can vary. As can be seen from Figures 1, 2, and 3, structure can vary by changing the parameter values but respecting the basic restrictions $a = 0$ and $b > 0$. By varying parameters in this way we change the structure of the system *quantitatively* but not *qualitatively*: we stretch, squeeze, or generally distort the basic configuration of Figure 1. There are, however, certain "critical" or threshold values of the parameters which, when passed, lead to qualitative changes in structure. For example, when $a$ becomes positive the closed curves of Figures 1, 2 and 3 become outgoing spirals; when $a$ is negative they become spirals that move inward. An even more startling result is obtained if we retain the restriction that $a = 0$ but change $b$ from positive to negative. The two clearly delineated regions of Figures 1, 2, and 3 are replaced by three: a "protracted conflict" region in which hostility and escalation wax and wane, a "peace" region in which all trajectories eventually lead to high levels of *de-escalation* of hostility, and a war region in which all trajectories go to war. This is illustrated by the phase portrait in Figure 4. The qualitative changes that occur in a phase portrait when parameters pass given thresholds can be thought of as *system transformations*. By analyzing the threshold values of all parameters, or equivalently all possible system transformations, one can form a complete catalogue of system types. Such a catalogue can be found in Zinnes et al. (1988).

Changes in parameter values, whether they lead to quantitative or qualitative consequences, produce changes in the structure of the hostility system. The focus of attention here is on how these changes in structure – i.e., variations in the parameters $a$, $b$, $c$, $p$, $H^*$, and $H^{**}$ – affect the propensity for the system to go to war. It would be ideal if we could examine how both quantitative and qualitative structural

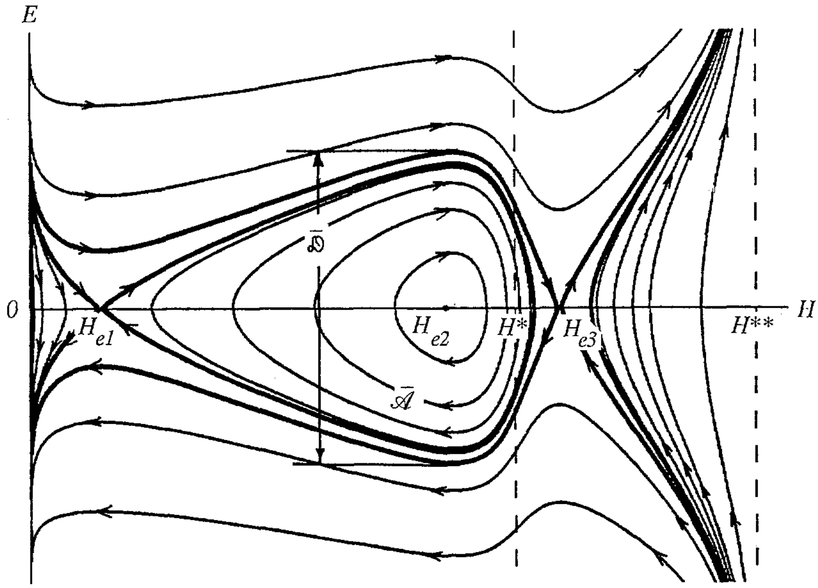


Fig. 4.

changes affect the propensity for war. Unfortunately the impact of system transformation on the likelihood of war is extremely difficult to analyze, and so we focus here only on quantitative changes.

Let us consider more carefully what is meant by the *propensity for war*. As noted previously, a phase portrait gives a graphical representation, for a specified system structure, of all behaviors of the hostility system modeled by equation (5). Each trajectory in that picture is a possible path of the system, giving the levels of hostility and escalation over the course of time. Which path a given system actually follows depends on initial conditions, i.e., on the levels of hostility and escalation at some initial time. For a given set of nations and a given system structure, we are not interested in a specific hostility path which the system might follow. Rather, our concern is with the full variety of hostility processes that the system is *likely* to follow. In order to specify what is likely, we assume that all points within a bounded part of the phase plane represent equally likely initial conditions for hostility processes. More precisely, assume there is a constant $E^*$ such that in practical cases the level of escalation of a system never exceeds $E^*$ and the level of de-escalation is never less than $-E^*$. Then *realistic* initial conditions $(H_0, E_0)$ satisfy the restrictions $0 < H_0 < H^{**}$ and $-E^* < E_0 < E^*$, and we assume that all such points give equally likely initial conditions for the model (5). Some of these will give rise to trajectories that eventuate in war, others will not, and the objective now is to develop a measure of the likelihood of war vs. non-war behavior.

We have noted that in Figures 1, 2, and 3 there is a bounded region of protracted conflict. All trajectories within this region can never cross the boundary, and thus never go to war; all trajectories outside this region always go to war. This shows that the likelihood that the hostility system will not go to war is the same as the likelihood that its initial hostility and escalation define a point in the region of protracted conflict. At the same time we have assumed that all "realistic" initial conditions are equally probable. Therefore, the larger the bounded region the less likely a hostility system will go to war, and so we may take the *size* of the bounded region to be a measure of the *propensity for war* for the given system structure. The greater the size of the bounded region the less war-prone is the system. Since this interpretation is based upon the somewhat imprecise notion of "realistic" initial values, we prefer to use the term propensity here rather than probability.

The preceding definition of propensity for war applies when there is a bounded region of protracted conflict. In particular it applies to systems with the structure represented by Figures 1, 2, and 3 (i.e., $a = 0$, $b > 0$). But we have already noted that these figures reflect only one "type" of system structure. When we pass parameter thresholds we obtain a "system transformation" and arrive at a qualitatively different portrait. Does this definition of the propensity for war generalize to other types of hostility systems?

Qualitative changes in the phase portraits and corresponding system transformations are obtained in this model for a number of different parameter combinations (see Zinnes et al. 1988). For example, when $b > 0$, $c > 0$, and $p > 0$, we obtain system transformations for $a < 0$, $a = 0$, $a > 0$ when the parameter $a$ passes these threshold values. However, in many system types there are no bounded regions. For example, consider Figure 5 which represents the case $b < 0$, $c > 0$, $p > 0$, and $a = 0$. We have four open regions. The dark line passing through the equilibrium point $H_e$ is a boundary which separates the space into two "peace" regions and two "war" regions. All trajectories beneath and to the left of the boundary go towards zero levels of hostility and negative levels of escalation. If we defined "peace" as infinite de-escalation at $H(t) = 0$, trajectories in these regions could be said to go off to peace. On the other hand, all trajectories above or to the right of the boundary go to war. Although one might argue that the larger the "peace" region the less likely war, since the regions are unbounded it is not possible to determine their relative sizes. Thus it is only possible to talk of the size of regions and whether they are protracted conflict regions, peace regions, or war regions, when these regions are bounded. The answer to our question then is that *we can study the propensity for war only in those systems in which there are bounded regions.*

There are only two types of systems which have bounded regions and thus permit a study of the relationship between variations in system structure and the propensity for war. The first of these is shown in Figures 1, 2, and 3 and is defined by the restrictions $a = 0$, $b > 0$, $c > 0$, and $p > 0$. The second is shown in Figure 4 and applies when $a = 0$, $b < 0$, $c > 0$, and $p > 0$ and very large.

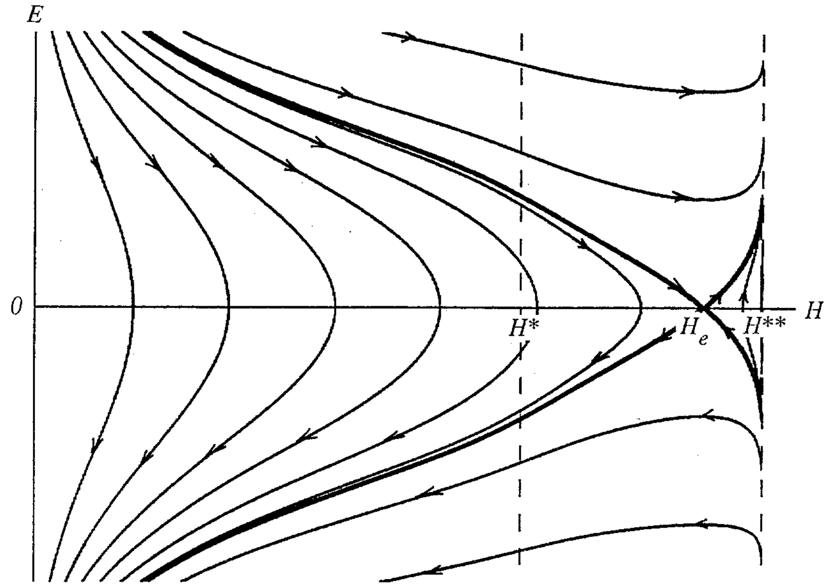


Fig. 5.

THE RELATIONSHIP BETWEEN WAR PROPENSITY AND SYSTEM STRUCTURE

Our concern in these two qualitatively different types of hostility systems is in how changes in the values of the parameters affect the area of the bounded region of protracted conflict. Let

$$\dot{E} = F(H) = \frac{b}{H} - \frac{c}{e + (H^* - H)^2} + \frac{p}{H^{**} - H}. \tag{6}$$

Then

$$\dot{E}E = F(H)E,$$

$$\left(\frac{1}{2}E^2\right)^{\cdot} = \left(\int F(H)\,dH\right)^{\cdot},$$

and so

$$\frac{1}{2}E^2 = \int F(H)\,\mathrm{d}H + C = A(H) + C,$$

$$E^2 = 2A(H) + 2C, \tag{7}$$

$$E = \pm\sqrt{2A(H) + 2C},$$

where $C$ is a constant of integration. This formula represents $E$ as a function of $H$ for the curves in each of our figures. Different curves correspond to different choices of the constant $C$.

Consider first Figure 1: $a = 0$, $b > 0$. We are interested in the boundary curve which passes through the equilibrium point $H_{e2}$. The equation for this particular curve can be obtained by letting

$$C = -A(H_{e2}):$$

i.e., the point $(H, E) = (H_{e2}, 0)$ must be on the curve defined by (7). The boundary curve is then given by

$$E(H) = \sqrt{2A(H) - 2A(H_{e2})}, \tag{8}$$

and thus the area within the boundary, by symmetry, is twice the area under the curve (8), namely

$$\mathscr{A} = 2\int_{H_m}^{H_{e2}} E(H)\,\mathrm{d}H. \tag{9}$$

The hostility level $H_m$ is shown in Figure 1.

Our basic premise is that *the larger the area $\mathscr{A}$ the smaller the propensity for war.* Our concern, then, is how changes in the parameters either increase or decrease $\mathscr{A}$. The answer to this question can be found by examining how changes in a generic parameter $k$, e.g., $b$, $c$, $p$, $H^*$, or $H^{**}$, affect $\mathscr{A}$. From (9) we obtain

$$\frac{\partial \mathscr{A}}{\partial k} = 2\int_{H_m}^{H_{e2}} \frac{\partial E}{\partial k}\,\mathrm{d}H + 2E(H_{e2})\frac{\partial H_{e2}}{\partial k} - 2E(H_m)\frac{\partial H_m}{\partial k}. \tag{10}$$

Since $E(H_{e2}) = E(H_m) = 0$, the second and third terms vanish. Using (8) we obtain

$$\frac{\partial E}{\partial k}(H) = \frac{1}{E}\left(\frac{\partial A}{\partial k}(H) - \frac{\partial A}{\partial k}(H_{e2}) - F(H_{e2})\frac{\partial H_{e2}}{\partial k}\right).$$

Since $F(H_{e2}) = 0$, we can now rewrite (10) as

$$\frac{\partial \mathscr{A}}{\partial k} = 2 \int_{H_m}^{H_{e2}} \left( \frac{\partial A}{\partial k}(H) - \frac{\partial A}{\partial k}(H_{e2}) \right) \frac{\mathrm{d}H}{E(H)}. \tag{11}$$

Equation (11) can be used to examine the effects of each of the parameters by choosing $k$ in turn to be $b$, $c$, $p$, etc. Since

$$A(H) = \int F(H)\,\mathrm{d}H,$$

we have the following explicit formula for $A(H)$:

$$A(H) = b \ln H + \frac{c}{\sqrt{e}} \operatorname{Tan}^{-1}\left( \frac{H^* - H}{\sqrt{e}} \right) - p \ln(H^{**} - H). \tag{12}$$

**Variations in grievance.** Consider first the effects of the parameter $b$. Directly from (12) we find that

$$\frac{\partial A}{\partial b}(H) = \ln H,$$

and so

$$\frac{\partial A}{\partial b}(H) - \frac{\partial A}{\partial b}(H_{e2}) = \ln\left(\frac{H}{H_{e2}}\right). \tag{13}$$

Since $H < H_{e2}$ within the boundary, we see that

$$\ln\left(\frac{H}{H_{e2}}\right) < 0,$$

and when substituted into (11) this gives

$$\frac{\partial \mathscr{A}}{\partial b} < 0.$$

As the parameter $b$ increases, the area within the boundary decreases: *as the long term memory of past grievances increases*, i.e., becomes more important in the hostility system, the region $\mathscr{A}$ decreases and *the propensity for war increases*.

**Variations in fear.** Consider next the impact of the fear parameter $c$. A simple calculation shows that

(14) $$\frac{\partial A}{\partial c}(H)-\frac{\partial A}{\partial c}(H_{e2})$$
$$=\frac{1}{\sqrt{e}}\left[\operatorname{Tan}^{-1}\left(\frac{H^*-H}{\sqrt{e}}\right)-\operatorname{Tan}^{-1}\left(\frac{H^*-H_{e2}}{\sqrt{e}}\right)\right].$$

Since $\operatorname{Tan}^{-1}$ is an increasing function and $H^*-H$ decreases as we increase $H$, the quantity in square brackets decreases in $H$. Clearly it vanishes when $H=H_{e2}$. Therefore, for $H<H_{e2}$ we have

(15) $$\frac{\partial A}{\partial c}(H)-\frac{\partial A}{\partial c}(H_{e2})>0.$$

Substituting in (11) we obtain

$$\frac{\partial \mathscr{A}}{\partial c}>0.$$

*As fear increases*, the area within the bounded region increases and *the propensity for war decreases*.

**Variations in the pull to war.** Consider the pull to war parameter $p$. From (12) we see that

(16) $$\frac{\partial A}{\partial p}(H)=-\ln(H^{**}-H),$$

and so

(17) $$\frac{\partial A}{\partial p}(H)-\frac{\partial A}{\partial p}(H_{e2})=-\ln\left(\frac{H^{**}-H}{H^{**}-H_{e2}}\right).$$

Since $H<H_{e2}$ within the boundary, we see that

$$\ln\left(\frac{H^{**}-H}{H^{**}-H_{e2}}\right)>0,$$

and substituting this into (11) we conclude that

$$\frac{\partial \mathscr{A}}{\partial p}<0.$$

*As the pull to war increases*, the bounded region decreases and *the propensity for war increases*.

**Variations in the level of war.** For the parameter $H^{**}$, the level of hostility at which the hostility system goes to war, we obtain

$$(18) \qquad \frac{\partial A}{\partial H^{**}}(H) = -\frac{p}{H^{**}-H},$$

and so

$$\frac{\partial A}{\partial H^{**}}(H) - \frac{\partial A}{\partial H^{**}}(H_{e2}) = \frac{p}{H^{**}-H_{e2}} - \frac{p}{H^{**}-H} = \frac{p(H_{e2}-H)}{(H^{**}-H_{e2})(H^{**}-H)}.$$

Since $H < H_{e2} < H^{**}$, we find using (11) that

$$(19) \qquad \frac{\partial \mathscr{A}}{\partial H^{**}} > 0.$$

*As the level of hostility at which war occurs increases, the propensity for war decreases.*

**Variations in the fear threshold.** We have one final parameter to examine in Figure 1, namely, the hostility level $H^*$ at which fear is most pervasive in the system. Unfortunately, the analysis for this parameter is not as straightforward as for the above three cases. This can be seen from the calculation

$$(20) \qquad \frac{\partial A}{\partial H^*}(H) - \frac{\partial A}{\partial H^*}(H_{e2}) = \frac{c}{e+(H^*-H)^2} - \frac{c}{e+(H^*-H_{e2})^2} = \frac{c(H-H_{e2})(2H^*-H-H_{e2})}{[e+(H^*-H)^2][e+(H^*-H_{e2})^2]}.$$

Both quantities in the denominator are positive, as is the first quantity in the numerator. The second quantity in the numerator, however, changes sign as we move from $H$ small to $H$ large. Consequently, there is no simple way to draw conclusions about increases or decreases in $\mathscr{A}$ as we vary $H^*$. While our principal interest lies with the area $\mathscr{A}$, by turning from it to an alternative measure some conclusions about variations in $H^*$ are possible. This new measure is the diameter $\mathscr{D}$ shown in Figure 1. $\mathscr{D}$ is simpler to analyze and yet may be expected to yield conclusions similar to those we have already drawn using the area $\mathscr{A}$. That is, we assume now that *as the diameter* $\mathscr{D}$ *increases, the propensity for war decreases.* A formula for $\mathscr{D}$ follows from (7) since the maximum height of the boundary occurs when $H = H_{e1}$. This formula is

$$\mathscr{D} = 2\sqrt{2A(H_{e1}) - 2A(H_{e2})},$$

and, by a calculation similar to that following (10), we find that

$$\text{(21)} \qquad \frac{\partial \mathscr{D}}{\partial k} = \frac{2}{\mathscr{D}}\left(\frac{\partial A}{\partial k}(H_{e1}) - \frac{\partial A}{\partial k}(H_{e2})\right).$$

Choosing $k$ to be $H^*$ and using (20), we conclude that the sign of $\partial \mathscr{D}/\partial H^*$ is the same as the sign of

$$\text{(22)} \qquad \tfrac{1}{2}(H_{e1} + H_{e2}) - H^*.$$

It can be shown from this that

$$\text{(23)} \qquad \frac{\partial \mathscr{D}}{\partial H^*} > 0 \text{ for } H^* \text{ small,}$$

and

$$\text{(24)} \qquad \frac{\partial \mathscr{D}}{\partial H^*} < 0 \text{ for } H^* \text{ large.}$$

We conclude that there is a critical value of $H^*$, say $H^*_c$, for which this derivative vanishes:

$$\text{(25)} \qquad \frac{\partial \mathscr{D}}{\partial H^*} = 0 \text{ when } H^* = H^*_c.$$

This critical value can be determined by requiring that the quantity (22) vanish. The calculation, however, is intricate. Since both $H_{e1}$ and $H_{e2}$ depend upon all the parameters, and in particular upon $H^*$, the requirement that (22) vanishes leads to a complex algebraic problem for $H^*_c$. Nevertheless, it can be shown[3] that the critical value of $H^*$ is given by

$$\text{(26)} \qquad H^*_c = \frac{bH^{**} + c}{b - p} \pm \left[\left(\frac{bH^{**} + c}{b - p}\right)^2 - \left(\frac{bH^{**} + c}{b - p}\right)H^{**} - e\right]^{1/2},$$

where the sign is to be chosen so that $0 < H^*_c < H^{**}$. The implications of this result are intriguing. Below the critical value $H^*_c$, *increases in the threshold of fear $H^*$ produce a decrease in the propensity for war.* Past the critical level $H^*_c$ *an increase in $H^*$ increases the propensity for war.* In brief, unlike the other parameters, there is an optimum fear level for any hostility system, $H^*_c$, which *minimizes* the propensity for war.

We turn next to Figure 4, the case for $a = 0$, $b < 0$, and $p > 0$ and large. Figure 4 is similar to Figure 1 in that we have a bounded protracted conflict region and an outer war region. But the two figures are completely different in other respects. First, there is a new type of region which we have argued earlier may be termed a "peace" region; all trajectories within this region "go to peace". Whereas the hostility system in Figure 1 is always conflictual, with only protracted conflict and war as possible options, in Figure 4 there are three different behaviors: war, peace, and protracted conflict. A second important difference is that a peace region encircles the protracted conflict region, separating it from war.

Figure 4 suggests that our initial question – how system structure affects the propensity for war – will have to be modified. If we are interested in the simple dichotomy of war/no-war, then the analysis of the propensity for war must take into account the peace regions. Unfortunately, as we saw earlier, it is not possible to determine the size of unbounded regions, and all of the peace regions are of this nature. Thus the question of how parameter changes affect the likelihood of going to war must be rephrased. The area of the protracted conflict region is still an important component, but the peace band surrounding the war region should also be considered. We propose two different measures. $\bar{\mathscr{A}}$ will denote the area of the bounded region of protracted conflict in Figure 4. $\bar{\mathscr{D}}$ will denote the diameter shown in Figure 4 which serves as a measure of the location of the boundary between the peace and war regions.

The analyses of $\bar{\mathscr{A}}$ and $\bar{\mathscr{D}}$ are completely analogous to steps we have presented previously when $b > 0$. A formula very similar to (11) holds for $\bar{\mathscr{A}}$ and one very similar to (21) holds for $\bar{\mathscr{D}}$. For this reason we omit the calculations and restrict our attention to the results.

**Variations in friendship.**

$$\frac{\partial \bar{\mathscr{A}}}{\partial b} > 0, \qquad \frac{\partial \bar{\mathscr{D}}}{\partial b} < 0.$$

Since $b < 0$ in this case, decreases in $b$ correspond to increases in friendship. Thus, *as friendship increases*, the area decreases and the diameter increases, and so *both the propensities for protracted conflict and war decrease and the propensity for peace increases.*

**Variations in fear.**

$$\frac{\partial \bar{\mathscr{A}}}{\partial c} < 0, \qquad \frac{\partial \bar{\mathscr{D}}}{\partial c} > 0.$$

*As the degree of fear increases* in a hostility system, the area decreases and the diameter increases, and so, as with friendship, *protracted conflict and war become less likely and the propensity for peace increases.*

**Variations in push to war.**

$$\frac{\partial \bar{\mathscr{A}}}{\partial p} > 0, \qquad \frac{\partial \bar{\mathscr{D}}}{\partial p} < 0.$$

Now the area increases and the diameter decreases as we increase $p$, so *increases in the push to war make protracted conflict and war more likely and decrease the propensity for peace.*

**Variations in the hostility level of war.**

$$\frac{\partial \bar{\mathscr{A}}}{\partial H^{**}} < 0, \qquad \frac{\partial \bar{\mathscr{D}}}{\partial H^{**}} > 0.$$

Increases in $H^{**}$ give rise to decreases in the area and increases in diameter, and so *the propensity for protracted conflict and war decreases and the likelihood of peace increases as the hostility level $H^{**}$ of war increases.*

## SUMMARY AND CONCLUSIONS

The foregoing pages present a somewhat different approach to the study and analysis of system structure and its relationship to war. The concept "international system" was defined by indicating both the set of nations under analysis and a specific relationship between the nations. While it was suggested that two kinds of international systems can be studied, one with respect to the observable attributes of nations (e.g., their resource of power) and one defined in terms of the behaviors of nations, the present paper focuses on the latter – the international hostility system.

A model of an international hostility system was presented in which specific values of the parameters constituted the structure of an international system. Any change in a parameter of the model constituted a change in the structure of the international hostility system. Two types of changes were noted: quantitative and qualitative. Quantitative changes are variations in the values of the parameters within specified restrictions and result in "distortions" but not dramatic changes of the basic characteristics of the hostility system (as represented by the phase portraits). Qualitative changes, in contrast, occur when parameter values pass certain thresholds and the behavioral patterns of the hostility system are drastically altered. It was suggested that qualitative changes can be interpreted as system transformations.

A system's propensity for war was defined in terms of regions within international hostility systems. Each of the hostility systems – where a particular hostility system is defined by a set of parameter values – contains "war regions", regions such that if the hostility system lies within that region it must be the case that that hostility system (i.e., the set of nations whose behavior is being examined) will go to war. A system's propensity for war is thus directly linked to the size of the war region. Since war regions in the phase portraits are typically unbounded, we cannot directly measure these war regions. We can, however, by an inverse logic, indicate how war regions are decreased, by showing how and when non-war regions increase. Thus the principal argument links the structure of the international system, in terms of specific sets of parameter values, to the propensities of war, in terms of the size of non-war regions.

Our analysis allows us to draw conclusions about two specific qualitatively different international hostility systems. In both hostility systems the parameter measuring short term memory is zero. The principal difference between the two systems lies in the parameter $b$. When $b > 0$ the system is affected by past grievances, and when $b < 0$ past grievances are replaced by the effects of friendship. We can summarize the results as follows:

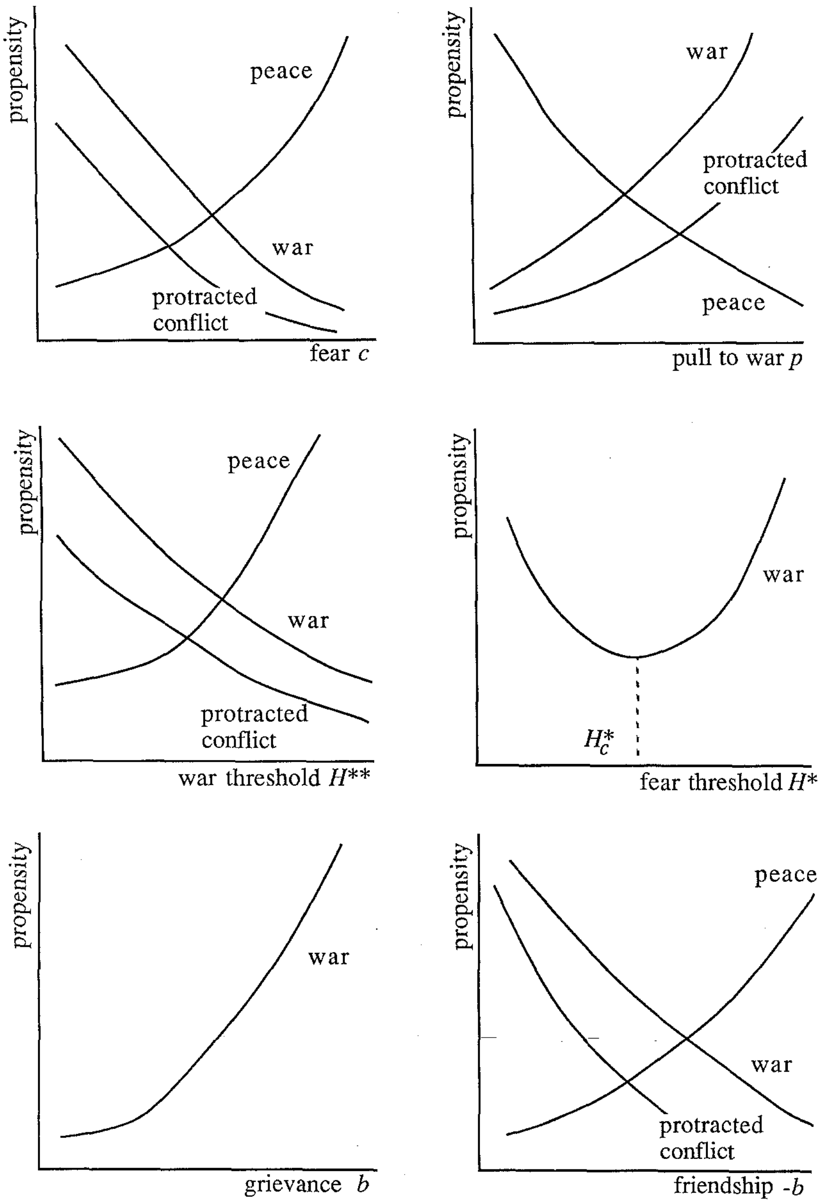
propensity
peace
war
protracted
conflict
fear $c$
propensity
war
protracted
conflict
peace
pull to war $p$
propensity
peace
war
protracted
conflict
war threshold $H^{**}$
propensity
war
$H_c^*$
fear threshold $H^*$
propensity
war
grievance $b$
propensity
peace
war
protracted
conflict
friendship $-b$

Fig. 6.

**Case 1. Systems influenced by past grievances.**

(1) As $b$ increases, the propensity for war increases.

(2) As the degree $c$ of fear increases, the propensity for war decreases.

(3) As the magnitude $p$ of the pull to war increases, the propensity for war increases.

(4) As the war level of hostility $H^{**}$ increases, the propensity for war decreases.

(5) For the fear level $H^*$ there is a critical value at which the propensity for war is a minimum. The likelihood of war decreases as $H^*$ increases up to this critical level and then increases after the critical level is reached.

**Case 2. Systems influenced by past friendships.**

(1) As $b$ decreases (i.e., $b$ becomes more negative or the absolute value of $b$ becomes larger), the propensity for war decreases; the propensity for peace increases; and the propensity for protracted conflict decreases.

(2) As the degree $c$ of fear increases, the propensity for war decreases; the propensity for peace increases; and the propensity for protracted conflict decreases.

(3) As the magnitude $p$ of the pull to war increases, the propensity for war increases; the propensity for peace decreases; and the propensity for protracted conflict increases.

(4) As the war level of hostility $H^{**}$ increases, the propensity for war decreases; the propensity for peace increases; and the propensity for protracted conflict decreases.

These conclusions are shown graphically in Figure 6.

NOTES

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[1] See Muncaster et al. (1983, 1988).

[2] This is in contrast to a similar but more complex model which has been developed by Zinnes et al. (1984).

[3] The fact that (22) vanishes tells us that there is a positive constant $K$ for which we can write $H_{e2} = H_c^* + K$ and $H_{e1} = H_c^* - K$. Since the function $F$ vanishes at these two equilibria, we obtain the two equations $F(H_c^* + K) = 0$ and $F(H_c^* - K) = 0$. By eliminating $K$ from these, we obtain a quadratic equation for $H_c^*$, whose solution is given by (26).

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